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**HAND-OFF ANALYSIS FOR CBWL SCHEMES IN CELLULAR
COMMUNICATIONS**

by

Hua Jiang and Stephen S. Rappaport
Department of Electrical Engineering
State University of New York
Stony Brook, N. Y. 11794-2350

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Hua Jiang and Stephen S. Rappaport
Department of Electrical Engineering
State University of New York
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Abstract

In recent work we suggested a new channel sharing method for cellular communications. The method, called Channel Borrowing Without Locking (CBWL) allows real-time borrowing of channels from adjacent cells without the need for channel locking in co-channel cells. CBWL provides enhanced traffic performance in homogeneous environments and also can be used to relieve spatially localized traffic overloads (tele-traffic "hot spots"). It can be applied in currently deployed as well as in next generation systems without additional costly infrastructure. CBWL permits simple channel management and easy implementation.

This paper is concerned with hand-off issues in CBWL. In CBWL, there are two types of hand-off: one occurs when a communicating mobile station in a cell moves to *another* cell; another occurs *within* a cell when a mobile station that uses a borrowed channel travels to a zone where the borrowed channel cannot be used, this require the mobile station to be transferred to a regular channel. We model both types of hand-off using multi-dimensional birth-death processes. Probabilities of blocking and forced termination are determined. The results, which are also validated by simulation, indicate that CBWL can provide significantly improved traffic and hand-off performance in comparison with non-borrowing cellular schemes.

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1 Introduction

A family of new channel assignment and sharing methods for cellular communication systems has been presented in [1]–[5]. The methods are called *channel borrowing without locking* (CBWL). CBWL can be used to enhance traffic capacity of cellular communication systems and to accommodate spatially localized communication traffic overloads (or “*hot spots*”). As in fixed channel assignment (FCA), each gateway (base station) is assigned a group of channels which are reused at gateways of other cells that are sufficiently distant for the co-channel interference to be tolerable. In CBWL, if all channels of the gateway of a cell are occupied when a new call arrives, channel borrowing is employed according to certain rules.

Channel locking has been suggested in other channel assignment strategies such as dynamic channel assignment (DCA) and hybrid channel assignment (HCA) [6] to limit co-channel interference. That is, the gateways within the required minimum reuse distance from a gateway that borrows a channel cannot use the same channel at the same time. Because of the difficulty in maintaining the reuse distance at the minimum value when channel locking is used, DCA and HCA generally perform less satisfactorily than FCA under high communication traffic loads [7], [8], [9].

In CBWL, a channel can be borrowed only from an adjacent gateway. Borrowed channels are temporarily transferred to the gateway that borrows the channel but are used with reduced transmitted power such that co-channel interference caused by channel borrowing is no worse than that of a non-borrowing scheme. Therefore, *channel locking* is not necessary in CBWL. The borrowed channels can be accessed only in part of the cell. To determine whether a mobile station is in the region that can be served by a borrowed channel, each gateway transmits a signal with the same reduced power as that on a borrowed channel. The signal is called borrowed channel sensing signal (BCSS). If the BCSS is not above some suitable threshold at a mobile station, a borrowed channel cannot be used by the mobile station; otherwise, the mobile station will use a borrowed channel if all its gateway’s channels are occupied and any of its neighboring gateways has a channel available for lending. Thus, there are two types of calls—those that are in parts of a cell in which a borrowed channel *can* be used if one is available, and those that are in parts of a cell where borrowed channels *cannot* be used. We denote these as *A*-type calls and *B*-type calls, respectively.

Neighboring gateways are identified in the following manner. With respect to the given gateway, choose the first adjacent gateway. The position of the reference adjacent gateway can be arbitrary, but once chosen for a given gateway, all other gateways label their neighbors in a corresponding manner. The remaining five adjacent gateways are numbered sequentially proceeding clockwise from the first. The given gateway is labeled gateway 0. The C channels of a gateway are divided into seven distinct groups. The seven groups are numbered 0, 1, . . . , 6. The channels of group 0 are reserved for exclusive use of the given gateway. Channels in the other six groups can be lent to neighbors. The i th neighbor can only borrow channels in the i th group. This channel layout structure is called *channel grouping and directional lending*. The number of channels in the i th group is denoted C_i , $i = 0, 1, \dots, 6$. For convenience we consider a symmetrical arrangement with $C_1 = C_2 = \dots = C_6 = l$. An example of the channel layout structure of CBWL is shown in Figure 1.

CBWL with the structure described above has three advantages: 1) In the scheme, a gateway does not need to transmit and receive on all channels of its neighboring gateways. It only needs to access the channels that are assigned to it and the borrowed channels of six groups, one group from each neighbor. Therefore, the transmitter of a gateway only needs to access a total of $C + 6l$ channels instead of $7C$ channels. The cost and complexity of a

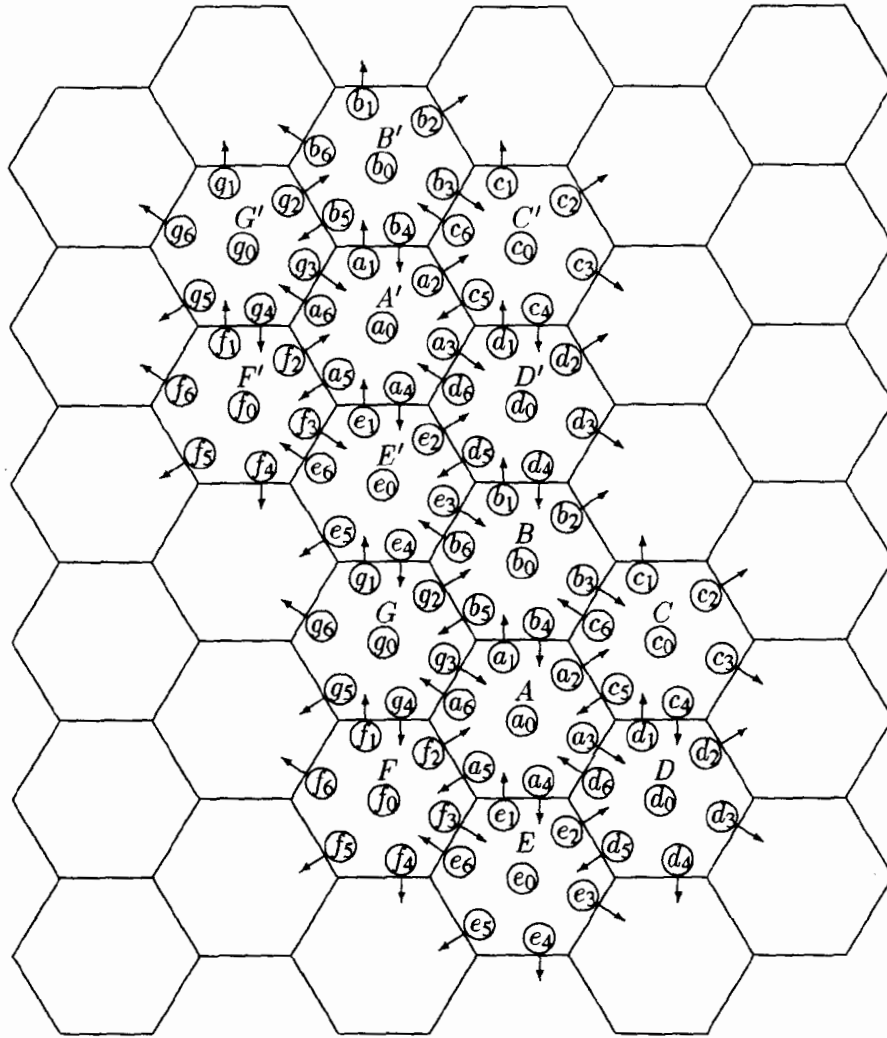


Figure 1: Channel grouping and directional lending. (cluster size = 7).

gateway are reduced. 2) The scheme eliminates the possibility that two co-channel gateways lend the same channel simultaneously to a pair of closely located gateways (which would result in unacceptable co-channel interference). 3) With careful organization, the scheme can ensure that no *adjacent channels* are used in a given cell even though channel borrowing is employed.

As described in [1], *channel rearrangement* can be used in CBWL. With channel rearrangements, if a new *B*-type call arrival finds all channels of its gateway occupied, the call is *still not necessarily blocked*. If at the same time an *A*-type call in the cell is using a regular channel, and at least one neighbor can *lend channels to the given gateway*, the *A*-type call will use a borrowed channel from a neighbor and give its regular channel to the *B*-type call. In this way, calls that cannot use borrowed channels directly also benefit from the borrowing scheme. Therefore, the difference of blocking probabilities between two types of calls is lessened and the number of calls that can use borrowed channels (directly or indirectly) is increased.

In [2] and [3], we suggested using cut-off priority for the calls that arise in its own cell and borrowed channel fast returning to further enhance the performance of CBWL. The two methods can reduce the probability that a gateway *simultaneously* borrows from and lends channels to its neighbors—even to the same neighbor from which

it has borrowed. With cut-off priority, the gateways that have more than $m (< C)$ channels occupied, will not lend any channel to neighbors. Thus at high traffic loading some channels will be available only for calls that are in the given cell. *Fast returning* of borrowed channels is another way to reduce unnecessarily borrowed channels in CBWL. Without fast returning, a borrowed channel is returned only after the call that uses the borrowed channel is completed. With *fast returning*, a call that is using a borrowed channel will be transferred to a regular channel as soon as one is available to service it. The borrowed channel is returned to its owner gateway. Thus, no call is served on a borrowed channel if a regular channel (that can accommodate it) is idle. We call the CBWL/CR that uses fast returning as CBWL/CR-FR.

If a gateway X receives a channel borrowing request from a neighbor Y , the request *is* or *is not* granted depending on the current channel occupancy of X . The following rules are observed.

1. X will deny the request, if the total number of occupied channels of gateway X is more than m . Thus gateway X gives a cut-off priority of $C - m$ channels to the calls arising in its cell.
2. X will deny the request, if the number of channels that are lent from X to Y is equal to l .
3. X will deny the request, if the number of total channels of cell X that are lent to other gateways (including Y) is equal to n .

Further discussion and comparison of the various channel assignment schemes including FCA, DCA, HCA, Generalized FCA, and Directed Retry is presented in [1]. The reader is referred to [7]–[12] for specific details of the schemes.

In [1]–[5], we did not consider the mobility of mobile stations. Analysis without considering of mobility does provide a general evaluation of the relative performance of CBWL. It applies most directly to various scenarios in the broad front of diverse applications that are developing [13]. In particular, systems with large cells and low user mobility, perhaps with many pedestrians with wireless keyboards are most representative. Nevertheless, such analysis cannot provide performance prediction of systems in which hand-off is significant. In consideration of this essence, the present paper augments our previous analyses of CBWL. In CBWL, the hand-off issue is more complex than in FCA. This is because in addition to the regular hand-off between cells, in CBWL, we must consider a new type of hand-off which occurs when a mobile station that uses a *borrowed* channel moves to a zone (in the same cell) where the borrowed channel cannot be used. Thus the call must be transferred to a *regular* channel.

Hand-off issues in different but related contexts have been modeled and analyzed in [14]–[18]. In those papers, multi-dimensional birth-death processes are used to characterize performance. We use this approach for CBWL schemes. However, we make use of the existence of product form solutions in some state variables of the CBWL characterization. An aggregation and decomposition method is used to find approximate but efficient solutions. The same method was used successfully in the analysis of CBWL without considering mobility [1]–[3].

In Section 2, the model of hand-off in CBWL/CR-FR is described. In Section 3, we use aggregation and decomposition method to analyze the model. The performance measures are given in Section 4. Numerical results from analysis and simulation are given in Section 5.

2 Model Description

For simplicity, we limit our analysis to a homogeneous system. That is, each gateway has the same number of channels and the same offered traffic. New calls in a cell arise at an average rate λ (new call arrivals per second per cell) according to a Poisson process, and calls originate uniformly throughout the service area. Call holding times have a negative exponential distribution with mean $1/\mu$. The ratio of the area covered by a borrowed channel to the area of a cell is denoted, p . With mobile units uniformly distributed throughout the spatial region, the fraction of calls that can use borrowed channels is p .

Dwell time in a cell is defined as the time that a mobile station is within communication range of a given gateway. *Dwell time in a zone A* is defined as the time that a mobile station is within the zone that is covered by borrowed channels from the given gateway. *Dwell time in a zone B* is defined as the time that a mobile station is within the remaining zone in the cell. We assume the mean dwell time in a cell is $1/\mu_d$. The parameter, μ_d depends on cell size as well as average speed and mobility characteristics of mobile units. Dwell time can be determined empirically. For given μ_d and p , mean dwell time in zone A and in zone B are of interested. We denote the mean dwell time in a zone A and a zone B as μ_{da} and μ_{db} , respectively, and we assume that $\mu_{da} = p^z/\mu_d$ and $\mu_{db} = (1 - p^z)/\mu_d$. The parameter, z , is determined by the mobility characteristics of mobile stations. We consider two extremes. If a mobile station always moves in a straight line and does not change direction in a cell, z is equal to 0.5. This is because \sqrt{p} is the ratio of the *radius* of zone A to the *radius* of a cell, and so the mean dwell time in zone A is directly proportional to this ratio. If a mobile station moves randomly, the mean dwell time in zone A is directly proportional to the fraction of zone A's area. Thus, $z = 1$. The most practical cases are between the two extremes. When $z = 1$, the mean dwell time in zone A is the least. Thus, with $z = 1$, the likelihood of the second type hand-off is the highest for given μ_d and p . So the case with $z = 1$ gives pessimistic results. We also assume the probability density functions (pdf) of dwell time in zone A and in zone B are negative exponential.

We assume that in a cell, zone A is proximate to its gateway and is enclosed by zone B. Thus, if a mobile station leaves a zone A it will definitely enter into the zone B of the cell. If a mobile station is in zone B, it moves into zone A with probability of q and enters into a zone B of another cell with probability of $1 - q$. The quantity of q can be determined from the given μ_{da} and μ_{db} (Appendix A).

We note that borrowing requests to a given gateway from an adjacent gateway arise from an overflow process (at the adjacent gateway) and therefore do not conform to a Poisson process [19]. However at the adjacent gateway, borrowing requests are randomly split into six parts, only one of which is directed to the given gateway. The random splitting tends to smooth the peakedness of the overflow traffic directed to the given gateway. Thus the borrowing requests directed to a given gateway from an adjacent gateway are approximated by a Poisson process. For now, it is assumed that the parameter, λ' , (intensity) of this process is known. Subsequently, we will show how λ' is determined.

2.1 PDF of Dwell Time in a Cell

In Appendix A, we show that the dwell time in a cell is a random sum of the dwell times in A and B. Thus, the pdf of dwell time in a cell is quite complex. In Appendix A, we find the variance of the dwell time in a cell, V_d . Since the mean dwell time in a cell is $1/\mu_d$, we find the ratio of the standard deviation to the mean dwell time

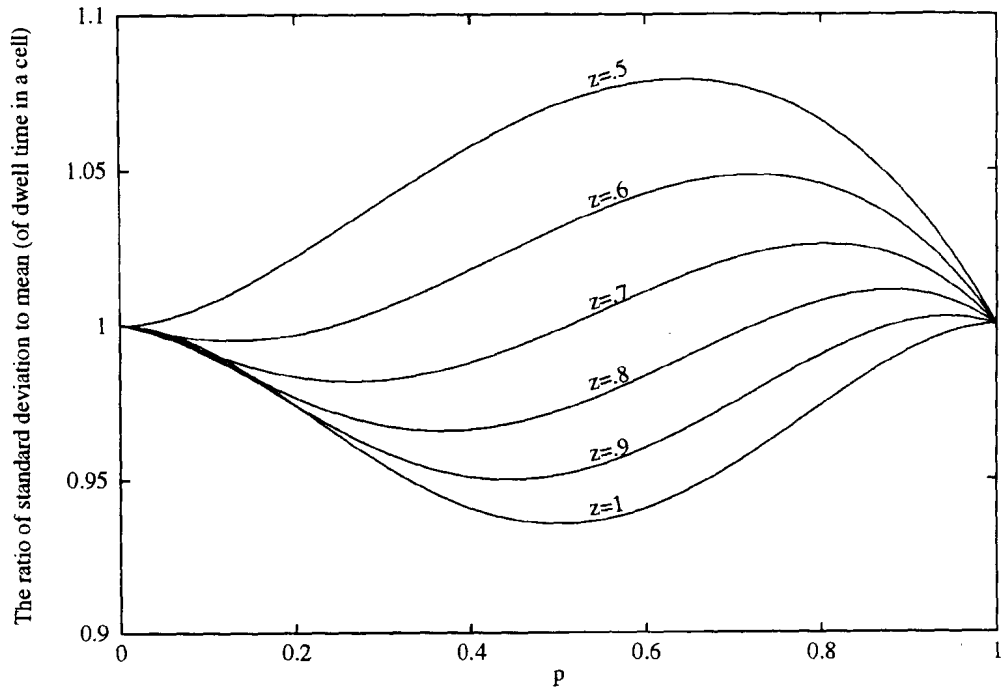


Figure 2: The ratio of standard deviation of dwell time to mean (for dwell time in a cell).

to be

$$\kappa = \frac{\sqrt{V_d}}{1/\mu_d}. \quad (1)$$

A plot is shown in Figure 2 for different values of p and z . It is seen from the figure that for any values of p and z , κ is close to 1. Thus it is reasonable to assume that the pdf of dwell time in a cell has a negative exponential density function with mean of μ_d^{-1} . Our simulation validated this assumption.

2.2 Channel Rearrangement for the Two Types of Hand-off Calls

A *communicating mobile* is one that has a call in progress. If a communicating mobile moves into another cell, a hand-off is needed. In the FCA scheme, if all channels of the target cell are occupied, the hand-off fails and the call is forced to terminate. In CBWL, if all channels of the target cell are occupied, it is possible to use channel borrowing to accommodate the hand-off call. Although the hand-off call enters zone B of the target cell and hence cannot use a borrowed channel directly, channel rearrangement can be used. With channel rearrangement, a call in zone A of the target cell that is using a regular channel of the target cell is reassigned to the borrowed channel and releases its original regular channel to accommodate the hand-off call.

The second type of hand-off occurs when a communicating mobile that is using a borrowed channel moves from zone A into zone B (of the same cell). Because the borrowed channel cannot be used in zone B , the communicating mobile needs a regular channel for successful hand-off. However, if all channels of the given gateway are occupied, there is no free regular channel for the hand-off call. In this case, channel rearrangement can also be used. With channel rearrangement, the gateway finds a user in zone A who is using a regular channel. The hand-off call and the A -zone call exchange channels. In this way, the hand-off call is assigned a regular channel and both calls continue. In this kind of channel rearrangement, no additional channel is borrowed. Thus, success of the hand-off does not depend on the availability of channels in adjacent cells.

If the number of *A*-zone calls that are using regular channels in a cell is zero, the channel rearrangements needed for the above situations are not possible. Because forced termination of calls is very annoying to users, it is useful to have some *users on regular channels in zone A* so that channel rearrangement can be used to accommodate hand-offs. To make this more likely, cut-off priority thresholds can be set so that in a given cell, no *new* call that requires a channel rearrangement is served if the number of second users in a cell is too few. Specifically, in each cell, we set up three thresholds, n_{h1} , n_{h2} , n_{h3} ($n_{h1} \geq 0$, $n_{h1} \geq n_{h2} \geq 0$, $n_{h1} \geq n_{h3} \geq 0$) to which the number, X , of *second users* is compared. Then

if $X \leq n_{h1}$, no *new* calls that require a channel rearrangement in that cell will be served.

If $n_{h3} = 0$ and $X \leq n_{h2}$, no hand-off call from zone *A* to zone *B* that requires a channel rearrangement will be accommodated.

If $n_{h2} = 0$ and $X \leq n_{h3}$, no hand-off call from an adjacent cell that requires a channel rearrangement will be accommodated.

Thus each gateway tends to keep its rearrangement chances for hand-off calls and can give highest priority to either type of hand-off call (depending on system parameters and on parameter choices).

If $n_{h2} = n_{h3} = 0$, there is equal priority for either type of hand-off call, but hand-offs are still favored over new calls if $n_{h1} > 0$. If $n_{h1} = n_{h2} = n_{h3} = 0$, there is no priority for hand-off calls.

With the new hand-off, CBWL require that a cell site has the processing ability to switch a call between a borrowed channel and a regular channel. However, the switch occurs within a cell site and can be processed distributively. Thus, the new hand-off will not increase significantly system processing loads.

3 Traffic and Hand-off Analysis of CBWL/CR-FR

3.1 State Vector

At any given time a gateway is in one of a finite number of states. A state is identified by a vector $\mathbf{I} = (i_a, i_b, i_1, i_2, i_3, i_4, i_5, i_6)$. The component i_a is the number of channels that are used by *A*-type calls in the cell and i_b is the number of channels that are used by *B*-type calls in the cell. The number of channels at the gateway that are lent to the k th neighbor is i_k , ($k = 1, 2, \dots, 6$), where $0 \leq i_k \leq C_k$.

The total number of channels that are (currently) lent to all adjacent gateways is

$$L(\mathbf{I}) = \sum_{k=1}^6 i_k . \quad (2)$$

From the discussion in the previous section, it follows that the maximum number of channels that a cell can lend at any given time is

$$L_{max} = \min(6l, m, n) . \quad (3)$$

3.2 Analysis Method

With eight dimensions, the number of states can be very large. To find the distribution of states, we use an aggregation and decomposition method. The method was successfully used to analyze traffic performance of CBWL in [1]–[3]. The method can be generally applied to analysis of multi-dimensional queuing problems

that have product form solution for some state variables. The basic idea of this method is to divide the multi-dimensional problem into several one-dimensional or two-dimensional problems and find their solution separately. The method has four steps:

Step 1: We aggregate state variable into 2 macro-variables: the first macro-variable consists of all variable components that have product-form solution; the second macro-variable consists of other variables.

Step 2: We then use convolution algorithm to find the transition rates of the first macro-variable.

Step 3: Find the distribution of macro-variables and marginal distribution of the first macro-variable.

Step 4: Divide the second macro-variable into original variables but retain the first macro-variable. Decompose the state space into several sub-spaces. Each of those subspaces is found by the original variables that replaced the second macro-variable, and a single fixed value of the the first macro-variable. All possible values of the first macro-variable must be considered. The distribution of each subspace is determined separately.

The method is developed from the aggregation method in [20]. If the interaction between the first macro-variable and the second macro-variable is much weaker than the interaction among the variables within each macro-variable, the method provides a good approximation.

3.3 Macro-variables

As in [1]–[3], we find that the distribution of numbers of channels that are lent to each neighbor is in product form. Thus we combine the six-dimensional variables that represent the number of channels lent to each of six neighbors into a macro-variable that represents the total number of lending channels. Denote the macro-variable as v . We combine i_a and i_b into another macro-variable and denote it as u which represents the number of channels that are occupied by the calls in the given cell. All permissible states of (u, v) are constrained by the following conditions:

$$\begin{aligned} 0 &\leq u \leq C \\ 0 &\leq v \leq L_{max} \\ 0 &\leq u + v \leq C . \end{aligned} \tag{4}$$

Let Ω be the set of permissible states (u, v) . Define a function, $Z(u, v)$, such that

$$Z(u, v) \triangleq \begin{cases} 1 & \text{if } (u, v) \in \Omega \\ 0 & \text{if } (u, v) \notin \Omega \end{cases} \tag{5}$$

Denote $p_2(u, v)$ as the equilibrium probability of state I. In statistical equilibrium, the probability flow out of each state (u, v) must equal the probability flow into that state. Application of this principle leads to a set equations which must be solved to find the state probabilities.

In those permissible states for which $0 \leq u + v < m$, the transitions out of (u, v) consist of six parts: that due to new call arrivals; that due to requests of first type hand-off calls; that due to channel lending to neighbors; that due to the completion of the calls in the given cell; that due to the departure of communicating mobile from the

cell; that due to the returning of channels that were lent to neighbors. The transition out of state (u, v) due to new call arrivals is given by

$$\{\text{transition out due to new call arrivals}\} = \lambda .$$

The transition out of state (u, v) due to the first type hand-off calls is

$$\{\text{transition out due to hand-off calls}\} = \lambda_h .$$

For now we assume that λ_h is known, but we will subsequently show how to calculate it from the underlying system parameters.

If the state of a gateway is such that channels can be lent ($0 \leq L(\mathbf{I}) \leq L_{max}$), the transition out of state (u, v) due to channel lending to neighbors is the sum of the channel borrowing rate from six neighbors given that v channels are lent. Note that if $v \geq l$, it is possible that a single specific neighbor borrows l channels from the given gateway—but that neighbor cannot borrow any additional channels from the given gateway. For a given v , there can be many different combinations of i_k 's ($k = 1, \dots, 6$) such that $\sum_{k=1}^6 i_k = v$. Each combination can have different channel lending rate (if $i_k < l$, the borrowing rate from the k th gateway is λ' , if $i_k = l$, the borrowing rate from the k th gateway is 0). Therefore, an *average* rate of borrowing requests from neighbors given that v channels are lent, $\rho(v)$, is used as the rate of transition out due to channel borrowing demands. The rate, which is determined in Appendix B is

$$\{\text{transition out due to channel borrowing demands}\} = \rho(v) .$$

The transition out due to completion of calls in the given cell is

$$\{\text{transition out due to completion of calls}\} = u\mu .$$

The transition out of state (u, v) due to the returning of channels that had been lent to neighbors is average channel return rate from all six neighbors given that v channels are lent. The average is on all states that compose the macro-state. We denote the rate as $\beta(v)$. The rate, which is determined in Appendix B is

$$\{\text{transition out due to returning of channels}\} = \beta(v) .$$

The transition out of state (u, v) due to the departure of communicating mobiles from the given cell is

$$\{\text{transition out due to departure of communicating mobiles}\} = u\mu_d .$$

Note that type 2 hand-offs do not cause macro-state transitions since $i_0 = i_a + i_b$.

Similarly we can find the probability transition components *into* state (u, v) with $0 \leq u + v < m$. The probability transitions *into* (u, v) consist of six components: that due to new call arrivals from a permissible state $(u - 1, v)$; that due to requests of first type hand-off calls from $(u - 1, v)$; that due to channel lending to neighbors from $(u, v - 1)$; that due to channel returned from neighbor from $(u, v + 1)$; that due to completions of calls from $(u + 1, v)$, and that due to the departure of communicating mobile from $(u + 1, v)$. The first component is

$$\{\text{transition in due to new call arrivals}\} = \lambda .$$

The transition into state (u, v) due to the first type hand-off calls is

$$\{\text{transition in due to hand-off calls}\} = \lambda_h .$$

The transition into (u, v) due to channel lending is given by

$$\{\text{transition in due to channel lending}\} = \rho(v - 1) .$$

The transition into (u, v) due to channel returns from adjacent gateways is given by

$$\{\text{transition in due to channel returning}\} = \beta(v + 1) .$$

The transition into (u, v) due to completions of calls that are served by the given gateway is

$$\{\text{transition in due to completions of calls}\} = (u + 1)\mu .$$

The transition into state (u, v) due to the departure of communicating mobiles from the given cell is

$$\{\text{transition in due to departure of communicating mobiles}\} = (u + 1)\mu_d .$$

In any permissible state (u, v) with $0 \leq u + v < m$, the flow balance equation is

$$\begin{aligned} [\lambda + \lambda_h + \rho(v) + u(\mu + \mu_d) + \beta(v)] p_2(u, v) &= (\lambda + \lambda_h)p_2(u - 1, v)Z(u - 1, v) \\ &+ \rho(v - 1)p_2(u, v - 1)Z(u, v - 1) + (u + 1)(\mu + \mu_d)p_2(u + 1, v) + \beta(v + 1)p_2(u, v + 1)Z(u, v + 1) \\ &(0 \leq u + v < m) \end{aligned} \quad (6a)$$

In any permissible state (u, v) with $u + v = m$, no channel is lent, the flow balance equation is

$$\begin{aligned} [\lambda + \lambda_h + u(\mu + \mu_d) + \beta(v)] p_2(u, v) &= (\lambda + \lambda_h)p_2(u - 1, v)Z(u - 1, v) \\ &+ \rho(v - 1)p_2(u, v - 1)Z(u, v - 1) + (u + 1)(\mu + \mu_d)p_2(u + 1, v) + \beta(v + 1)p_2(u, v + 1)Z(u, v + 1) \\ &(u + v = m) \end{aligned} \quad (6b)$$

In any permissible state (u, v) with $m < u + v < C - 1$, the transition in due to channel borrowing demands is zero. The flow balance equations are

$$\begin{aligned} [\lambda + \lambda_h + u(\mu + \mu_d) + \beta(v)] p_2(u, v) &= (\lambda + \lambda_h)p_2(u - 1, v)Z(u - 1, v) \\ &+ (u + 1)(\mu + \mu_d)p_2(u + 1, v) + \beta(v + 1)p_2(u, v + 1)Z(u, v + 1) \\ &(m < u + v < C) \end{aligned} \quad (6c)$$

In any permissible state (u, v) with $u + v = C$, all channels of the given gateway are occupied. Thus some borrowed channels may be used in these states. We assume that the gateway may use some borrowed channels with probability of $1 - p_{nb}$, or it may not borrow any channels with probability of p_{nb} . The quantity p_{nb} is determined in Appendix B. State transitions are different for the two cases. If no channel is borrowed and a channel is released (due to a call completion or the departure of a communicating mobile), the state is changed to $(u - 1, v)$. Its transition rate is $p_{nb}u(\mu + \mu_d)$. If no channel is borrowed and a lending channel is returned, the state is changed to $(u, v - 1)$. Its transition rate is $p_{nb}\beta(v)$. If some channels are borrowed and a channel is released, the released channel is at once given to a call that uses a borrowed channel, and the borrowed channel is returned to its owner, thus state is not changed. If some channels are borrowed and a lending channel is returned, with fast channel returning, the returned channel is used to replace a borrowed channel, thus state is changed to

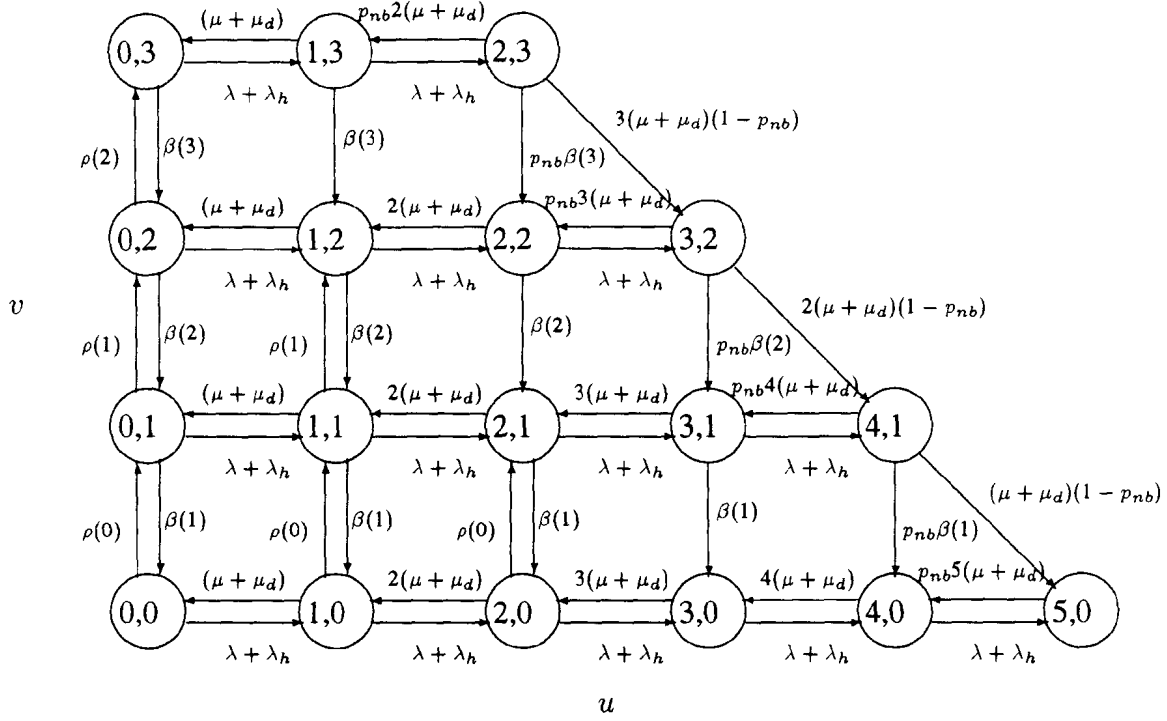


Figure 3: State-transition diagram of two-dimensional macro-state (u, v) for a CBWL/CR-FR with $C = 5, m = 4$.

$(u + 1, v - 1)$. The transition rate is $(1 - p_{nb})\beta(v)$. Thus, in any permissible state (u, v) with $u + v = C - 1$, the flow balance equation is

$$\begin{aligned}
 [\lambda + \lambda_h + u(\mu + \mu_d) + \beta(v)] p_2(u, v) &= (\lambda + \lambda_h) p_2(u - 1, v) Z(u - 1, v) \\
 &+ p_{nb}(u + 1)(\mu + \mu_d) p_2(u + 1, v) + p_{nb}\beta(v + 1) p_2(u, v + 1) Z(u, v + 1) \\
 &(u + v = C - 1)
 \end{aligned} \tag{6d}$$

In any permissible states (u, v) with $u + v = C$, the flow balance equations are

$$\begin{aligned}
 [p_{nb}u(\mu + \mu_d) + \beta(v)] p_2(u, v) &= (\lambda + \lambda_h) p_2(u - 1, v) Z(u - 1, v) \\
 &+ (1 - p_{nb})\beta(v + 1) p_2(u - 1, v + 1) Z(u, v + 1) \\
 &(u + v = C)
 \end{aligned} \tag{6e}$$

Figure 3 shows the state-transition diagram of an example of CBWL/CR-FR with $C = 5, m = 4$.

We can use Gauss-Seidel iteration [21] or other numerical methods to find the solution of the balance equations. Since the number of states has been reduced greatly in the two-dimensional model, the required computation effort is considerably reduced.

3.4 Decomposition to find $p(i_a, i_b)$

To find the equilibrium distribution of (i_a, i_b) , $p(i_a, i_b)$, we use a *decomposition* method. The method is to divide 7-variable state space into $L_{max} + 1$ subspaces, each of which corresponds to a fixed value of $v = \sum_{k=1}^6 i_k$ ($v = 0, 1, \dots, L_{max}$). The conditional distribution, $\Pr(i_a, i_b | v)$ can be calculated separately for each fixed v .

However, because i_a and i_b cannot be completely separated from the other variables, the decomposition method is an approximation. If $\lambda \gg \lambda'$, the interactions between i_a and i_b are much stronger than the interactions between i_0 and other i_k 's ($k \geq 1$), we can calculate $p_v(i_a, i_b)$ separately for each fixed v and *neglect* the interactions between i_a, i_b and other i_k 's ($k \geq 1$) as if those interactions do not exist [20]. The agreement between results of simulation and analysis validates this approximation.

Denote $p_v(i_a, i_b)$ as the equilibrium distribution of (i_a, i_b) given that v channels are lent. If v channels have been lent, all permissible states of i_a and i_b are constrained by following conditions:

$$\begin{aligned} 0 &\leq i_a \leq C - v \\ 0 &\leq i_b \leq C - v \\ 0 &\leq i_a + i_b \leq C - v. \end{aligned} \quad (7)$$

Denote λ_1 as the transition rate of A -type calls due to new call arrival. That is,

$$\lambda_1 = p\lambda, \quad (8)$$

and denote λ_2 as the transition rate of B -type calls due to new call arrivals and hand-off request. That is,

$$\lambda_2 = (1 - p)\lambda + \lambda_h. \quad (9)$$

According to our assumption, only B -type calls can leave a cell for hand-off. The transition rate from (i_a, i_b) to $(i_a, i_b - 1)$ includes the rate of a call completion and the departure rate of communicating mobile. That is

$$i_b\mu_2 = i_b[\mu + (1 - q)\mu_d/(1 - p^z)]. \quad (10)$$

The transition rate from (i_a, i_b) to $(i_a - 1, i_b)$ is the completion rate of i_a calls. That is $i_a\mu$. The transition rate from (i_a, i_b) to $(i_a - 1, i_b + 1)$ is the rate of communicating mobiles travel from zone A into zone B . That is

$$i_a\xi_o = i_a\mu_d/p^z. \quad (11)$$

The transition rate from (i_a, i_b) to $(i_a + 1, i_b - 1)$ is the rate of communicating mobiles travel from zone B into zone A . That is

$$i_b\xi_i = i_b(1 - q)\mu_d/(1 - p^z). \quad (12)$$

If a gateway is in a state (i_a, i_b) with $i_a + i_b = C - v$ and $i_a > n_{h2}$, a channel will be borrowed through channel rearrangement for a new B -type call or a first type hand-off call. If the channel borrowing request is accepted by an adjacent gateway, an A -type call is transferred to the borrowed channel and the released regular channel is for the B -type call. Thus, the gateway's state is changed to $(i_a - 1, i_b + 1)$. The second type hand-off calls (a communicating mobile that uses a borrowed channel travels from zone A to B zone) also use channel rearrangement. However, no more channel is borrowed. Denote the transition rate due to channel rearrangement in these states as λ_3 . Thus,

$$\lambda_3 = [\lambda(1 - p) + \lambda_h] \sum_{j=0}^{\delta l} \frac{p_1(C + j)}{p_c} p_{bs}(j) + \frac{\mu_d}{p^z} \sum_{j=0}^{\delta l} j p_1(C + j). \quad (13)$$

If a gateway is in a state (i_a, i_b) with $i_a + i_b = C - v$ and $n_{h2} \geq i_a > n_{h1}$, no new B -type call can use channel rearrangement. only the first and second types of hand-off calls can use channel rearrangement. Denote the transition rate due to channel rearrangement in these states as λ_4 . Thus,

$$\lambda_4 = \lambda_h \sum_{j=0}^{6l} \frac{p_1(C+j)}{p_c} p_{bs}(j) + \frac{\mu_d}{p^z} \sum_{j=0}^{6l} j p_1(C+j). \quad (14)$$

If a gateway is in a state (i_a, i_b) with $i_a + i_b = C - v$ and $n_{h1} \geq i_a > 0$, only the first type of hand-off calls can use channel rearrangement. Denote the transition rate due to channel rearrangement in these states as λ_5 . Thus,

$$\lambda_5 = \lambda_h \sum_{j=0}^{6l} \frac{p_1(C+j)}{p_c} p_{bs}(j). \quad (15)$$

Now we consider the channel releasing rate in a state with $i_a + i_b = C - v$. In those states, the gateway may use some borrowed channels with probability of $1 - p_{nb}$, or it may not borrow any channels with probability of p_{nb} (B.10). State transitions are different for the two cases. If no channel is borrowed and a channel is released by an A -type call, the state is changed to $(i_a - 1, i_b)$ with the rate of $p_{nb} i_a \mu$. If no channel is borrowed and a channel is released by a B -type calls, the state is changed to $(i_a, i_b - 1)$ with the rate of $p_{nb} i_b \mu_2$. If the gateway borrows at least one channel and a channel is released, the released channel replace a borrowed channel and the borrowed channel is returned to its owner. If the channel is released by an A -type call, the state is not changed. If the channel is released by a B -type call, the state is changed to $(i_a + 1, i_b - 1)$ with the rate of $(1 - p_{nb}) i_b \mu_2$.

The state-transition diagram of i_a and i_b for a example with $C - v = 3$, $n_{h1} > n_{h2} > 0$ and n_{h3} is shown in Figure 4.

The flow balance equations of $p_v(i_a, i_b)$ are as follows:

$$\begin{aligned} & [\lambda + \lambda_h + i_a(\mu + \xi_o) + i_b(\mu_2 + \xi_i)] p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\ & \quad + (i_a + 1) \mu p_v(i_a + 1, i_b) + (i_b + 1) \mu_2 p_v(i_a, i_b + 1) \\ & \quad + (i_a + 1) \xi_o p_v(i_a + 1, i_b - 1) + (i_b + 1) \xi_i p_v(i_a - 1, i_b + 1) \\ & \quad (0 \leq i_a + i_b < C - v - 1) \\ & [\lambda + \lambda_h + i_a(\mu + \xi_o) + i_b(\mu_2 + \xi_i)] p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\ & \quad + (i_a + 1) p_{nb} \mu p_v(i_a + 1, i_b) + (i_b + 1) p_{nb} \mu_2 p_v(i_a, i_b + 1) \\ & \quad + (i_a + 1) \xi_o p_v(i_a + 1, i_b - 1) + (i_b + 1) \xi_i p_v(i_a - 1, i_b + 1) \\ & \quad (i_a + i_b = C - v - 1) \quad (16) \\ & [\lambda_3 + i_a(p_{nb} \mu + \xi_o) + i_b(\mu_2 + \xi_i)] p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\ & \quad + [\lambda_3 + (i_a + 1) \xi_o] p_v(i_a + 1, i_b - 1) + (i_b + 1) [(1 - p_{nb}) \mu_2 + \xi_i] p_v(i_a - 1, i_b + 1) \\ & \quad (i_a + i_b = C - v, i_a > n_{h2}) \\ & [\lambda_4 + i_a(p_{nb} \mu + \xi_o) + i_b(\mu_2 + \xi_i)] p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\ & \quad + [\lambda_3 + (i_a + 1) \xi_o] p_v(i_a + 1, i_b - 1) + (i_b + 1) [(1 - p_{nb}) \mu_2 + \xi_i] p_v(i_a - 1, i_b + 1) \\ & \quad (i_a + i_b = C - v, i_a = n_{h2}) \\ & [\lambda_4 + i_a(p_{nb} \mu + \xi_o) + i_b(\mu_2 + \xi_i)] p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\ & \quad + [\lambda_4 + (i_a + 1) \xi_o] p_v(i_a + 1, i_b - 1) + (i_b + 1) [(1 - p_{nb}) \mu_2 + \xi_i] p_v(i_a - 1, i_b + 1) \end{aligned}$$

$$\begin{aligned}
& (i_a + i_b = C - v, n_{h2} > i_a > n_{h1}) \\
& [\lambda_5 + i_a(p_{nb}\mu + \xi_o) + i_b(\mu_2 + \xi_i)]p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\
& + [\lambda_4 + (i_a + 1)\xi_o]p_v(i_a + 1, i_b - 1) + (i_b + 1)[(1 - p_{nb})\mu_2 + \xi_i]p_v(i_a - 1, i_b + 1) \\
& (i_a + i_b = C - v, i_a = n_{h1}) \\
& [\lambda_5 + i_a(p_{nb}\mu + \xi_o) + i_b(\mu_2 + \xi_i)]p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\
& + [\lambda_5 + (i_a + 1)\xi_o]p_v(i_a + 1, i_b - 1) + (i_b + 1)[(1 - p_{nb})\mu_2 + \xi_i]p_v(i_a - 1, i_b + 1) \\
& (i_a + i_b = C - v, n_{h1} > i_a > 0) \\
& i_b(\mu_2 + \xi_i)p_v(i_a, i_b) = \lambda_2 p_v(i_a, i_b - 1) + [\lambda_5 + (i_a + 1)\xi_o]p_v(i_a + 1, i_b - 1) \\
& (i_a = 0, i_b = C - v)
\end{aligned}$$

where $p_v(x, y) = 0$, if $x < 0$ or $y < 0$. The balance equations are solved by Gauss-Seidel Iteration.

Now we can calculate some important probabilities from $p_v(i_a, i_b)$. First we determine the probability that a *new B-type* call cannot use channel rearrangement given that v channels are lent. Denote $p_a(v)$ as the conditional probability. The event is the set of states, (i_a, i_b) , with $i_a + i_b = C - v$ and $i_a \leq n_{h2}$. Thus,

$$p_a(v) = \sum_{i_a=0}^{n_{h2}} p_v(i_a, C - v - i_a) \quad v = 0, 1, \dots, L_{max}. \quad (17)$$

Denote $p_l(v)$ as the probability that a gateway lends v channels to neighbors. The latter probability can be calculated from $p_2(u, v)$ by

$$p_l(v) = \sum_{u=0}^{C-v} p_2(u, v) \quad v = 0, 1, \dots, L_{max}. \quad (18)$$

Denote p_a as the unconditional probability that a *new B-type* call cannot use channel rearrangement. Thus,

$$p_a = \sum_{v=0}^{L_{max}} p_a(v)p_l(v) = \sum_{v=0}^{L_{max}} p_l(v) \sum_{i_a=0}^{n_{h2}} p_v(i_a, C - v - i_a). \quad (19)$$

Similarly, we can determine the probability that a second type of hand-off call cannot use channel rearrangement, p_w , and the probability that a first type of hand-off call cannot use channel rearrangement p_m . That is

$$p_w = \sum_{v=0}^{L_{max}} p_l(v) \sum_{i_a=0}^{n_{h1}} p_v(i_a, C - v - i_a). \quad (20)$$

and

$$p_m = \sum_{v=0}^{L_{max}} p_l(v)p_v(0, C - v). \quad (21)$$

In (19), (20) and (21), $L_{max} + 1$ groups of equations of (16) with v from 0 to L_{max} must be solved. However, the number of equation groups to be solved can be reduced greatly. Since fast returning reduces the usage of borrowed channels, the probability that a gateway lends a lot of channels is quite low. In our algorithm, when $p_l(v)$ is less than a desired precision, it is not necessary to solve the equations that corresponds to that v .

4 Some Important Probabilities and Performance Measures

From state probability $p_2(u, v)$, some important probabilities and blocking probabilities of each type of calls can be computed.

Probability that all channels of a gateway are occupied p_c

The probability that all channels of a gateway are occupied is the sum of state probabilities with $u + v = C$. Thus,

$$p_c = \sum_{u=0}^C p_2(u, C - u). \quad (22)$$

Probability that a borrowing request of the given gateway is denied by a specific adjacent gateway: p_f

The probability was determined in [3].

$$p_f = p_c + \sum_{u=0}^{m-1-L_{max}} p_2(u, L_{max}) + \sum_{v=0}^{L_{max}} \sum_{u=m-v}^{C-1-v} p_2(u, v) + \sum_{v=l}^{L_{max}-1} \sum_{s=1}^6 \frac{b(6-s, v) s}{b(v)} \frac{1}{6} \sum_{u=0}^{m-v-1} p_2(u, v). \quad (23)$$

The average rate borrowing requests to a neighbor, λ'

First we consider the average channel borrowing rate of the given gateway from a specific neighbor. Denote the rate as λ'' . The channel borrowing rate of the given gateway from the neighbor given j channels are borrowed is $\zeta(C + j)$ (B.5). Thus, the average channel borrowing rate is

$$\lambda'' = \frac{1}{6} \sum_{j=0}^{6l} p_1(C + j) \zeta(C + j). \quad (24)$$

Denote λ' as the average borrowing *request* rate of the given gateway to the specific neighbor. The probability that those requests are accepted by the neighbor is $1 - p_f$. That is

$$\lambda'(1 - p_f) = \lambda''. \quad (25)$$

From (25), we find

$$\lambda' = \frac{1}{6(1 - p_f)} \sum_{j=0}^{6l} p_1(C + j) \zeta(C + j). \quad (26)$$

The average arrival rate of first type hand-off calls λ_h

For a homogeneous system in statistical equilibrium, the hand-off arrival rate in a cell must be equal to the hand-off departure rate in the cell. That is

$$\lambda_h = \sum_{u=1}^C u \mu_d \sum_{v=0}^{\min(C-u, L_{max})} p_2(u, v). \quad (27)$$

Blocking probability

The blocking probability of *A*-type calls is denoted as α_{CR-FR} . *A*-type calls are blocked if all channels of a gateway are occupied and their borrowing requests are rejected by all neighbors. If the given gateway has borrowed j channels, the probability that the borrowing requests are denied by neighbors is $1 - p_{bs}(j)$ (see [2]). Thus,

$$\alpha_{CR-FR} = \sum_{j=0}^{6l} p_1(C + j) [1 - p_{bs}(j)]. \quad (28)$$

If a B -type call can use channel rearrangement, it has the same blocking probability as an A -type call. If a new B -type call find all channels occupied and it cannot use channel rearrangement, it will be blocked. Thus

$$\beta_{CR-FR} = \left(1 - \frac{p_a}{p_c}\right)\alpha_{CR-FR} + p_a. \quad (29)$$

The overall blocking probability in a gateway is

$$B_{CR-FR} = p\alpha_{CR-FR} + (1 - p)\beta_{CR-FR}. \quad (30)$$

Hand-off failure probability

The first type of hand-off call is really a B -type call but with high priority for channel rearrangement. The probability that it cannot use channel rearrangement is p_m . Thus,

$$p_h = \left(1 - \frac{p_m}{p_c}\right)\alpha_{CR-FR} + p_m. \quad (31)$$

The second type of hand-off calls do not need borrowed channels. Thus the failure probability of the second type of hand-off calls is exactly the probability that they cannot use channel rearrangement, p_w .

Forced termination probability

The forced termination probability is defined as the probability that a call type hand-off failure as P_{FT1} and P_{FT2} , respectively. Because a call may involve two types of hand-off, it is very complicated to determine the forced termination probability in CBWL. We use a signal flow diagram in Appendix C to describe the process of a call and use Mason's formula [24] to find the probabilities.

The forced termination probability due to first type hand-off, P_{FT1} , is the probability gain from node a to node f . The forced termination probability due to second type hand-off, P_{FT2} , is the probability gain from node a to node g in Figure 14. Using Mason's formula, we can find

$$P_{FT1} = \frac{\{1 - p + p[r_o + r_{h2}(1 - p_w)]\}r_{h1}p_h}{1 - r_i[r_o + r_{h2}(1 - p_w)] - r_{h1}(1 - p_h)}, \quad (32)$$

and

$$P_{FT2} = \frac{p_w r_{h2} \{p[1 - r_{h1}(1 - p_h)] + (1 - p)r_i\}}{1 - r_i[r_o + r_{h2}(1 - p_w)] - r_{h1}(1 - p_h)}. \quad (33)$$

Hand-off activity factor

We define the hand-off activity factors, η_1, η_2 as the expected number of the first type and second type hand-off attempts for a non-blocked call, respectively.

Let φ_1 be the probability that a call which require a first type hand-off will not require an additional the same type hand-off. The probability, φ_1 , can be found from the gains from node h to nodes d, e, f and g after open of the path r_{h1} . Using Mason's formula, we find

$$\varphi_1 = p_h + (1 - p_h) \frac{r_i(r_{h2}p_w + r_{ac}) + r_{bc}}{1 - r_i[r_o + r_{h2}(1 - p_w)]}. \quad (34)$$

Let $B_1(0)$ be the probability that a call makes its first first-type hand-off. This can be found from the gain from node a to node h in Figure 14 after opening the path $1 - p_h$. That is

$$B_1(0) = \frac{p[r_o + r_{h2}(1 - p_w)] + 1 - p}{1 - r_i[r_o + r_{h2}(1 - p_w)]} r_{h1}. \quad (35)$$

Let B_1 be the probability that a call makes its another first-type hand-off after a successful hand-off. This can be found from the gain from node b to node h after opening the path $1 - p_h$. That is

$$B_1 = \frac{1}{1 - r_i[r_o + r_{h2}(1 - p_w)]} r_{h1}. \quad (36)$$

Let $U_1(k)$ be the probability that a call requires exactly k hand-off attempts before ending either by completion or by forced termination. Then we find

$$\begin{aligned} U_1(1) &= B_1(0)\varphi_1 \\ U_1(2) &= B_1(0)(1 - p_h)B_1\varphi_1 \\ &\dots \\ U_1(k) &= B_1(0) [(1 - p_h)B_1]^{k-1} \varphi_1 \end{aligned} \quad (37)$$

The hand-off activity factor is

$$\eta_1 = \sum_{k=1}^{\infty} k \cdot U_1(k). \quad (38)$$

This can be compactly written in closed form as

$$\eta_1 = B_1(0)\varphi_1/[1 - (1 - p_h)B_1]^2. \quad (39)$$

To find η_2 , let φ_2 be the probability that a call which require a second type hand-off will not require an additional the same type hand-off. The probability, φ_2 , can be found from the gains from node o to nodes d , e and f after opening the path r_{h2} . Using Mason's formula, we find

$$\varphi_2 = p_w + \frac{(1 - p_w)(r_{bc} + r_{h1}p_h + r_{ac}r_i)}{1 - r_o r_i - r_{h1}(1 - p_h)}. \quad (40)$$

Let $B_2(0)$ be the probability that a call makes its first second-type hand-off. This can be found from the gain from node a to node o in Figure 14 after opening the path $1 - p_w$. That is

$$B_2(0) = r_{h2} \frac{p[1 - r_{h1}(1 - p_h)] + (1 - p)r_i}{1 - r_o r_i - r_{h1}(1 - p_h)} r_{h2}. \quad (41)$$

Let B_2 be the probability that a call makes its another second-type hand-off after a successful second-type hand-off. This can be found from the gain from node b to node o after opening the path $1 - p_w$. That is

$$B_2 = \frac{r_i r_{h2}}{1 - r_o r_i - r_{h1}(1 - p_h)}. \quad (42)$$

Using the similar procedure as for η_1 , we find

$$\eta_2 = B_2(0)\varphi_2/[1 - (1 - p_w)B_2]^2. \quad (43)$$

5 Numerical Results And Discussion

In our numerical examples, we consider a CBWL/CR-FR scheme with 24 channels in each gateway.

Figure 5 and 6 shows blocking probability and forced termination probabilities of CBWL/CR-FR obtained by numerical computation plotted against offered traffic in a cell. Simulation confidence intervals of 95% are also

shown. From the figures, we can see that the results of analysis agree with those by simulation. It is seen that CBWL/CR-FR can improve the forced termination probability significantly and can slightly improve blocking probability in light traffic load in comparison with FCA.

Figure 7, 8 and 9 show blocking probability, first type forced termination probability and the second forced termination probability with different cut-off priorities, respectively. Also shown is the blocking probability of fixed channel assignment. It is seen that if there is no any cut-off priority ($n_{h2} = n_{h1} = 0$), CBWL/CR-FR improves blocking probability and first-type forced termination probability in comparison with FCA. That is because the borrowed channels can benefit both new call arrivals and hand-off calls. The second-type forced termination probability can also be kept very small. Cut-off priority can be used to improve further the first-type forced termination probability. However, the degree of the improvement is limited. Because the cut-off priority is for channel rearrangement, if adjacent cells cannot lend any channel for the hand-off call, the hand-off request is still denied even it can make channel rearrangement. The reduce of forced termination probability is at the cost of increasing blocking probability. With $n_{h2} = n_{h1} = 2$, blocking probability is almost as that of FCA scheme. However, blocking probability does not change much if we continue to increase n_{h2} and n_{h1} . When n_{h2} fixed, we can select a value of n_{h1} such that $n_{h1} < n_{h2}$. Thus the first type of hand-off calls get highest priority to make channel rearrangement. As the result, the first type forced termination probability can be reduced further at the cost of increasing the second type forced termination probability.

Figure 10 shows dependence of blocking probability and forced termination probabilities on μ_d/mu . That is inverse of normalized mean dwell time. The same performance measures of FCA ($p = 0$) are also plotted for comparison. As μ_d/μ increased (mean dwell time is reduced), hand-off attempts are increased hence forced termination probabilities are also increased. The blocking probability is slowly decreasing with the increase of μ_d/μ because more hand-off failure tends to increase the chance that a new call arrival gets a channel assignment.

Figure 11 shows dependence of hand-off activity factor on, μ_d/μ . It is seen that hand-off activity factors have a linear relationship with μ_d/μ . The first type hand-off activity factor is steeper than the second type hand-off activity factor. The second type hand-off activity factor is much less than the first type hand-off activity factor. Thus CBWL will not cause too many second type hand-off attempts.

6 Conclusion

Our analysis and simulation of CBWL with hand-off have shown that CBWL/CR-FR can improve blocking and forced termination probabilities of cellular communication systems in comparison with FCA schemes. Cut-off priority can be used to further reduce forced termination probability at the cost of increasing blocking probability. CBWL/CR-FR introduces a new type hand-off which occurs when a communicating mobile that is using a borrowed channel moves from zone *A* into zone *B* (of the same cell). However, the forced termination probability caused by the new type hand-off is quite small. We also show that these hand-off attempts occur much less frequently in comparison with regular hand-off attempts. Thus it will not increase significantly the processing task. The new hand-off also can be processed distributely in a base station.

A systematic method using aggregation and decomposition is described. The method can be applied to multi-dimensional queuing systems where groups of state variables have product form solution.

Appendix A. Variance of Dwell Time PDF in a Cell

To determine the variance of dwell time in a cell experimentally, we randomly choose some sample vehicles in a given cell and record the time in which the vehicles spend in the cell until they leave the cell. These data are used to find variance of dwell time in a cell. There are two cases: case α and case β . In case α , a sampled vehicle is in zone A . In case β , a sampled vehicle is in zone B . Denote T_α as the dwell time in the cell for case α and T_β as the dwell time for case β . T_α and T_β are random variables (RVs). From assumptions 4 and 5 in Section 2.1, we can visualize T_α in Figure 12, in which T_A and T_B are dwell times in the A and B zones in the cell, respectively. Because we assume that the dwell time in zone A is exponentially distributed, the mean and variance of its pdf are $1/\mu_{da}$ and $1/\mu_{da}^2$, respectively. Similarly, the mean and variance of dwell time pdf in are $1/\mu_{db}$ and $1/\mu_{db}^2$, respectively. We define

$$T \triangleq T_A + T_B. \quad (\text{A.1})$$

Since T_A and T_B are independent, the pdf of T_0 is the convolution of the pdf's of T_A and T_B . The mean of T is $1/\mu_d$ and the variance of T is

$$\sigma_T^2 = 1/\mu_{da}^2 + 1/\mu_{db}^2 = (1 - 2p^z + 2p^{2z})/\mu_d^2. \quad (\text{A.2})$$

Denote T_k as the k th independent realization of the random variable, T_0 . From Figure 12, we know

$$T_\alpha = \sum_{k=1}^i T_k. \quad (\text{A.3})$$

where i is a discrete random variable. Specifically, i is the number of times that a vehicle moves back from zone B to zone A before it leaves the cell. The pdf of n is geometric type. That is

$$P\{i = s\} = (1 - q)q^{s-1} \quad s = 1, 2, \dots \quad (\text{A.4})$$

The mean of i , $E\{i\}$, is $1/(1 - q)$. The 2nd moment of i , $E\{i^2\}$, is $(1 + q)/(1 - q)^2$.

The mean and the 2nd moment of (A.3) can be determined using the method in [23]. The results are

$$E\{T_\alpha\} = E\{T\}E\{i\} = 1/\mu_d(1 - q), \quad (\text{A.5})$$

$$E\{T_\alpha^2\} = E\{T\}^2E\{i^2\} + \sigma_T^2E\{i\} = \frac{1 + q + (1 - q)[1 - 2p^z(1 - p^z)]}{\mu_d^2(1 - q)^2} \quad (\text{A.6})$$

In case β , the sampled vehicle is in zone B . The corresponding dwell time, T_α , is shown in Figure 13 From the figure, we find

$$T_\beta = T_B + \sum_{k=0}^j T_k. \quad (\text{A.7})$$

where j is a discrete random variable with following geometric distribution.

$$P\{j = s\} = (1 - q)q^s \quad s = 0, 1, \dots \quad (\text{A.8})$$

Note j starts with $k = 0$ while i starts with $k = 1$ (A.4). Thus their means and 2nd moments are different. The mean of j , $E\{j\}$, is $q/(1 - q)$. The 2nd moment of j , $E\{j^2\}$, is $(1 + q)q/(1 - q)^2$.

The mean and the 2nd moment of T_β can be determined from (A.7). The results are

$$E\{T_\beta\} = 1/\mu_{db} + E\{T\}E\{j\} = (1 - p^z)/\mu_d + q/\mu_d(1 - q), \quad (\text{A.9})$$

$$\begin{aligned} E\{T_\beta^2\} &= E\{T_B^2\} + 2E\{j\}E\{T_B\}E\{T\} + E\{T\}^2E\{j^2\} + \sigma_T^2E\{j\} \\ &= \frac{1}{\mu_d^2} \left[2(1 - p^z)^2 + \frac{q(1 + q) + q(1 - q)(3 - 4p^z + 2p^{2z})}{(1 - q)^2} \right]. \end{aligned} \quad (\text{A.10})$$

Denote the dwell time in a cell as T_d . From assumption 3, case α and β occur with probability of p and $1 - p$, respectively. Thus, the mean of T_d is

$$E\{T_d\} = pE\{T_\alpha\} + (1 - p)E\{T_\beta\} = \frac{1 - p + p^2 + q(p - p^2)}{(1 - q)\mu_d}. \quad (\text{A.11})$$

Recall $E\{T_d\} = 1/\mu_d$ from assumption 4, thus,

$$q = \frac{(1 - p)p^z}{1 + (1 - p)p^z}. \quad (\text{A.12})$$

The quantity is the probability that a mobile station in zone B enters A zone and $1 - q$ is the probability that the mobile goes to another cell under the assumption 3, 4 and 5.

The 2nd moment of T_d is

$$E\{T_d^2\} = pE\{T_\alpha^2\} + (1 - p)E\{T_\beta^2\}. \quad (\text{A.13})$$

The variance of T_d can be determined from

$$V_d = E\{T_d^2\} - E^2\{T_d\}. \quad (\text{A.14})$$

Substituting (A.11) and (A.13) into (A.14) and using (A.12), we find

$$\begin{aligned} V_d &= (1 + 2p^z - 4p^{z+1} + p^{2z} - 4p^{2z+1} + 5p^{2z+2} + 2p^{3z} - 2p^{3z+3} \\ &\quad + 4p^{4z} - 6p^{4z+1} + 2p^{4z+3} + 2p^{5z} - 6p^{5z+1} + 6p^{5z+2} - 2p^{5z+3}) \\ &\quad / \mu_d^2(1 + p^z - p^{z+1})^2. \end{aligned} \quad (\text{A.15})$$

For $z = 1$, V_d has a simple form

$$V_d = (1 - 2p^2 + 4p^3 - 2p^4)/\mu_d^2. \quad (\text{A.16})$$

Appendix B. Transition Rates of Macro-States (u, v)

Because the variables that comprise v have product form solution, we use a modified convolution algorithm to find $\rho(v)$ and $\beta(v)$ effectively. The convolution algorithm is described in [2] and [3]. The formulas are given here without derivation.

Average lending rate, $\rho(v)$

Denote $b(v)$ as the normalization constant in convolution algorithm that is described in [2] and $b(t, v)/b(v)$ as the probability that exactly $6 - t$ adjacent gateways have each borrowed exactly l channels from a given gateway given that a total v channels of that gateway are lent. If $6 - t$ gateways have each borrowed l channels, they cannot borrow any more channels from the given gateway. The rate of borrowing requests from all adjacent gateways becomes $t\lambda'$. Thus

$$\rho(v) = \frac{\lambda'}{b(v)} \sum_{t=1}^6 tb(t, v). \quad (\text{B.1})$$

Average channel returning rate from all neighbors, $\beta(v)$

In [3], we have shown that $\beta(v)$ can be easily computed from $b(t, v)/b(v)$. That is

$$\beta(v) = \frac{\lambda'}{b(v)} \sum_{t=1}^6 tb(t, v-1) \quad v = 1, \dots, L_{max} \quad (\text{B.2})$$

Probability of not borrowing channels

To calculate p_{nb} , we construct a one-dimension death-birth process, M/M/1 queuing model. The model is constructed by letting $s = u + v$ and expanding state of $u + v = C$ to $6l + 1$ states that correspond to $0, 1, \dots, 6l$ of borrowed channels. Thus the total number of states is $C + 6l + 1$. Denote the birth rate and death rate in state s as $\zeta(s)$ and $\theta(s)$, respectively. We can find the rates from $p_2(u, v)$.

For any state (u, v) with $0 \leq u + v < m$, the birth rate is $\lambda + \lambda_h + \rho(v)$. Thus, $\zeta(s)$ is the average birth rate on all (u, v) 's with $u + v = s$. That is

$$\zeta(s) = \lambda + \lambda_h + \sum_{v=1}^s p_2(s-v, v)\rho(v) \quad 0 \leq s < m. \quad (\text{B.3})$$

For any state (u, v) with $m \leq u + v < C$, the birth rate is $\lambda + \lambda_h$. Thus,

$$\zeta(s) = \lambda + \lambda_h \quad m \leq s < C. \quad (\text{B.4})$$

For any state s with $C \leq s < C + 6l$, the gateway will borrow channels for new call arrival and hand-off calls. The birth rate in state s is the channel borrowing rate of a gateway given that the gateway has borrowed $s - C$ channels. The rate is given by

$$\zeta(s) = \lambda_b p_{bs}(s - C) \quad C \leq s < C + 6l. \quad (\text{B.5})$$

in which $p_{bs}(j)$ is the probability that a borrowing request can be accepted by adjacent gateways given that j channels have been borrowed. It is determined in [3]. The quantity, λ_b is channel borrowing traffic from the given cell. That is

$$\lambda_b = \lambda[p + (1-p)(1-p_a/p_c)] + \lambda_h(1-p_m/p_c). \quad (\text{B.6})$$

The first and the second parts in (B.6) are the channel borrowing traffic due to new call arrivals and hand-off calls, respectively. The p_a is the probability that a new call cannot use channel rearrangement and p_m is the probability that a hand-off call cannot use channel rearrangement. They will be determined subsequently.

Now we find death rate in state s . For any state (u, v) with $0 < u + v \leq C$, the death rate is $u(\mu + \mu_d) + \beta(v)$. Thus, $\theta(s)$ is the average death rate on all (u, v) 's with $u + v = s$. That is

$$\theta(s) = \sum_{v=1}^s p_2(s-v, v)[u(\mu + \mu_d) + \beta(v)] \quad 0 < s \leq C. \quad (\text{B.7})$$

For any state s with $C < s \leq C + 6l$, the death rate includes the completion and departure rates in the C regular channels of the given gateway as well as the returning rate of $s - C$ borrowed channels. The death rate can be approximated by following formula:

$$\theta(s) = \theta(C) + (s - C)(\mu + \mu_{da}p_w/p_c) \quad C < s \leq C + 6l. \quad (\text{B.8})$$

where μ_{da} is the second-type hand-off attempt rate of borrowed channels and p_w is the probability that the second-type hand-off calls cannot use channel rearrangement. Thus $\mu_{da}p_w/p_c$ is the forced termination rate of a call due to failure of the second type hand-off.

Having found all birth and death rates of each states, we can calculate the pdf of s . From queuing theory, we can find equilibrium state distribution of M/M/1 system. That is,

$$p_1(s) = \prod_{k=0}^{s-1} \frac{\zeta(k)}{\theta(k+1)} / \sum_{i=0}^{C+6l-1} \prod_{k=0}^{i-1} \frac{\zeta(k)}{\theta(k+1)}. \quad (\text{B.9})$$

From the probabilities, we can find p_{nb} by

$$p_{nb} = p_1(C) / \sum_{s=C}^{C+6l} p_1(s). \quad (\text{B.10})$$

Appendix C. Signal Flow Diagram of a Call

The signal flow diagram for a call life is show in Figure 14. In the diagram, the nodes represent every *states in a process of call*. They are:

- a: The start of a call.
- b: The call is in zone *B*.
- c: The call is in zone *A*.
- d: The call is successfully completed in zone *B*.
- e: The call is successfully completed in zone *A*.
- f: The call is forced to termination due to failure of the first hand-off.
- g: The call is forced to termination due to failure of the second hand-off.
- h: The call makes a first type hand-off attempt.
- o: The call makes a second type hand-off attempt.

The gains in the paths are transition probabilities between states. The probability that a communicating mobile in zone *A* completes its call before it moves to zone *B* is

$$r_{ac} = \frac{\mu}{\mu + \mu_{da}} = \frac{\mu p^z}{\mu p^z + \mu_d}. \quad (\text{B.1})$$

The probability that a call in zone *B* completes its call before it moves out of zone *B* is

$$r_{bc} = \frac{\mu}{\mu + \mu_{db}} = \frac{\mu(1 - p^z)}{\mu(1 - p^z) + \mu_d}. \quad (\text{B.2})$$

The probability that a call in zone *B* travels to an *adjacent cell* hence needs the first type of hand-off is $(1 - q)\mu_{db}/(\mu_{db} + \mu)$. That is,

$$r_{h1} = \frac{1}{1 + p^z(1 - p)} \frac{\mu_d}{\mu(1 - p^z) + \mu_d}. \quad (\text{B.3})$$

The probability that the attempt fail is p_h . The probability that a communication mobile in zone A travels to zone B before its call is completed is $\mu_{da}/(\mu + \mu_{da})$. Let $E(BR)$ and $E(A)$ are the average number of calls that use borrowed channels and the average number of A -type calls in a cell, respectively. Thus,

$$E(BR) = \sum_{k=1}^{6l} k p_1(C+k), \quad (\text{B.4})$$

and

$$E(A) = \sum_{v=0}^{L_{max}} p_l(v) \sum_{i_a=1}^{C-v} i_a \sum_{i_b=0}^{C-v-i_a} p_v(i_a, i_b). \quad (\text{B.5})$$

$E(BR)/[E(BR) + E(A)]$ is the average number of calls in zone A that use borrowed channels. Thus, the probability that a communicating mobile makes a second type of hand-off attempt is

$$r_{h2} = \frac{\mu_d}{\mu p^z + \mu_d} \frac{E(BR)}{E(A) + E(BR)}. \quad (\text{B.6})$$

The probability that a second type hand-off fails is p_w . The probabilities that a call moves from zone A to zone B without the second type hand-off is

$$r_o = \frac{\mu_d}{\mu p^z + \mu_d} \frac{E(A)}{E(A) + E(BR)}, \quad (\text{B.7})$$

The probability of a call from zone B to zone A is

$$r_i = \frac{\mu_{db}}{\mu_{db} + \mu} q = \frac{\mu_d}{\mu(1-p^z) + \mu_d} \frac{p^z(1-p)}{1+p^z(1-p)}. \quad (\text{B.8})$$

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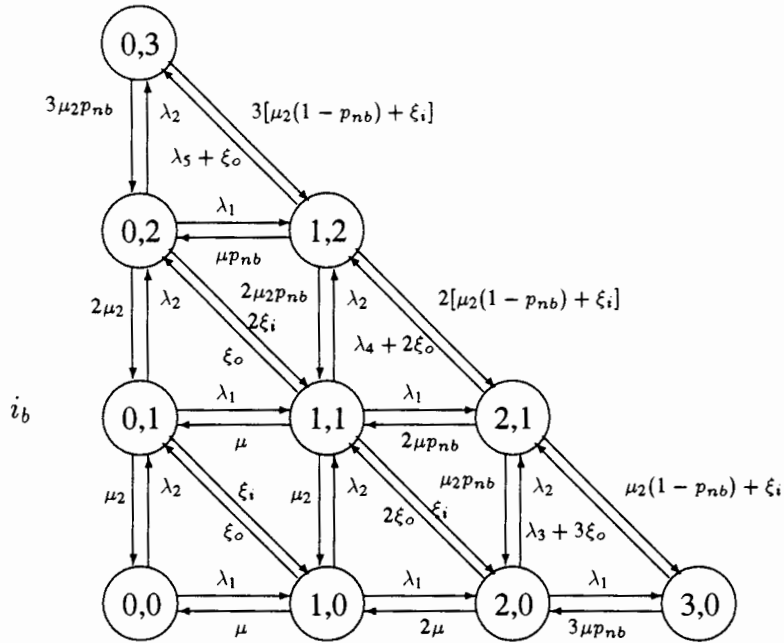


Figure 4: An example of transition diagram for macro-state (i_a, i_b) of CBWL/CR-FR, ($C - v = 3, n_{h2} = 2, n_{h1} = 1$).

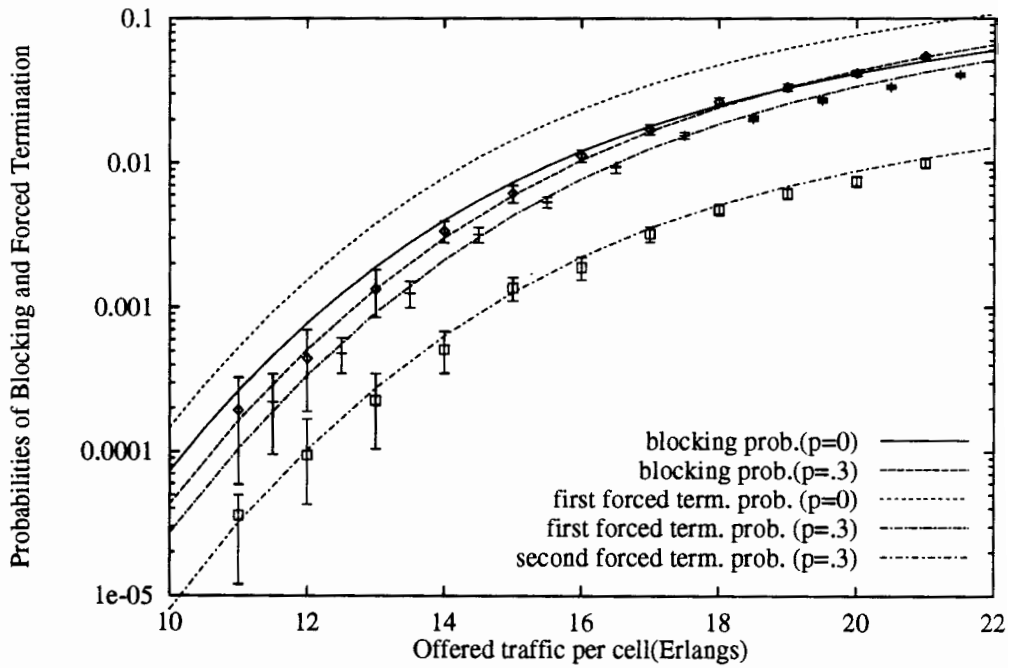


Figure 5: Blocking probabilities and forced termination probabilities of CBWL/CR-FR ($C = 24, m = 22, l = 4, n_{h2} = n_{h1} = 2, z = 1$).

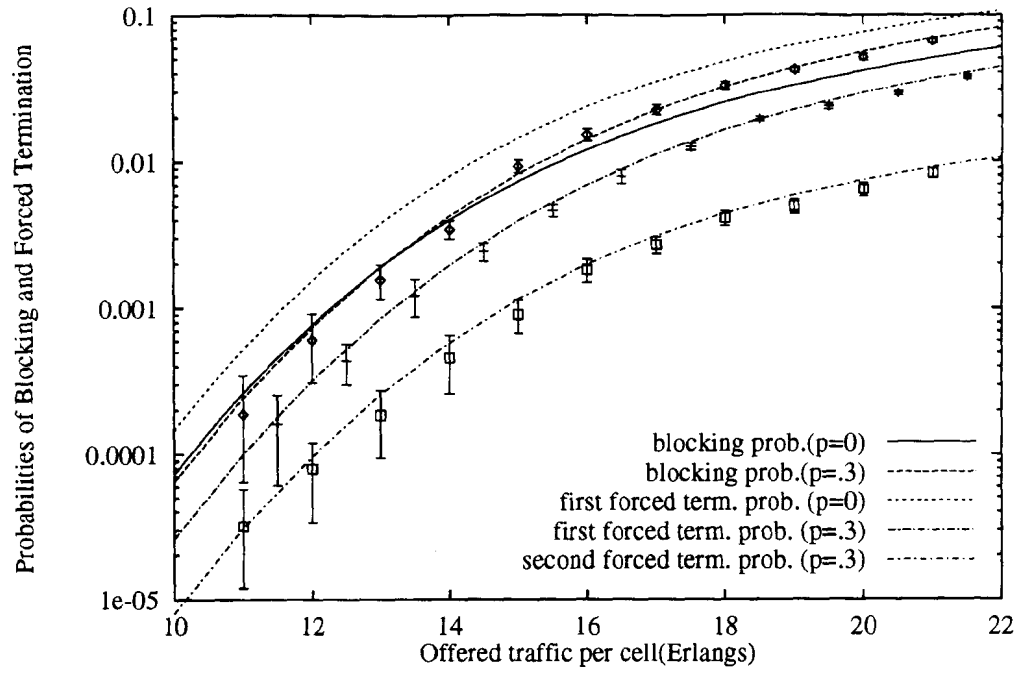


Figure 6: Blocking probabilities and forced termination probabilities of CBWL/CR-FR ($C = 24, m = 22, l = 4, n_{h2} = n_{h1} = 4, z = 1$).

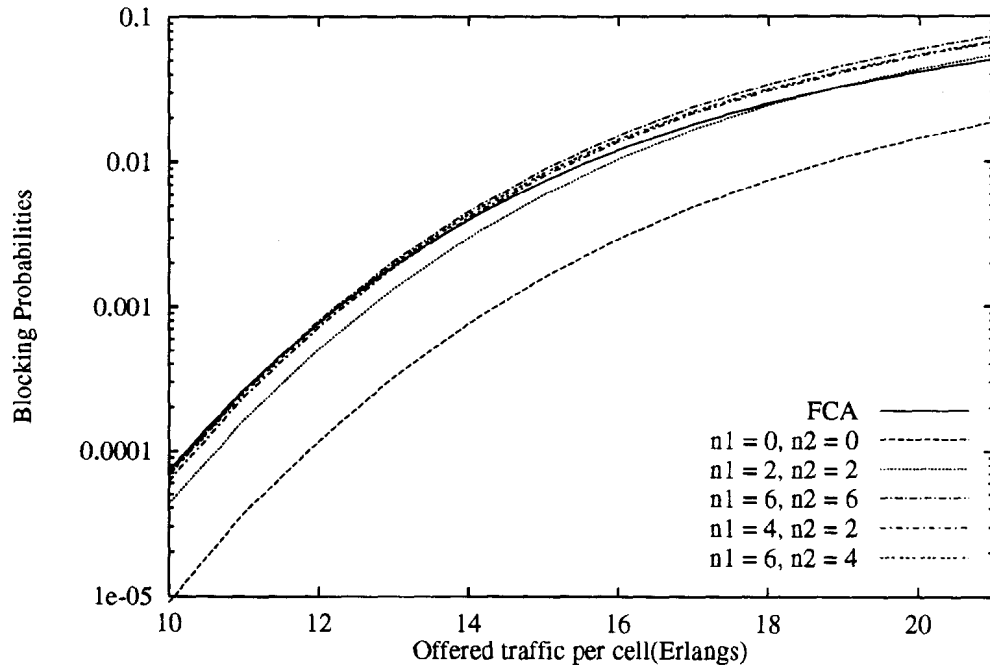


Figure 7: Blocking probabilities of different cut-off priorities in CBWL/CR-FR ($C = 24, m = 22, l = 4, p = .3, z = 1$).

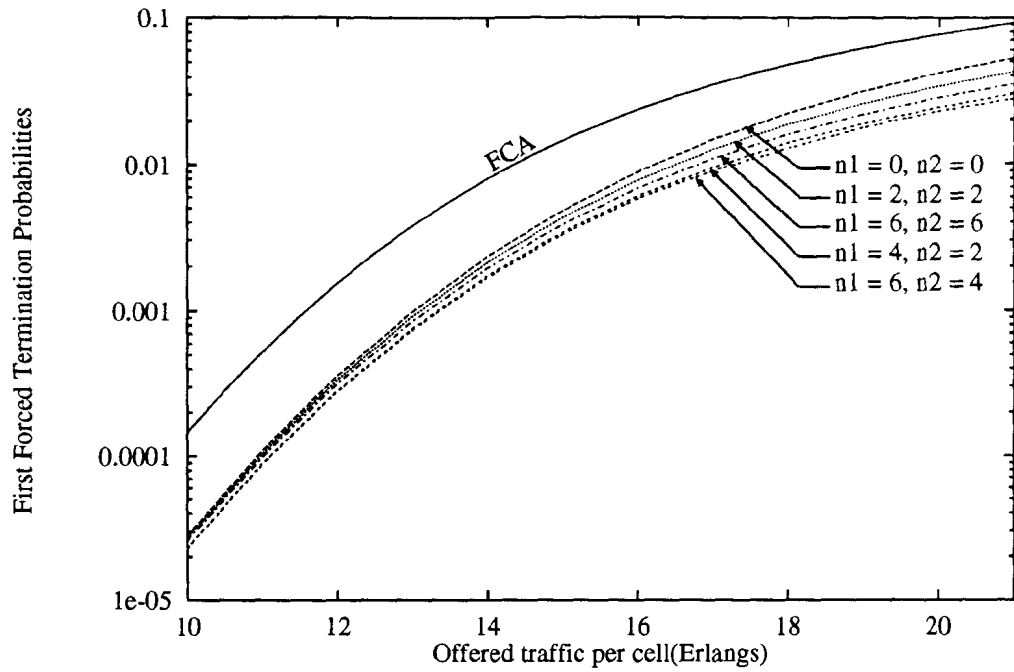


Figure 8: The first type forced-termination probabilities of different cut-off priorities in CBWL/CR-FR ($C = 24, m = 22, l = 4, p = .3, z = 1$).

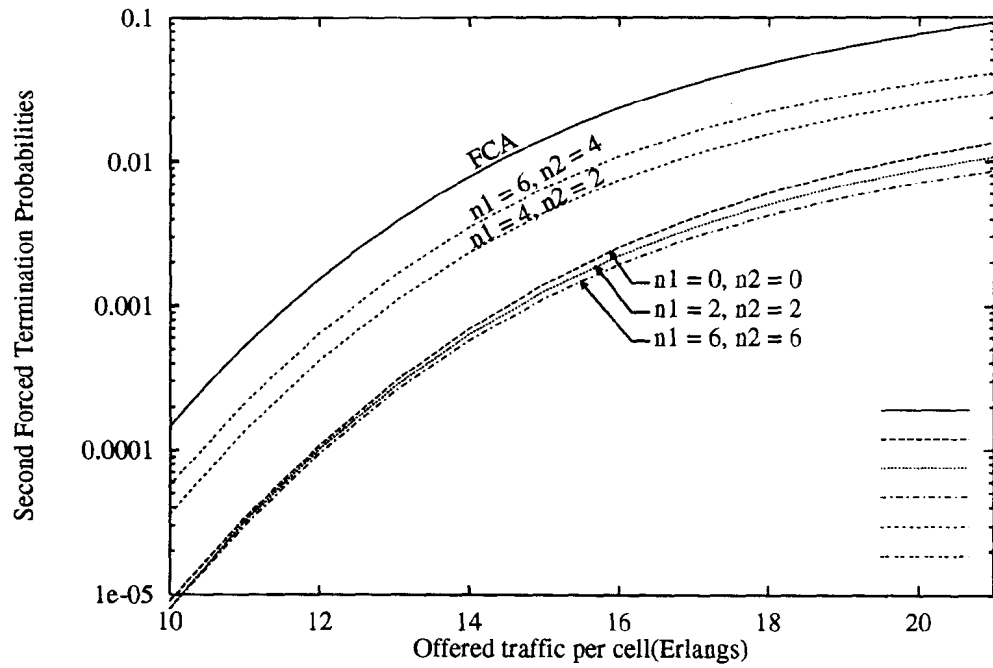


Figure 9: The second type forced-termination probabilities of different cut-off priorities in CBWL/CR-FR ($C = 24, m = 22, l = 4, p = .3, z = 1$).

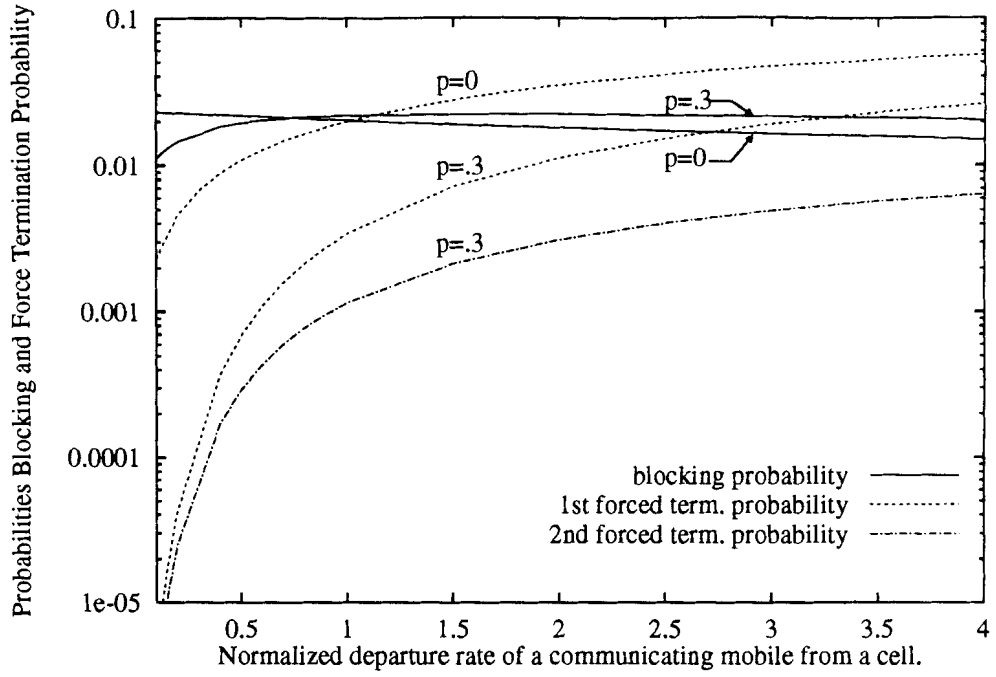


Figure 10: Blocking and forced termination probabilities of CBWL/CR-FR depend on μ_d/μ . ($C = 24, m = 22, l = 4, p = .3, n_{h2} = n_{h1} = 4, z = 1$, offered traffic in a cell = 17 erlangs.).

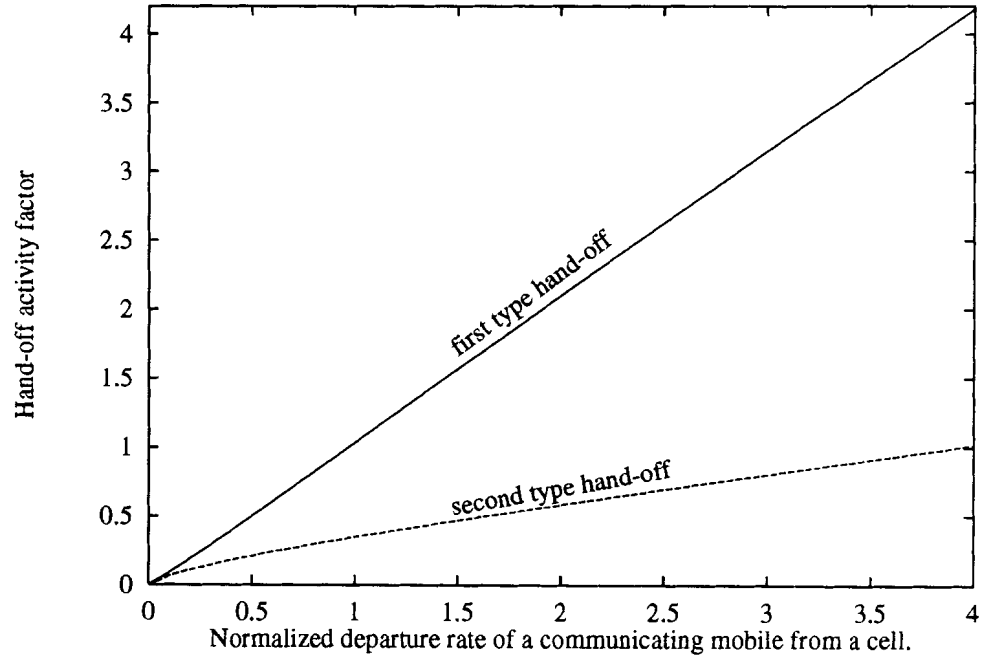


Figure 11: Hand-off activity factor of CBWL/CR-FR depend on μ_d/μ . ($C = 24, m = 22, l = 4, p = .3, n_{h2} = n_{h1} = 4, z = 1$, offered traffic in a cell = 17 erlangs.).

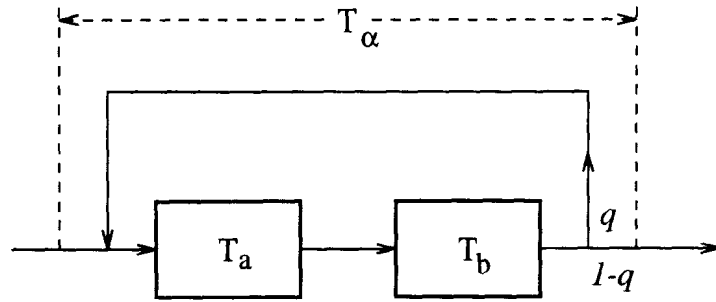


Figure 12: Dwell time in a cell for case α .

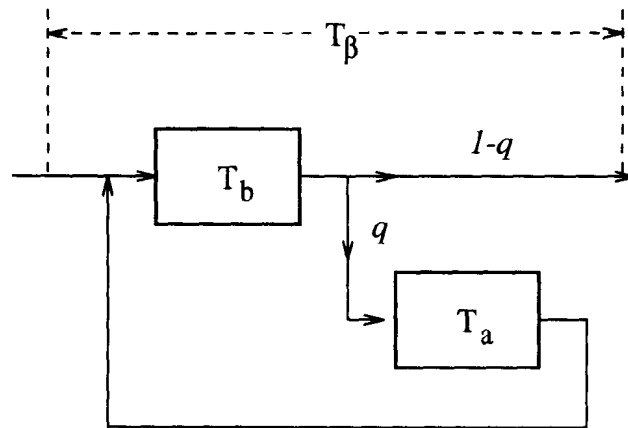


Figure 13: Dwell time in a cell for case β .

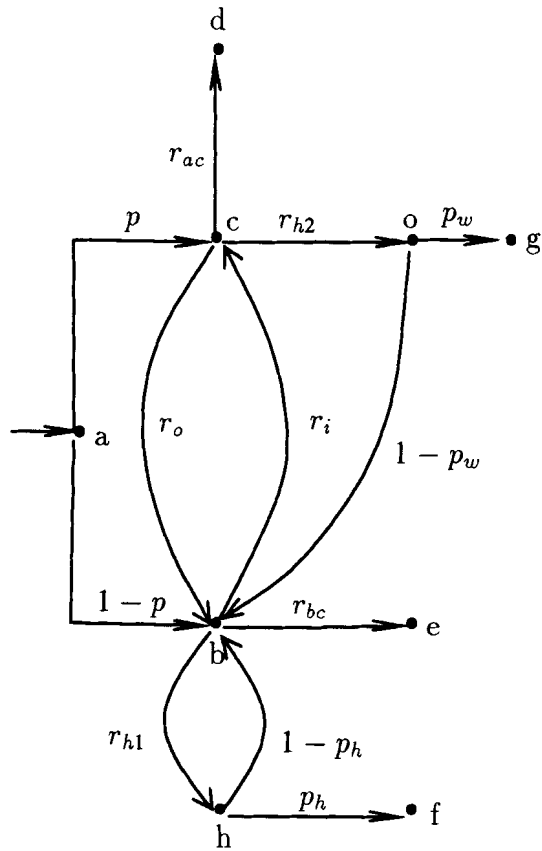


Figure 14: Signal flow diagram of a call in CBWL.