

A STUDY OF FIXED AND DEMAND ASSIGNED CODE
DIVISION MULTIPLE ACCESS USING TREE CODES

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ABSTRACT: Schemes for multiple user - point to point communication, using orthogonal and quasi-orthogonal tree codes are presented. The packet error probability resulting from mutual interference between users' transmissions is first evaluated for fixed assigned multiple access.

A reservation scheme which uses tree coded request and message channels is developed.

The model incorporates (tree coded) request channels of collision type for deterministic as well as random message lengths. Traffic and detection performance characteristics and interactions are investigated under steady state conditions. The advantage of certain quasi-orthogonal schemes is demonstrated.

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A. Comparison of Multiple Access Schemes Using Tree Codes

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I. INTRODUCTION:

Use of Tree Codes for Multiple Access.

Bounds on error probability for orthogonal and quasi-orthogonal tree codes were developed in an earlier paper and the use of such codes was briefly discussed[1]. Here, two distinct schemes with good anti-intercept properties for multipoint user-user communication on a common wideband channel are considered. Each of these schemes can use either orthogonal or quasi-orthogonal signal sets. First, the previously derived bounds are used to compare performance of these schemes in a fixed assigned multiple access situation. A reservation scheme for multipoint message and/or packet switching is then described and a steady state analysis of traffic behaviour is presented. Finally, the interactions of traffic and detection performance for these schemes are discussed.

We describe two schemes for the use of tree codes in multiple access.

- (i) Common Signal Set Scheme (CSSS): Here all users transmit their tree coded data (either orthogonal or quasi-orthogonal) simultaneously, using the same signal set. For codes of constraint length K , the encoder for each user adds (mod 2^K) a K -bit portion of a unique pseudo-random sequence to the K bit shift register into which the data enters. The sum is used to select a signal $s_j(t)$ ($j = 0, 1 \dots 2^K - 1$). This randomization makes other user interference appear noise-like to a receiver which has a synchronous copy of the PN sequence used at the corresponding encoder. This scheme has been discussed in [1,2] for certain quasi-orthogonal & orthogonal tree codes respectively.
- (ii) Multiple Signal Set Scheme (MSSS): In this scheme, each transmitting user is assigned a unique signal set $\{s_j(t), j = 0, 1 \dots 2^K - 1\}$ from a family of mutually quasi-orthogonal or orthogonal signals. Thus, a

receiver which has a matched filter bank corresponding to transmitter i 's signal set can decode his transmission. Clearly, the orthogonal case is equivalent to the use of separate channels for each coded transmission. On the other hand, reasonable bandwidth expansion, though with some mutual interference, can be achieved if quasi-orthogonal signals are used. With quasi-orthogonal signals, the performance will depend on the correlation properties of the signal family.

Performance curves for the above schemes are derived under the assumption that other user interference is the dominant cause of channel errors. The approach is to find an effective signal to noise ratio at any given receiver and to use this to compute the error probability for a packet 'L' bits long, using the results obtained in [1]. For the sake of brevity, only continuous incoherent reception is considered here, although the corresponding coherent cases can be similarly analysed.

selected signal of the set and a particular signal (s_0). This will hence be equal to the cross-correlation between two different signals $\langle s_i \cdot s_j \rangle$ when $s_i \neq s_0$, and will be equal to the auto-correlation $\langle s_i \cdot s_i \rangle$ when $s_i = s_0$. The probability of s_{i_j} being s_0 is $1/2^K$ since it is randomly selected from any of 2^K signals, and the probability of s_{i_j} being other than s_0 is $1 - 1/2^K$. Thus, we introduce random variables x_k, z_k where x_k is equal to 0 with probability $1 - 1/2^K$ and A with probability $1/2^K$, as in the orthogonal case, and z_k is K_c with probability $1 - 1/2^K$ and A_c with probability $1/2^K$, as in the quasi-orthogonal case. (K_c, A_c are the quasi-

II. PACKET ERROR PROBABILITY FOR PROPOSED SCHEMES:

A) Common Signal Set Scheme (CSSS) (Incoherent):

We assume here that M users transmit simultaneously on a common channel, and that receivers detect all the signals with equal strength. In a signalling interval, user j sends $s_{i_j}(t) \cos(\omega_0 t + \theta_j)$. $s_{i_j}(t)$ is the signal on the code tree sent out by user j and is assumed to be any of the signals $\{s_i, i = 0, 1, \dots, 2^K-1\}$ with equal probability. θ_j is a random phase associated with user j's carrier and is assumed uniformly distributed. The composite received signal $r(t)$ at any receiver is given by:

$$r(t) = s_{i_1}(t) \cos(\omega_0 t + \theta_1) + s_{i_2}(t) \cos(\omega_0 t + \theta_2) + \dots + s_{i_M}(t) \cos(\omega_0 t + \theta_M) \tag{1}$$

Referring to the analysis in [1], we recall that the optimum decoder forms the 'branch metric' corresponding to signal i by passing the received signal through a matched filter for s_i followed by a quadratic detector in the incoherent case (Fig. 1). Proceeding as in [1], we consider receiver 1 where the signal sent by transmitter 1 is $s_{i_1} = s_0$. Let $r_1^0(t_0)$ be receiver 1's matched filter output on the 'correct' channel (corresponding to the signal s_0), at the decision time t_0 . This output will be composed of a desired term produced by $s_{i_1} (= s_0)$ and M-1 undesired interference terms produced by the other users' transmissions. Referring to (1) we observe that $r_1^0(t_0)$ is given by

$$r_1^0(t_0) = \langle s_{i_1} \cdot s_0 \rangle \cos(\omega_0 t_0 + \theta_1) + \langle s_{i_2} \cdot s_0 \rangle \cos(\omega_0 t_0 + \theta_2) + \dots + \langle s_{i_M} \cdot s_0 \rangle \cos(\omega_0 t_0 + \theta_M) \tag{2}$$

and

$$\begin{aligned}
 y_c^0 &= A_c \cos \theta_1 + \sum_{i=2}^M z_i \cos \theta_i \triangleq A_c \cos \theta_1 + n_{cq} \\
 y_s^0 &= A_c \sin \theta_1 + \sum_{i=2}^M z_i \sin \theta_i \triangleq A_c \sin \theta_1 + n_{sq}
 \end{aligned}
 \tag{4b}$$

for quasi-orthogonal CSSS.

Hence, the noise variance σ_n^2 can be found from

$$\sigma_{n^0}^2 = E \left\{ \sum_{i=2}^M x_i \cos \theta_i \right\}^2
 \tag{5a}$$

for orthogonal signals

$$\sigma_{nq}^2 = E \left\{ \sum_{i=2}^M z_i \cos \theta_i \right\}^2
 \tag{5b}$$

for quasi-orthogonal signals.

Using the assumption that θ_i 's are uniformly distributed and are statistically independent of each other and x_i , z_i , we obtain

$$\sigma_{n^0}^2 = \frac{(M-1) A_c^2}{2^{K+1}}
 \tag{6a}$$

for orthogonal signals

$$\sigma_{nq}^2 = \frac{(M-1)}{2^{K+1}} \{A_c^2 + (2^K - 1) K_c^2\}
 \tag{6b}$$

for quasi-orthogonal signals

This permits computation of an effective signal to noise ratio (S) where the 'noise' is entirely other user interference. Defining $\mu \triangleq A_c / K_c$ as the auto-correlation to cross correlation ratio we obtain

$$S_0 = \left[\frac{2^{K+1}}{M-1} \right]^{1/2}
 \tag{7}$$

for orthogonal CSSS

$$S_q = [2^{K+1} / \{ (1 + (2^K - 1) / \mu^2) (M-1) \}]^{1/2} \quad (8)$$

for quasi-orthogonal CSSS.

If now we assume that the noise produced by a large number of interfering users can be modelled as Gaussian, we can use the above effective signal to noise ratios to compute the packet error probability, P_E , using formulas described in Appendix I. This approach has been used to obtain specific performance curves for the orthogonal CSSS and quasi-orthogonal CSSS with Gold codes used to form the signal set. These results will be discussed later and are shown in Fig. 2.

B) Multiple Signal Set Scheme (MSSS) (Incoherent):

We analyse this scheme using assumptions similar to those for the CSSS. However in this scheme, signals emitted by any two transmitting users are either orthogonal or quasi-orthogonal. If they are orthogonal, there is no other user interference and the situation is equivalent to ordinary coded transmission on separate channels. This will require a very large bandwidth to support a number of users. There will be no errors due to other user interference.

However, if the signal sets are from a family of mutually quasi-orthogonal signals all of user j 's signals are quasi-orthogonal to those of user 1, and $\langle s_0(t), s_{i \neq 1}^j(t) \rangle = K_c$. The peak matched filter output on the $i=1$ 'correct' channel of receiver 1 is given by

$$r_1^0(t) = A_c \cos(\omega_0 t_0 + \theta_1) + \sum_{i=2}^M K_c \cos(\omega_0 t_0 + \theta_i) \quad (9)$$

Thus, the quadrature components y_c^0 and y_s^0 are given by

$$y_c^0 = A_c \cos \theta_1 + \sum_{i=2}^M K_c \cos \theta_i \triangleq A_c \cos \theta_1 + n_c \quad (10)$$

$$y_s^0 = A_c \sin \theta_1 + \sum_{i=2}^M K_c \sin \theta_i \triangleq A_c \sin \theta_1 + n_s$$

The noise variance σ_n^2 can thus be found from

$$\sigma_n^2 = E \left\{ \sum_{i=2}^M K_c \cos \theta_i \right\}^2 = \frac{K_c^2}{2} (M-1) \quad (11)$$

assuming θ_i 's to be independent of each other and uniformly distributed.

Hence, the signal to noise ratio S is given by

$$S = A_c / \sigma_n = \sqrt{2} \mu / \sqrt{M-1} \quad (12)$$

Once again, if the interference noise is assumed Gaussian, the packet error probability can be computed from formulae in Appendix I. Specific performance curves using Gold codes as the quasi-orthogonal signal family are shown in Fig. 2.

III. COMPARATIVE PERFORMANCE OF SCHEMES (using Gold codes as the quasi-orthogonal signal family):

Gold codes have been widely used for multiple access because they provide large families of sequences with relatively low mutual cross-correlation. We consider here a specific realization for the schemes described using Gold sequences and obtain comparative performance curves. A measure of the multiple access capability of a system is the bandwidth expansion per user (E) needed to maintain a given error probability P_E . Such curves were obtained for the quasi-orthogonal CSSS in [1]. Here, we consider the CSSS and MSSS, both orthogonal and quasi-orthogonal.

In [6] it is shown that $2^n + 1$ distinct Gold sequences, each of length $2^n - 1$ can be generated with a 2 register Gold code generator of length n . The corresponding bandwidth expansion is hence $2^n - 1$. The cross correlation and out of phase auto-correlation are strictly bounded by

$$|K_c| < \begin{cases} 2^{(n+1)/2} + 1 & n \text{ odd} \\ 2^{(n+2)/2} + 1 & n \text{ even} \end{cases} \quad (13)$$

$n \neq \text{mod } 4.$

This gives the largest possible cross-correlation for any two members of the family. If the above value of K_c is used in the expression for effective signal to noise ratio S , derived in the previous section, performance prediction would be excessively pessimistic. In a multiple access environment interfering users will be transmitting signals randomly selected from the Gold code family. With a large number of users, an average over the family would be more indicative. In [7], Gold shows that the cross-correlation between two sequences a and b is given by

$$\begin{aligned}
|\theta(a,b) (\tau)| &= 1 \quad [\text{when } a (\tau) = 1] \\
&= 2^{(n+1)/2} + 1 \quad [\text{when } a (\tau) = -1] \\
&\text{(n odd) or } 2^{(n+1)/2} - 1
\end{aligned} \tag{14}$$

Hence, in an average sense, we can use the approximation,

$$\bar{K}_c \approx (2^{(n+1)/2} + 1)/2 \tag{15}$$

since $a (\tau) = \pm 1$ with nearly the same frequency for large n . Using the above results for Gold codes, we compute error probability, P_E as a function of bandwidth expansion per user E , as outlined below.

CSSS Orthogonal: S (and hence P_E for a packet length L) can be found using (7) for a given number of simultaneous transmissions, M and constraint length K . The bandwidth expansion corresponding to a constraint length K is equal to 2^K , the number of orthogonal signals required; hence $E = 2^K/M$. A curve showing P_E vs. E can therefore be obtained.

CSSS Quasi-orthogonal: In this scheme, we note that 2^K signals are required and that with timing information $2^n - 1$ cyclic shifts of $2^n + 1$ sequences can be obtained to form $(2^n + 1) \cdot (2^n - 1)$ usable signals with a Gold code generator of length n . We have $2^{2n} - 1 \geq 2^K$, which yields the result that n must be at least the integer greater than $K/2$ [1]. A larger n means a larger bandwidth expansion ($= 2^n - 1$, the length of each sequence), but also a higher auto-correlation to cross-correlation ratio (μ). The bandwidth expansion per user $E = (2^n - 1)/M$ and $\mu = A_c/K_c$ is found from (15) since $A_c = 2^n - 1$. Thus, computing S from (8), P_E vs. E can be obtained.

MSSS Orthogonal: As mentioned earlier, there is no error due to interference. The error probability in presence of Gaussian noise can be found from [1].

MSSS Quasi-orthogonal: In this scheme, in order to support M active users, $M \cdot 2^K$ usable quasi-orthogonal signals are required, for a given K . The value of n required is thus determined by $(2^n + 1)(2^n - 1) \geq M \cdot 2^K$. Therefore for a specified n and K , the maximum number of users supportable, even without considering error probability is $M_{\max} = 2^{2n-K} - 1$. Here again $E = (2^n - 1)/M$ and μ is found from (15). S is given by (12) and consequently P_E can be found as a function of E with K and n supplied.

IV. DISCUSSION OF RESULTS FOR FIXED ASSIGNED SCHEMES

Figure 2 shows curves of packet error probability ($L = 1000$) vs. bandwidth expansion per user (E) for various schemes. It can be seen that these schemes, both orthogonal and quasi-orthogonal, permit several users to transmit simultaneously with a reasonable bandwidth expansion. It might be noted that the bit error probability is upper bounded by packet error probability. A few general observations about the performance of the schemes can be made here.

(i) The bandwidth expansion per user required to maintain a given error probability decreases as K increases. This reflects increased coding efficiency for higher constraint length codes.

(ii) The quasi-orthogonal schemes are more bandwidth efficient than orthogonal ones. This is of course, highly dependent on the choice of signal family and its correlation properties. The results justify the use of quasi-orthogonal schemes in preference to orthogonal ones.

(iii) The quasi-orthogonal schemes need smaller bandwidth expansion per user as n is decreased for a given K ($n > K/2$). However, a smaller n means that fewer simultaneous transmissions can be supported. A reasonable way to choose n and K for a quasi-orthogonal scheme is to select as large a K as decoder complexity permits and then select the smallest n which can support the required number of users below a specified P_E .

(iv) The MSSS Quasi-orthogonal is more bandwidth efficient than CSSS. However, it has an additional restriction on the maximum number of users $M_{\max} \leq 2^{2n-K} - 1$. It also requires a receiver to have i banks of matched filters each with 2^K filers to decode transmissions from i different users.

- V. A reservation scheme for multipoint message/packet switching using quasi-orthogonal tree codes and collision type request channels:

In previous sections, it has been shown that quasi-orthogonal tree codes can be used for code division multiple access on a wideband channel in a fixed assigned context. We now consider the use of tree codes schemes (MSSS and CSSS) for demand assignment or reservation multiple access.

The question of how to use these tree code schemes to permit multipoint user to user communication among a large number of low duty cycle subscribers arises next. Messages among users will be assumed to be formatted into fixed length (length = L) packets and could consist of one or more of these packets. Clearly, if the subscribers are not always active, it is inefficient to assign them to a fixed signal set in the MSSS. Again in CSSS, it may be desirable to limit the maximum number of active users in order to keep the error probability within acceptable limits. A possible solution that requires some network discipline is to demand assign the tree code schemes in much the same way as TDM or FDM channels. We assume that the system configuration allows at most ' c ' simultaneous coded transmissions. In the MSSS, this corresponds to ' c ' unique signal sets and in the CSSS, to ' c ' unique randomizing PN-sequences. A subscriber wishing to transmit must request a 'channel' and be assigned a unique signal set (in MSSS) or a unique PN sequence (in CSSS). We shall refer to the assignment of such a signal set or PN-sequence as a 'channel' for the subscribers transmission.

To realize such demand assignment, a mechanism for handling request traffic is required. For this purpose, we shall use c_r tree coded 'channels'

(out of the available 'c') for requests and c_m tree coded 'channels' for messages. All the channels are slotted and the c_r tree coded request channels are collision type - an user who wishes to place a reservation unconditionally transmits his request packet at the next request slot on any one of the ' c_r ' channels. If his packet collides with another, he retransmits after a random scheduling delay. Distributed control is assumed, where each user continually decodes all c_r request channels and maintains a data base from which he can unambiguously determine his transmission and reception times. A reservation packet will therefore contain the user ID# of the caller, the user ID# of the called party, the length of the message in packets as well as optional information such as priority. For traffic management, each subscriber maintains a list of active users, and a waiting user list as shown in Fig. 3. These lists are continuously updated on the basis of received request packets; active user entries are deleted at the end of transmissions.

These lists are sufficient for any subscriber to keep track of the state of the system. Moreover, they permit the use of an arbitrary scheduling algorithm other than FCFS or LCFS since each user has a list of waiting requests and their message lengths. Another advantage of maintaining the above list is that any potential caller knows in advance whether his intended called party will be busy when his turn arrives. This can eliminate the need for busy signals or acknowledgement traffic for this purpose.

A simpler realization would be to store only the number of packets for transmission using a LCFS or FCFS queue. This means that a user would need to set a flag to keep track of messages intended for him on hearing the request packet on a reservation channel. Another flag would

be needed to identify his own positions in the transmit queue. The data base would then be of the form shown in Fig. 4.

The latter scheme does not account for busy receivers and would mean an increase in traffic due to wasted transmissions, or some capacity dedicated to busy signal traffic. The benefit achieved is reduced storage at each subscriber, which could be substantial if a large number of users exist.

Before continuing, we mention another simplification of the data base if all messages consist of a single packet. Then, c_m requests are serviced every message channel slot. This allows a simple data base as shown in Fig. 5. Whenever the transmit or receive registers have 1's in any of the last c_m bits, the user must be ready to transmit or receive in the next message slot. The code (for CSSS) or signal set (MSSS) to be used is determined from the position of this '1'- i^{th} bit = 1 means i^{th} code or signal set is to be used.

VI. STEADY STATE ANALYSIS OF DEMAND ASSIGNED MODELS:

The traffic carrying behaviour of multiple channel demand assigned systems with collision type request channels has been considered in [4]. We note that in the type of system being considered here, the 'channels' are mutually interfering tree-coded transmissions (CSSS or MSSS) and there will hence be some interaction between traffic and detection performance.

We focus here on two basic cases which are important in the context of the previous sections. These are (a) multiple packets per message (b) single packet per message. The data bases required to maintain distributed control have already been outlined for the two cases. We shall assume sufficiently large storage at each user's queue to permit the assumption of an infinite waiting room.

For these configurations, we seek a characterization of traffic carried, packet error probability and delay encountered. The delay (d) will be taken as the time between first initiation of a request and the start of message transmission. The error probability (P_E) is the error probability for any packet in a message, where all packets are L bits long.

In Fig. 6 is shown the schematic representation of the flow of requests through the system. Collision errors in the request channels increase the effective traffic offered to them. We note that a fraction of the message traffic is wasted because of busy receivers, and rejoins the new requests thus increasing the effective request arrival rate.

Let c_m be the number of tree coded message 'channels' and c_r the number of tree coded request channels. λ_T is the rate at which new requests arrive and λ_a is the rate at which they are assigned. T_m is the average duration of a message and τ_m, τ_r are the message slot and

request.slot times respectively. The message channel occupancy or message traffic intensity (ρ_m), is hence equal to $\lambda_T T_m / c_m$. Referring to Fig. 6, we find that the rate at which requests (old and new) enter the system is given by:

$$\lambda_r = \frac{\lambda_T}{(1 - P_0)(1 - P_{\text{busy}})} \quad (16)$$

P_0 is the probability of a detectable error in the request packet.

Assuming that a collision will generate a detectable error

$$P_0 \approx P_c + P_{\text{RE}}(1 - P_c) \quad (17)$$

where P_c : probability of collision

P_{RE} : probability of detectable error in request packet due to 'noise'.

Typically $P_c \gg P_{\text{RE}}$, so that $P_0 \approx P_c$. It is known from [5] that the collision probability for a slotted Aloha channel with total offered traffic G is $(1 - \exp(-G))$. Thus, if we assume that a request channel is chosen randomly from the c_r available.

$$P_c = 1 - \exp(-\lambda_r \tau_r / c_r) \quad (18)$$

If we now define an activity factor called $R_a = 1 / (1 - P_c)(1 - P_{\text{busy}})$ and follow the analysis of [4]. We can express (16) as

$$\rho_m \gamma R_a \exp(-\rho_m \gamma R_a) = \rho_m \gamma / (1 - P_{\text{busy}}) \quad (19)$$

where $\gamma = (\tau_r / T_m) / (c_r / c_m)$

P_{busy} is the probability of a given subscriber being busy and can be assumed equal to ρ_m / M_T [4], where M_T is the total number of subscribers per message channel. The above equation then becomes

$$y \exp(-y) = \beta$$

$$\text{where } \beta = \rho_m \gamma / (1 - \rho_m / M_T) \quad (20)$$

$$\text{and } y \triangleq \rho_m \gamma R_a$$

This equation can be solved iteratively for y given β , and gives the required 'throughput' of each collision channel to maintain a traffic intensity ρ_m . Note that β is always $< 1/e$ for a stable collision type channel. It can be seen that the required throughput per channel decreases as the messages become longer or as the number of request channels is increased.

Next, we obtain error probability (P_E) and delay (d) for the two models being considered.

(i) Calculation of Delay:

The delay experienced by a request for transmission will be the sum of a request delay and a queuing delay. The request delay is the time taken by a request to enter the system queue and is due to the round trip delay (which can be substantial if a satellite is used) together with retransmission delays due to collisions. The queuing delay is the time for which a request must wait on the queue (after a successful reservation) before transmission. The request delay can be found from an analysis of delay in slotted Aloha channels, and is given by [11]

$$d_r = R + 1.5 + \left(\frac{\gamma}{\beta} - 1 \right) \left(R + .5 + \frac{K_t + 1}{2} \right) \quad (21)$$

where

R : Round trip delay in request packets

K_t : Retransmission interval

γ, β are as defined by (20)

This permits the request delay to be computed as a function of traffic intensity ρ_m . Note that $\gamma \triangleq (\tau_r/T_m)/(c_r/c_m)$ is given by $(\tau_r/n_p \tau_m)/(c_r/c_m)$ for the multiple packet message case (n_p = average number of packets per message), and by $(\tau_r/\tau_m)/(c_r/c_m)$ for the single packet case.

Queuing delay for multiple packet messages:

The number of request slots available in the length of an average message is (τ_m/τ_r) . $n_p \triangleq N \cdot n_p$. N is the ratio of message slot time and is typically of the order of 10-50 depending on the length of the message packets and the amount of information in the request packet. For a specific case when the message packet is 1000 bits long and the request packet 50 bits, $N = 20$. Thus if $n_p \gg 1$, $N \cdot n_p$ is fairly large and a queuing model with Poisson arrivals is justified, even though the arrivals are actually constrained to fixed time instants and to a finite number. Again, if the service time (number of packets per message) is assumed exponentially distributed, though only discrete values are possible, the queue under consideration can be modelled as a $M/M/c_m$ queue with infinite waiting room. The average delay experienced by arrivals to such a queue is well known [11]. In terms of the quantities of interest here, the queuing delay (d_q) is given by

$$d_q = \frac{C(c_m, \rho_m c_m)}{c_m (1 - \rho_m)} \cdot n_p \text{ message packets} \quad (22)$$

where

$$C(s, a) \triangleq \frac{a^s [(s-1)! (s-a)]}{s-1 \sum_{k=0}^{s-1} (a^k/k!) + a^s [(s-1)! (s-a)]} \quad (0 < a < s)$$

is the second Erlang function.

Queuing delay for single packet messages:

If all messages consist of a single packet, every message slot c_m calls can be serviced. However, now if we consider the arrival process, only $N = \tau_m/\tau_r$ request slots are available for requests to the system in the duration of a message slot. Hence, unless N is very large, the assumption of Poisson arrivals is not justified. This arrival process can be modelled as binomial with N trials and probability of a successful request in a given slot p_s . The service time is deterministic and service can start only at message slot times. The queue can then be modelled as a sampled binomial $/D/c_m$ queue. In [8,9], the sampled $M/D/c_m$ queue has been treated, but no closed form expression for average delay is available. The sampled $G/D/c_m$ case is also not well known. We have therefore obtained an upper bound for average delay for such a queue (Appendix II), and it has been found to be fairly accurate for $c_m \gg 1$, on the basis of numerically computed results in [8]. For binomial arrivals, the formula used is

$$d_q \leq 0.5 + \frac{1 - \rho_m c_m / N}{c_m (1 - \rho_m)} \quad (23)$$

(i) Utilization:

Since a fraction of the available bandwidth is used for request traffic, it is useful to consider a measure of the efficiency with which message traffic is carried. In a tree coded scheme, a given bandwidth expansion provides coding gain and the ability to multiplex a number of low duty cycle users. We define utilization (S) as:

$$S \triangleq \frac{\text{message traffic carried}}{\text{total bandwidth expansion}} \quad (24)$$

It can be noted that S approaches unity, in the case of perfectly scheduled uncoded TDM or FDM transmission. S can be used as a basis for comparing

different tree coded schemes.

In [4], maximization of utilization over ρ_m for blocking and holding systems is considered. Here we have a holding system for which maximum utilization is always achieved at $\rho_m = 1$. This may not, however, be the suitable operating point for the system because of intolerably large delay.

To calculate S for the demand assigned schemes using quasi-orthogonal CSSS and MSSS, we note that for a given n and K , the bandwidth expansion (using Gold codes) is $2^n - 1$. Thus, for a given ρ_m and c_m , the value of S is $\rho_m c_m / (2^n - 1)$. Using this with (21), (22), and (23), we have obtained delay for various schemes (different n , K ; CSSS & MSSS) as a function of the utilization (Fig. 7).

(iii) Calculation of error probability:

In Section III, we have indicated how to compute packet error probability for the fixed assigned quasi-orthogonal MSSS and CSSS multiple access schemes. In each case, the effective signal to noise ratio (S) at any receiver (given that M simultaneous transmissions are taking place) can be computed and error probability determined from S .

For the demand assigned schemes being considered, the error probability can be found by two methods. The first method computes P_E conditioned on M , the number of active users, and then averages over the distribution of M . The second method is to evaluate M_{av} , the average number of active users and use it in (8), (12) to find S and hence the error probability. The second approach is simpler and more compatible with the assumption of interference being modelled as Gaussian noise; accordingly, we evaluate P_E from M_{av} .

Now, for a given traffic intensity ρ_m , the total message traffic (a_m) offered is $\rho_m c_m / (1 - \rho_m / M_T)$; there is also some request traffic a_r , which is

given by $y \cdot c_r$, since y is the effective traffic being offered to the request channels. (Note that only $\beta \cdot c_r$ is useful request traffic, where β is the throughput of each collision channel as in (20)). The average number of active users is then equivalent to the total offered traffic and is given by

$$M_{av} = a_r + a_m = \frac{\rho_m c_m}{(1 - \rho_m / M_T)} + y \cdot c_r \quad (25)$$

Using the above, the packet error probability has been computed for both the common signal set scheme and the multiple signal set scheme as a function of utilization S . The difference between the single packet case and the multiple packet case is reflected by different y for the same ρ_m , since longer messages imply fewer requests and hence lower traffic on the request channels. Fig. 8 shows some results of the above calculations.

VII. DISCUSSION OF RESULTS FOR DEMAND ASSIGNED SCHEMES:

Figures 7 and 8 together can be used to characterize the performance of the schemes being considered. Both delay and error probability increase with utilization for any given choice of CSSS/MSSS, n and K . The delay is independent of whether CSSS or MSSS is used, but error performance differs. It can be observed that delay decreases, for low traffic intensity, as the number of packets per message increases, because of smaller request delay. In Fig. 7 it can be seen that to operate at a reasonable utilization and delay in the single packet scheme, a relatively large number of request channels are required. Stable operation of the request channels and system queue require $\beta = \rho_m \gamma / (1 - \rho_m / M_T) < 1/e$ and $\rho_m < 1$ respectively. Thus, the request channel capacity limits the utilization for small c_r/c_m and single packet messages. On the other hand, for long messages ($n_p \gg 1$), the utilization is limited by $\rho_m < 1$: the queuing delay then predominates and total delay is not substantially altered by changing c_r .

The error probability curves in Fig. 8 show that the error rate is lower for multiple packet ($n_p > 1$) messages, since the request channels need to carry less traffic, (24). Again, as for the fixed assigned schemes, we observe that higher K for a given n is more efficient, giving a lower P_E for a given utilization. Also the MSSS is found to be more efficient than the CSSS.

Fig. 9 combines the results of Figs. 7 and 8 to show delay-throughput curves with P_E as a parameter. It may be noted that the curves are not continuous and represent achievable values with different schemes. This can be of use in selecting system parameters such as n and K .

VIII: CONCLUSIONS:

The applicability of orthogonal and quasi-orthogonal tree codes to multiple access has been demonstrated. Quasi-orthogonal tree codes have been shown to be suitable for certain fixed assigned and demand assigned schemes.

APPENDIX I: CALCULATION OF PACKET ERROR PROBABILITY FOR ORTHOGONAL AND QUASI-ORTHOGONAL TREE CODES

Orthogonal tree codes are a special class of convolutional codes for which an optimum decoding algorithm has been presented by Viterbi in [3]. The performance of this code has been analysed in [3] for a channel with additive white Gaussian noise and coherent detection. An upper bound for the error probability for a packet of length L followed by $K-1$ synchronizing bits ($K =$ constraint length of the code) has been derived. In [1] we show that this bound can be generalized to a more general channel and formulae for a number of coherent and incoherent detection schemes were obtained. For example, coherent and incoherent detection for continuous, hard quantizing and 'greatest of' detection have been considered.

The formula used to compute the packet error probability is

$$P_E \leq LP_{1,K} + \sum_{j=1}^{L-1} (L-j) P_{2^{j-1} K+j} \quad (1)$$

where $P_{m,n}$ depends on the channel and detection scheme used. For additive white Gaussian noise and continuous incoherent detection,

$$P_{m,n} = \int_0^{\infty} dx \exp(-x) \cdot h(x) \quad (2)$$

where $h(x) = \{x^{n-1} / \Gamma(n)\} \{G_{2n}(2x)\}^{m-1} \{1 - Q_n(\sqrt{ns}, \sqrt{2x})\} \cdot m$

S: Signal to noise ratio

$G_{2n}(x)$: χ^2 c.d.f. with $2n$ degrees of freedom.

$Q_M(\alpha, \beta)$: Generalized Q-function.

Similar formulas were derived for hard quantizing and 'greatest of' detection, thus enabling error probability to be computed as a function of signal to noise ratio.

In [1], we suggest the use of quasi-orthogonal tree codes as a potential bandwidth saving artifice. The bound for error probability is still given by (1), but different expressions for $P_{m,n}$ apply. For continuous incoherent detection, using a quasi-orthogonal signal set with auto-correlation to cross-correlation equal to μ ,

$$P_{m,n} = \int_0^{\infty} dx p(x) \exp(-x) \quad (3)$$

where

$$p(x) = \left(\frac{2x}{n^2}\right)^{\frac{n-1}{2}} \exp\left(-\frac{n^2}{2}x\right) I_{n-1}(n\sqrt{2x}) \cdot \{1 - Q_n(n, \sqrt{2x})\}^{m-1} \\ \{1 - Q_n(\sqrt{nS}, \sqrt{2x})\} \cdot m$$

$$n \triangleq \frac{\sqrt{nS}}{\mu}$$

S: Signal to noise ratio

$Q_M(\alpha, \beta)$: Generalized Q-function

APPENDIX II: AVERAGE WAIT IN A SAMPLED G/D/c QUEUE

Consider a system in which 'c' servers synchronously process waiting 'customers' every T seconds. The service time is then deterministic (equal to T) and service can commence only at specific instants of time (sampled). Let us denote the number of arrivals between the i^{th} clock instant and the $i+1^{\text{th}}$ instant as k_i (input arrival process), and the number of departures as α_i . If x_j is the number of customers in the queue

just prior to the j^{th} clock instant,

$$x_{i+1} = x_i - \alpha_i + k_i \quad (1)$$

define

$$v_i \triangleq x_i - \alpha_i \quad (2)$$

now

$$\begin{aligned} \alpha_i &= x_i \text{ if } x_i < c \\ &= c \text{ if } x_i \geq c \end{aligned} \quad (3)$$

Hence, the generating function of v_i is

$$G_{v_i}(z) = \sum_{v_i=0}^{\infty} p(v_i) z^{v_i} = \sum_{x_i=0}^{c-1} p(x_i) z^0 + \sum_{x_i=c}^{\infty} p(x_i) z^{x_i-c} \quad (4a)$$

In the steady state, the subscript i can be dropped, yielding

$$G_v(z) = \sum_{x=0}^{c-1} p(x) + \sum_{x=c}^{\infty} p(x) z^{x-c} \quad (4b)$$

Defining p_c as the probability that $(x < c)$, we obtain for the generating function of x

$$G_x(z) = G_v(z) G_k(z)$$

$$\begin{aligned} &= \frac{G_k(z) [p_c - z^{-c} \sum_{x=0}^{c-1} p(x) z^x]}{1 - z^{-c} G_k(z)} \\ &= \frac{G_k(z) \sum_{x=0}^{c-1} (z^c - z^x) p(x)}{z^c - G_k(z)} \end{aligned} \quad (5)$$

Now, the expected value of x is given by $\frac{d}{dz} G_x(z) \Big|_{z=1}$. This can be written as

$$G'_x(z) = N/D \quad (6)$$

where

$$\begin{aligned} N = & [G_k(z) \sum_{x=0}^{c-1} (cz^{c-1-x} - xz^{x-1}) p(x) \\ & + G'_k(z) \sum_{x=0}^{c-1} (z^c - z^x) p(x)] [z^c - G_k(z)] \\ & + [G_k(z) \sum_{x=0}^{c-1} (z^c - z^x) p(x)] [cz^{c-1} - G'_k(z)] \end{aligned} \quad (7)$$

and

$$D = (z^c - G_k(z))^2 \quad (8)$$

Now, since $G_k(1) = 1$, $\{N/D\}_{z=1}$ is of the form $0/0$. Applying L'Hospital's rule twice and performing considerable algebraic manipulation, we obtain

$$\begin{aligned} N'' \Big|_{z=1} = & [G_k(1) \gamma - G_k(1)\beta + 2G'_k(1)\beta] [c - G'_k(1)] \\ & - [G_k(1)\beta] [c^2 - c - G''_k(1)] \end{aligned} \quad (9)$$

$$\text{and } D'' \Big|_{z=1} = 2(c - G'_k(1))^2 \quad (10)$$

where

$$\beta \triangleq \sum_{x=0}^{c-1} (c-x) p(x) \quad (11a)$$

and

$$\gamma \triangleq \sum_{x=0}^{c-1} (c^2 - x^2) p(x) \quad (11b)$$

Noting that $G_k(1) = 1$, $G''_k(1) = \mu_k^2 \triangleq E[k]$ and $G'_k(1) = \sigma_k^2 + \mu_k^2 - \mu_k$ we obtain

$$E[x] \triangleq \mu_x = \frac{\{\gamma + 2\mu_k\beta - c^2 + \sigma_k^2 + \mu_k^2\}\beta}{2(c - \mu_k)^2} \quad (12)$$

Now, β can be determined from steady state conditions. Since $E(x_{i+1}) = E(x_i)$

$$E(x_{i+1}) = E(x_i - \alpha_i + k_i) \quad (13a)$$

$$E(\alpha_i) = E(k_i) \quad (13b)$$

Thus $c - \sum_{x=0}^{c-1} (c-x) p(x) = \mu_k$

and

$$\beta \triangleq \sum_{x=0}^{c-1} (c-x) p(x) = c - \mu_k \quad (13c)$$

Substituting this in (12), we obtain

$$\mu_x = \frac{\gamma - c^2 + \sigma_k^2 + 2c\mu_k - \mu_k^2}{2(c-\mu_k)} \quad (14)$$

γ cannot be easily evaluated from steady state conditions. However, if we can upper bound γ , an upper bound on μ_x is obtained. Consider,

$$E(x_{i+1}^2) = E(x_i^2 + \alpha_i^2 + k_i^2 - 2k_i \alpha_i - 2x_i \alpha_i + 2x_i k_i) \quad (15)$$

Since

$$E(x_{i+1}^2) = E(x_i^2) \text{ is steady state}$$

$$E(\alpha_i^2) + E(k_i^2) - 2E(k_i) E(\alpha_i) - 2E(\alpha_i x_i) + 2E(x_i k_i) = 0 \quad (16a)$$

Noting that $E(\alpha_i) = E(k_i)$ in steady state

$$\sum_{x=0}^{c-1} x^2 p(x) + (1-p_c)c^2 + \sigma_k^2 + \mu_k^2 - 2\mu_k^2 - 2E\{x_i \alpha_i - x_i k_i\} = 0 \quad (16b)$$

$$\sum_{x=0}^{c-1} (c^2 - x^2)p(x) = c^2 + \sigma_k^2 - \mu_k^2 - 2\{E(x_i \alpha_i) - E(x_i k_i)\}$$

Again

$$E(x_{i+1} x_i) = E(x_i^2 - x_i \alpha_i + x_i k_i) \quad (17)$$

$$2E\{x_i \alpha_i - x_i k_i\} = 2E\{x_i^2 - x_i x_{i+1}\} \geq 0$$

Hence

$$\gamma \leq c^2 + \sigma_k^2 - \mu_k^2 \quad (18)$$

Substituting in (12), μ_x can be upper bounded as

$$\mu_x \leq \mu_k + \frac{\sigma_k^2}{c - \mu_k} \quad (19)$$

Average waiting time is found invoking Little's formula which gives

$$\frac{t_w}{T} \leq \frac{\mu_x}{\lambda T} \quad (20)$$

Poisson arrivals:

For the case of Poisson arrivals, $\mu_k = \lambda T$ and $\sigma_k^2 = \lambda T$. Thus,

$$\frac{t_w}{T} \leq 1 + \frac{1}{c(1 - \frac{\lambda T}{c})} \quad (21)$$

If we take into account the fact that new arrivals wait only half a clock interval on the average and not one clock interval as assumed in the analysis before they qualify for service.

$$\frac{t_w}{T} \leq .5 + \frac{1}{c(1 - \frac{\lambda T}{c})} \quad (22)$$

Binomial Arrivals:

Consider the case of binomial arrivals. If N arrival 'slots' are available during one clock interval, where the probability of an arrival in any given one is p_s , the arrival process k is binomial with the distribution

$$\text{prob}\{k=m\} = {}^N C_m p_s^m (1-p_s)^{N-m} \quad (23)$$

Then, $\mu_k = Np_s$ and $\sigma_k^2 = Np_s(1-p_s)$. Also $\lambda T = \mu_k$ since λT is the average number of arrivals in a block interval. Hence,

$$\frac{t_w}{T} = 1 + \frac{1 - \frac{\lambda T}{N}}{c - \lambda T} \quad (24a)$$

$$= 1 + \frac{1 - \rho \frac{c}{N}}{c(1 - \rho)} \quad (24b)$$

where $\rho = \frac{\lambda T}{c}$ is the traffic intensity.

Correcting for the random arrival time within a clock duration,

$$\frac{t_w}{T} \leq .5 + \frac{1 - \rho \cdot c/N}{c(1 - \rho)} \quad (25)$$

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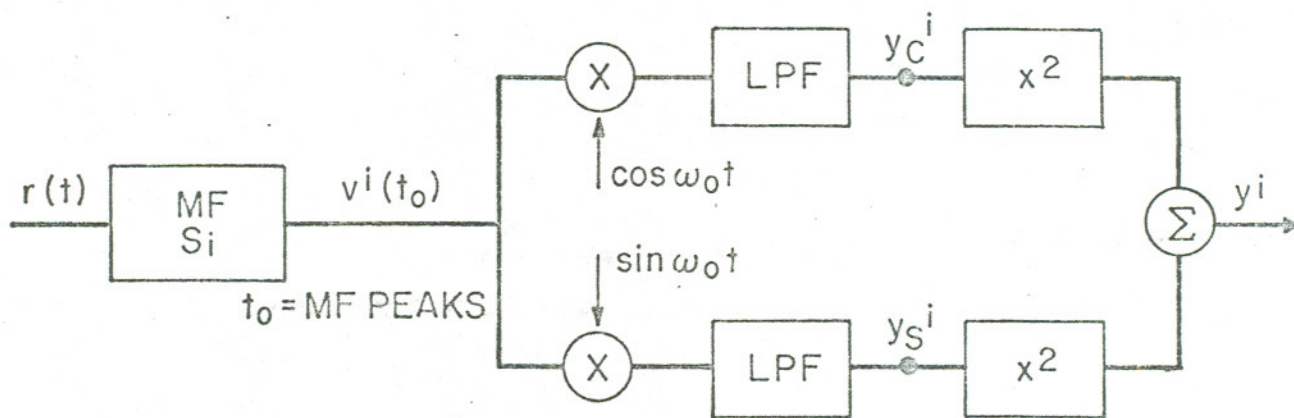


Fig. 1 EXPANDED VIEW OF CHANNEL "i" FOR INCOHERENT RECEIVER

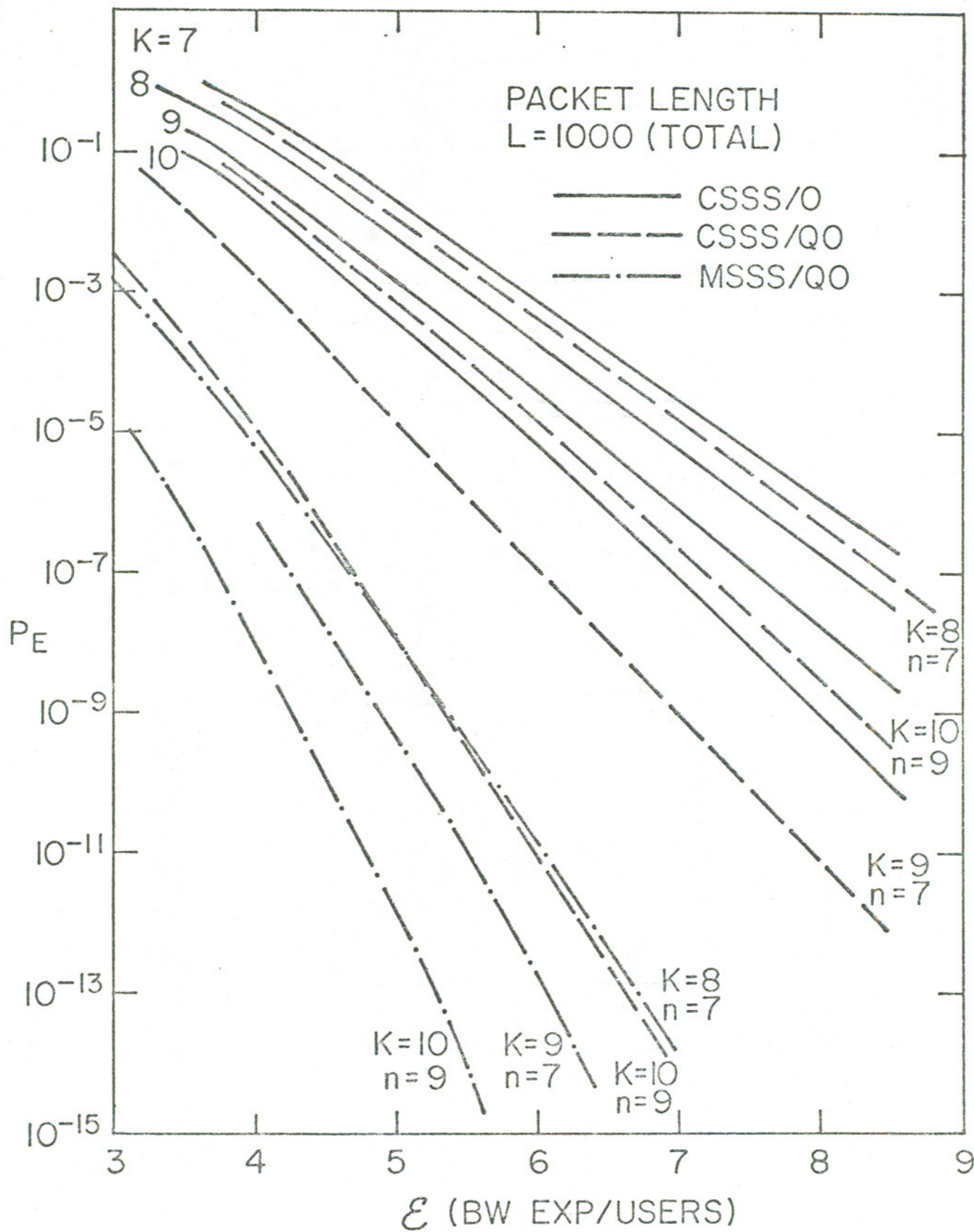


FIG. 2. PERFORMANCE OF FIXED ASSIGNED SCHEMES

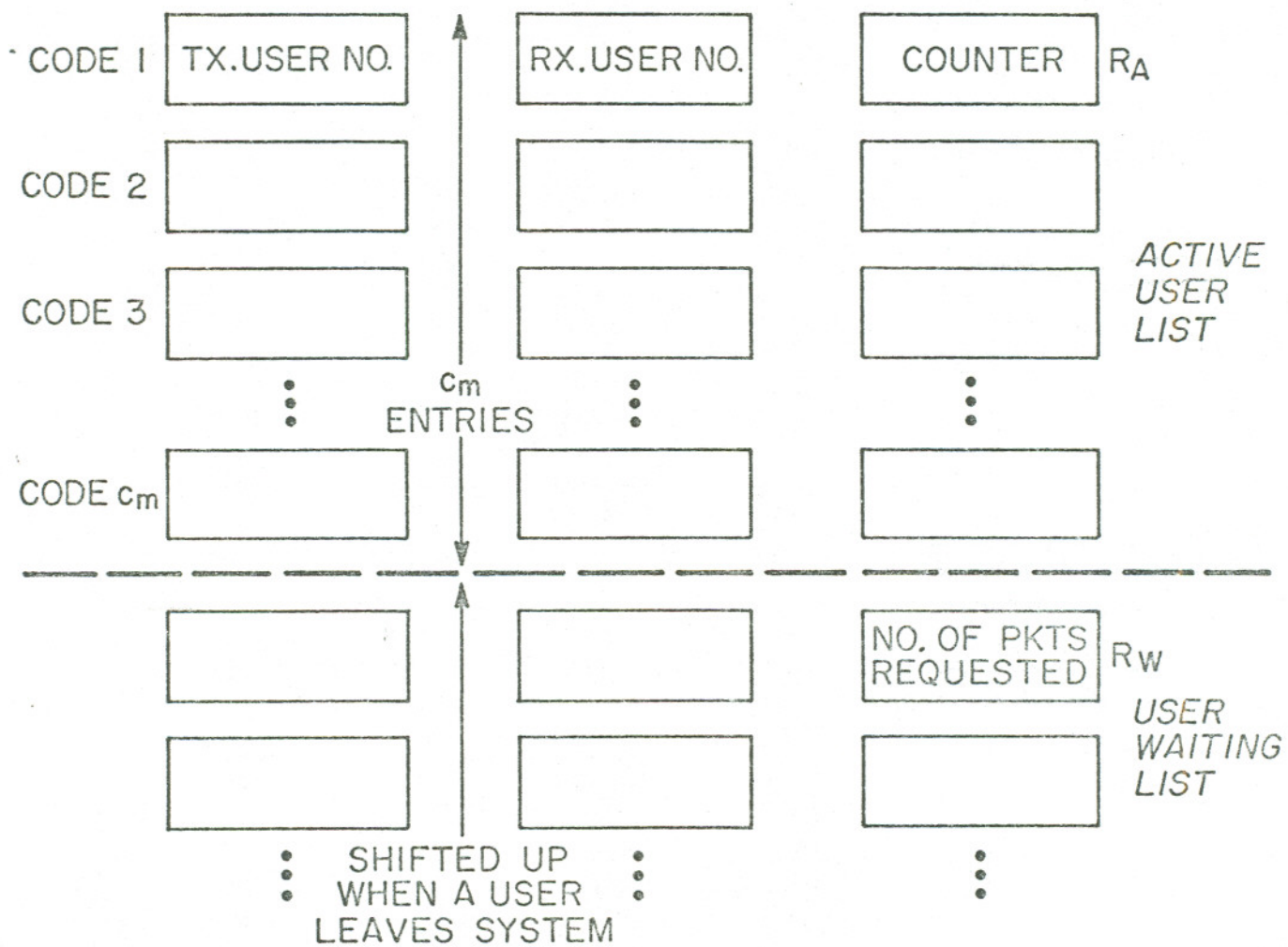


FIG. 3. DETAILED DATA BASE FOR DISTRIBUTED CONTROL

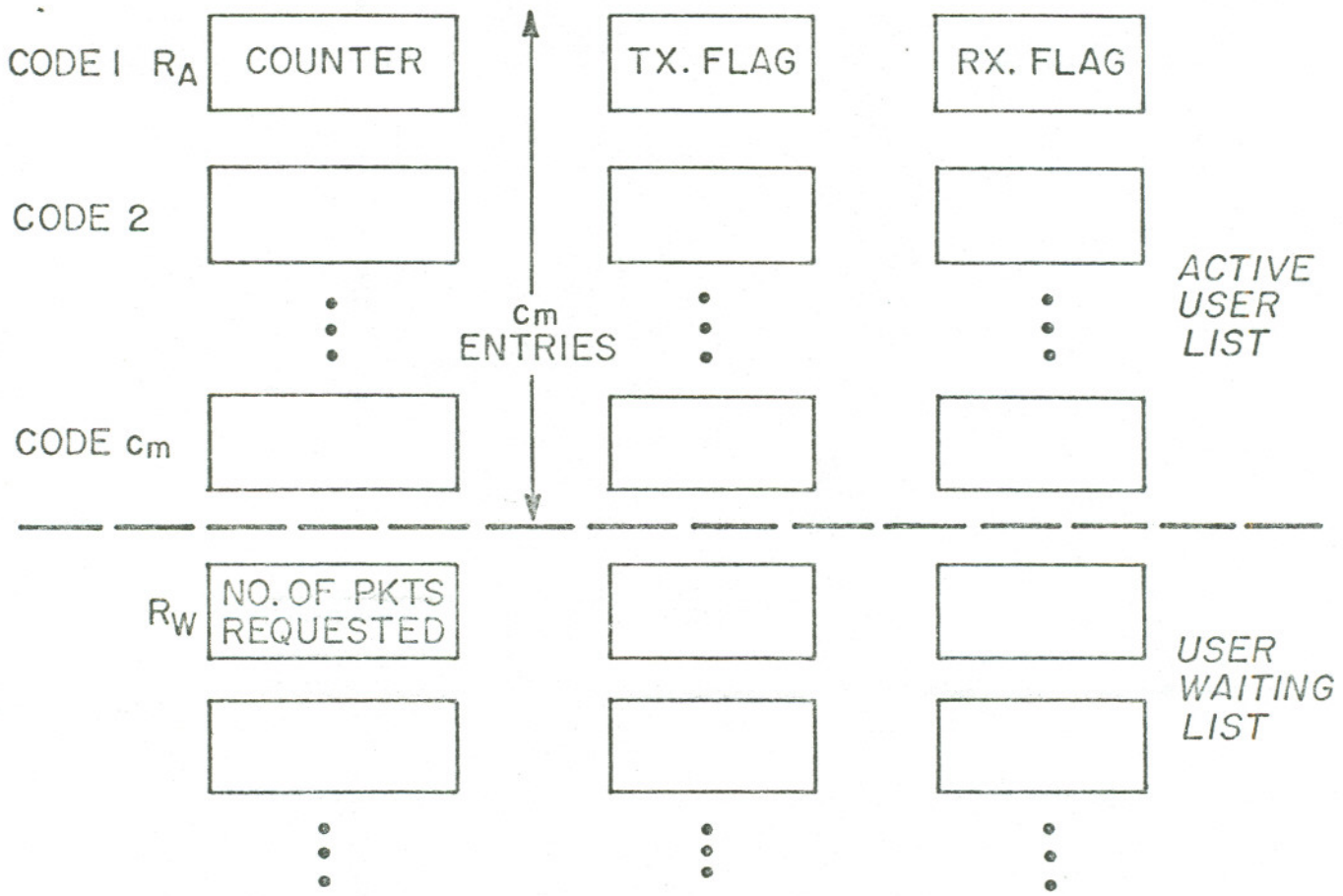


FIG. 4. SIMPLIFIED DATA BASE FOR MULTIPLE PACKET MESSAGES

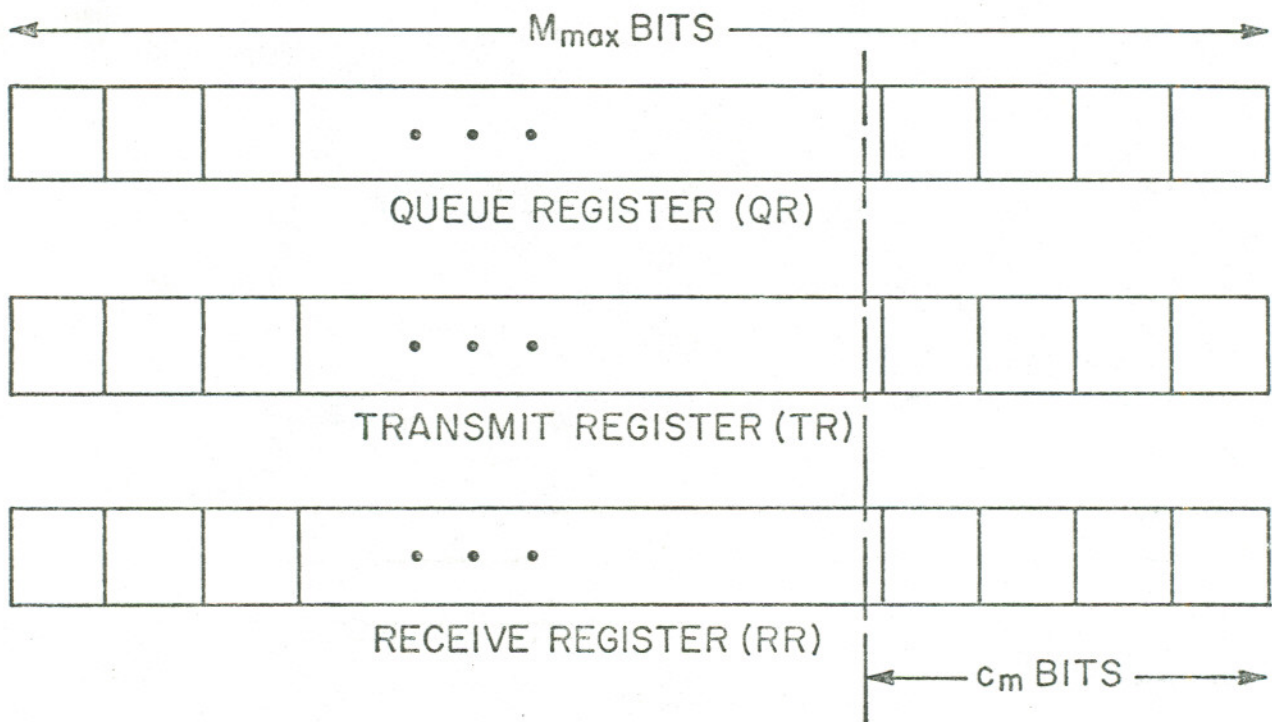


FIG. 5. SIMPLIFIED DATA BASE FOR SINGLE PACKET MESSAGES

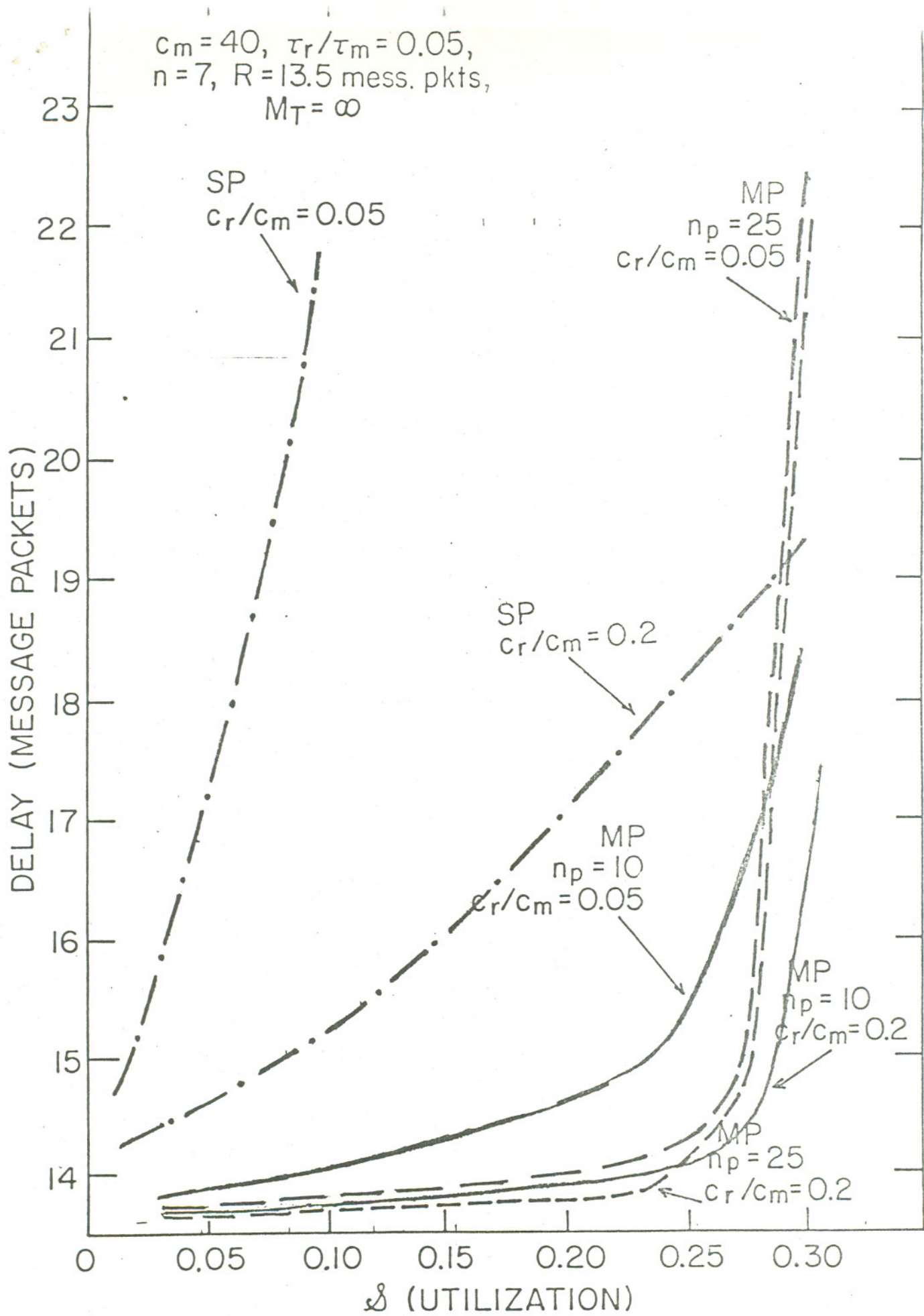


FIG. 7. DELAY CHARACTERISTICS OF DEMAND

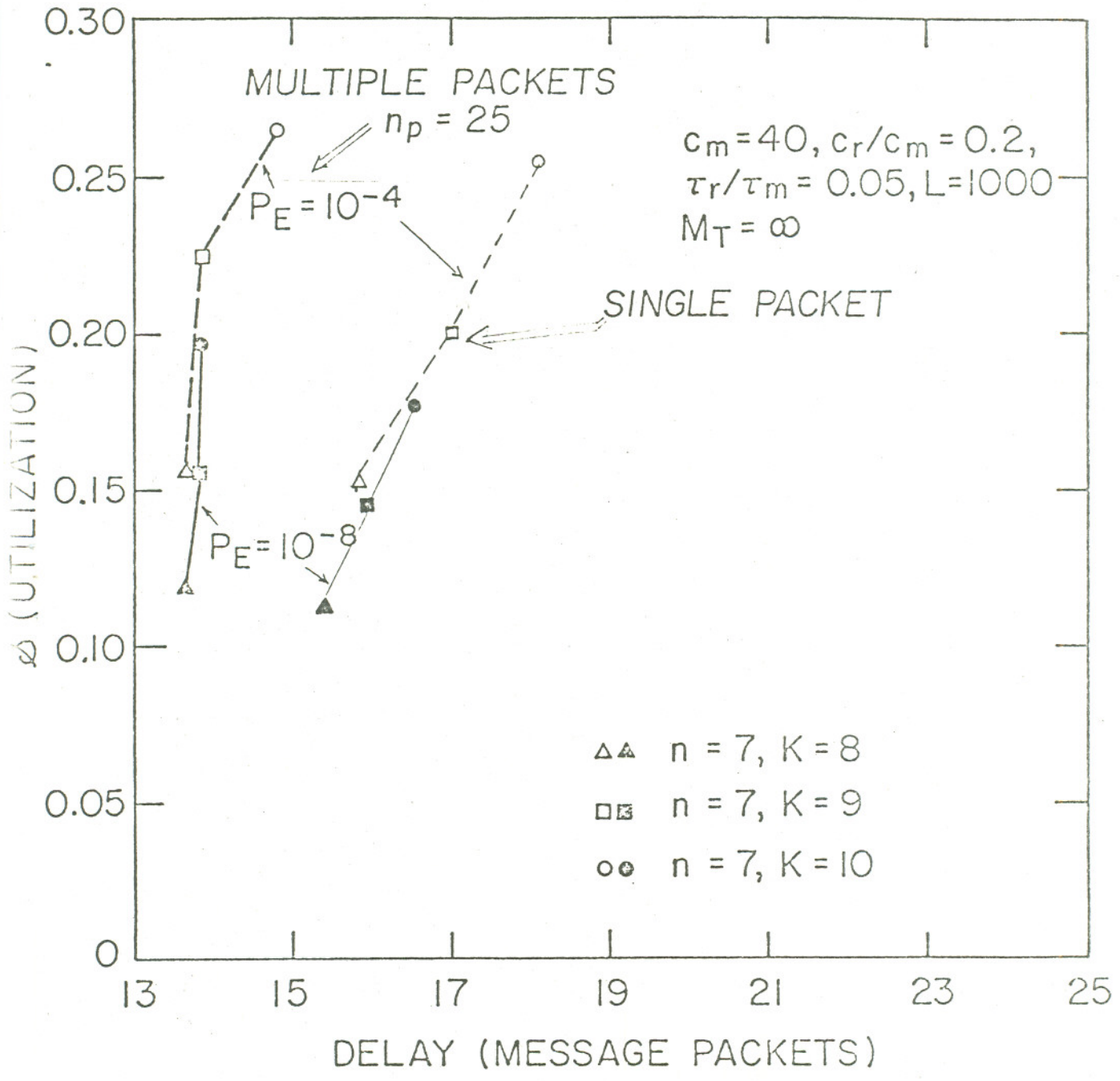


FIG. 9. UTILIZATION-DELAY CHARACTERISTICS OF DEMAND ASSIGNED (CSSS QUASI-ORTHOGONAL) SCHEMES.