



STATE UNIVERSITY OF NEW YORK
AT STONY BROOK

COLLEGE OF
ENGINEERING

Report No. 5

NOTE ON THE SCALAR SPECTRUM TRANSPORTED

BY AN

ARTIFICIAL TURBULENT SPECTRUM

by

Edward E. O'Brien and Thore Omholt

September 1963

Sp. Coll.

TAL

.N532

no. 5

o. 2

Note On the Scalar Spectrum Transported by an
Artificial Turbulent Spectrum

Edward E. O'Brien and Thore Omholt
State University of New York at Stony Brook
Stony Brook, New York

A recent analysis⁽¹⁾ has shown that the evolution of a scalar correlation in a homogeneous turbulent fluid in the absence of molecular diffusion can be represented by

$$f(\underline{r}, t) = \int_{\underline{r}_0} f(\underline{r}_0, 0) q(\underline{r}, t/\underline{r}_0, 0) d\underline{r}_0 \quad (1)$$

where $f(\underline{r}, t)$ is the correlation at time t and $q(\underline{r}, t/\underline{r}_0, 0)$ is the probability that two fluid particles separated by a vector distance \underline{r}_0 at time zero will be separated by a vector \underline{r} at time t .

Unfortunately information about $q(\underline{r}, t/\underline{r}_0, 0)$ in realizable turbulent flows is extremely meagre and certainly there is no information about the whole range of separations \underline{r}_0 . Certain attempts⁽¹⁾⁽²⁾ have been made to use the asymptotic behavior in time of this Lagrangian function but in both cases the region near the origin is too awkward. For example Roberts⁽¹⁾ is interested in the small time approximation when the fluid point can be assumed to move with the Eulerian velocity at the point over the time of interest. For the velocity field he made a Gaussian assumption and obtained

$$q(\underline{r}, t/\underline{r}_0, 0) = p(\underline{r}_0, t) \exp - (A(\underline{r}_0, t)(r_1^2 + r_2^2) + B(\underline{r}_0, t)r_3^2)$$

Without considering the details of this result it is evident that $q(0, t/\underline{r}_0, 0)$ is in general nonzero for $\underline{r}_0 \neq 0$. Continuum theory of

course rejects such a notion and instead we should demand

$$\lim_{r \rightarrow 0} q(\underline{r}, t/r_{\underline{r}_0}, 0) = \delta(\underline{r}_0, t) \quad (2)$$

and

$$\lim_{r_{\underline{r}_0} \rightarrow 0} q(\underline{r}, t/r_{\underline{r}_0}, 0) = \delta(\underline{r}, t)$$

where $\delta(r, t)$ is the Dirac delta function. Further for $q(\underline{r}, t/r_{\underline{r}_0}, 0)$ to be a legitimate probability density we require

$$\int_{r_{\underline{r}_0}} q(\underline{r}, t/r_{\underline{r}_0}, 0) dr_{\underline{r}_0} = 1 \quad (3)$$

and

$$\int_r q(\underline{r}, t/r_{\underline{r}_0}, 0) dr = 1$$

To obtain a simply integrable function satisfying conditions (2) and (3) we add an assumption which, in detail, is unreasonable but probably not so violent as to compromise the spectral evolution seriously. Namely we consider q as a function only of the magnitudes of \underline{r} and \underline{r}_0 . This is evidently not true in general but Batchelor's quasi asymptotic assumption⁽¹⁾ amounts to something similar after a time of order $(\frac{\nu}{\epsilon})^{\frac{1}{2}}$, the Kolmogoroff time scale.

A well behaved function which satisfies all the above conditions can be written

$$q(\underline{r}, t/r_{\underline{r}_0}, 0) = \frac{g(t)^{\frac{1}{2}}}{(r r_{\underline{r}_0} \pi)^{\frac{1}{2}}} \exp \left\{ 2g(t) - g(t) \left[\frac{r}{r_{\underline{r}_0}} + \frac{r_{\underline{r}_0}}{r} \right] \right\} \quad (4)$$

Here we notice a further disadvantage and that is the symmetry in r and r_0 a property which we do not anticipate. However, both

course rejects such a notion and instead we should demand

$$\lim_{\underline{r} \rightarrow 0} q(\underline{r}, t / \underline{r}_0, 0) = \delta(\underline{r}_0, t) \quad (2)$$

and

$$\lim_{\underline{r}_0 \rightarrow 0} q(\underline{r}, t / \underline{r}_0, 0) = \delta(\underline{r}, t)$$

where $\delta(r, t)$ is the Dirac delta function. Further for $q(\underline{r}, t / \underline{r}_0, 0)$ to be a legitimate probability density we require

$$\int_{\underline{r}_0} q(\underline{r}, t / \underline{r}_0, 0) d\underline{r}_0 = 1 \quad (3)$$

and

$$\int_{\underline{r}} q(\underline{r}, t / \underline{r}_0, 0) d\underline{r} = 1$$

To obtain a simply integrable function satisfying conditions (2) and (3) we add an assumption which, in detail, is unreasonable but probably not so violent as to compromise the spectral evolution seriously. Namely we consider q as a function only of the magnitudes of \underline{r} and \underline{r}_0 . This is evidently not true in general but Batchelor's quasi asymptotic assumption⁽¹⁾ amounts to something similar after a time of order $(\frac{\nu}{\epsilon})^{\frac{1}{2}}$, the Kolmogoroff time scale.

A well behaved function which satisfies all the above conditions can be written

$$q(\underline{r}, t / \underline{r}_0, 0) = \frac{g(t)^{\frac{1}{2}}}{(\underline{r}\underline{r}_0\pi)^{\frac{1}{2}}} \exp\left\{2g(t) - g(t)\left[\frac{\underline{r}}{\underline{r}_0} + \frac{\underline{r}_0}{\underline{r}}\right]\right\} \quad (4)$$

Here we notice a further disadvantage and that is the symmetry in \underline{r} and \underline{r}_0 a property which we do not anticipate. However, both

the small r behavior and the large r behavior seem to be crudely correct. Quadratic functions of r would have been preferable but we were unable to arrive at a simple expression satisfying all the necessary conditions. We note in passing that $g(t)$ should be a monotonically decreasing function of t which is infinite initially and zero asymptotically. Furthermore the time scale should be $O(\frac{\lambda}{v'})$ where λ is the turbulence microscale and v' the root mean square turbulence velocity.

Numerical calculation of equation (f) for the cases $g(t) = 6, 3, 1$ and $.001$ and for $f(r_0, 0) = e^{-\frac{r_0^2}{L^2}}$ have been carried out with the results as graphed in Figure 1.

It has also been established⁽³⁾ that molecular diffusion up to terms $O(t)$ can be included in the decay evolution and that the appropriate integral relation with which to replace (1) is, if k is the diffusion coefficient

$$f(r, t) = \int_{r_0} \left[1 + 2kt\tau^2 \right] f(r, 0) q(r, t/r_0, 0) dr_0 \quad (5)$$

$$e^{-\frac{r_0^2}{L^2}}$$

or using $f(r_0, 0) = e^{-\frac{r_0^2}{L^2}}$ and the previous assumptions on q

$$f(r, t) = \int_0^\infty e^{-\frac{r_0^2}{L^2}} q(r, t/r_0, 0) dr_0 + \left[f_D^1(r, t) + f_D^2(r, t) \right] \frac{4kt}{L^2} \quad (6)$$

Thus the first contribution of molecular diffusion can be determined by examining the second and third term of Equation (6).

In Figure 2(a & b) plots have been made of the resultant of these two contributions and it is found that, initially at least, molecular diffusion tends to reinforce the turbulent diffusion effect on the scalar spectrum.

Fig. 1

$$f(r, t) = \frac{P(x+\epsilon)P(x)}{P^2(x)}$$

- $t = 0.0$
- $g(t) = 6.0$
- $g(t) = 3.0$
- $g(t) = 1.0$
- $g(t) = 0.001$

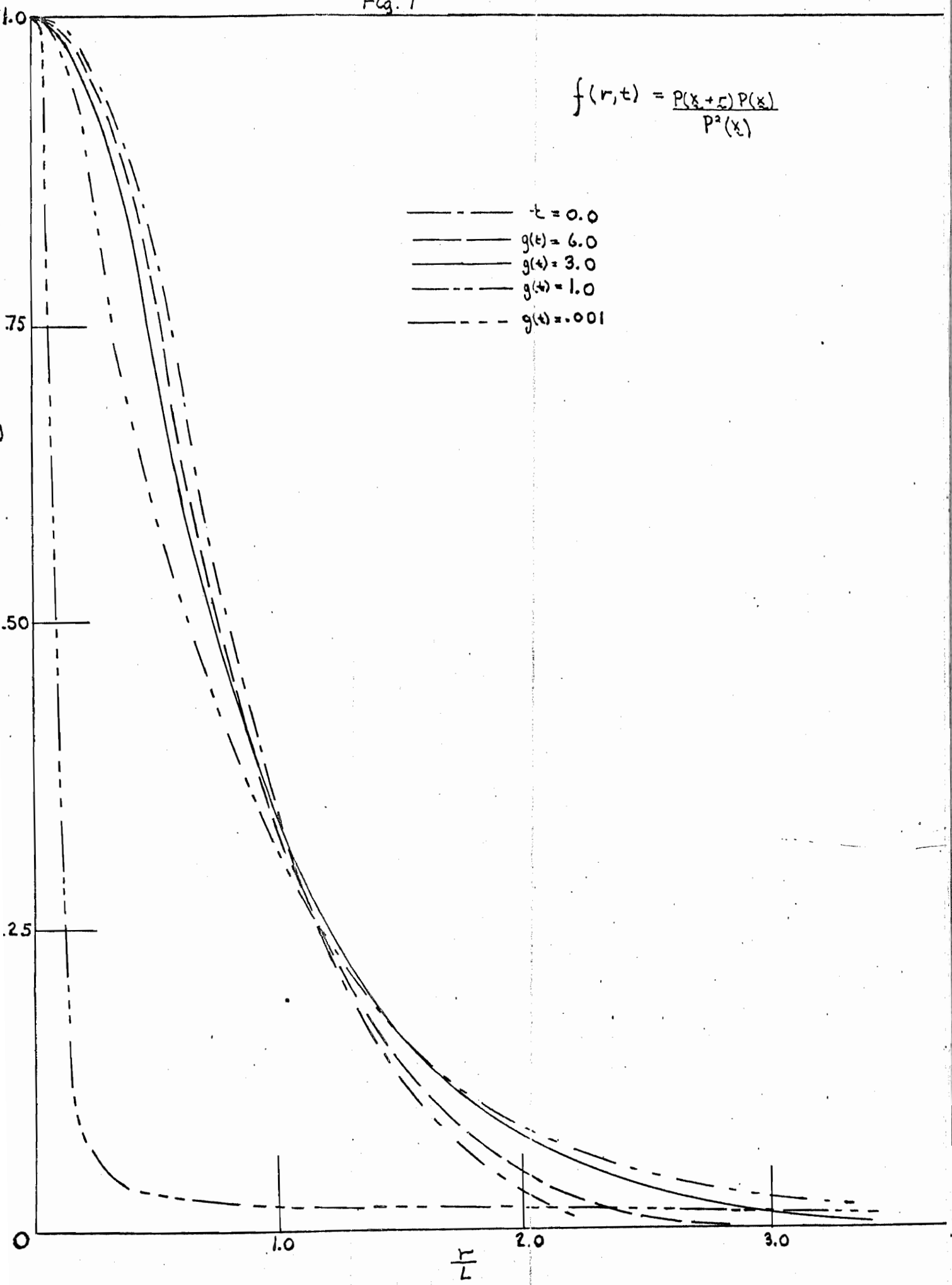
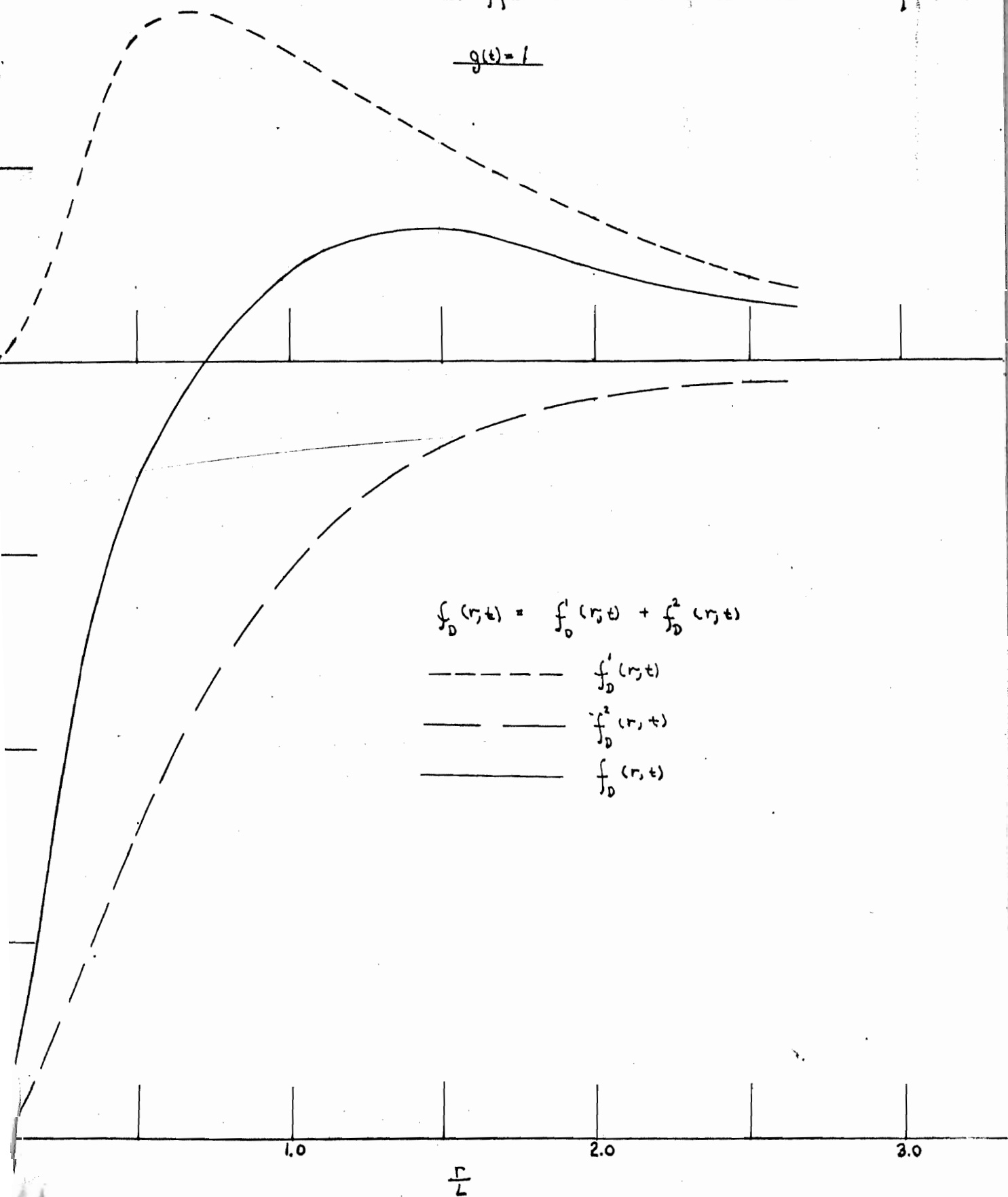


fig. 2a

Diffusive contribution to $f(r,t)$

$g(t) = 1$



$$f_0(r,t) = f_0^1(r,t) + f_0^2(r,t)$$

- $f_0^1(r,t)$
- $f_0^2(r,t)$
- · — · — $f_0(r,t)$

fig. 2b

Diffusive contribution to $f(r,t)$

$g(t) = 6$

