CEAS TECHNICAL REPORT NO. 652 January 7, 1993

GENERALIZED FIXED CHANNEL ASSIGNMENT WITH HAND-OFF PRIORITY IN MICRO-CELLULAR COMMUNICATION SYSTEMS

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Abstract

Overlapping coverage areas of nearby base stations arise naturally in cellular communication systems - especially in small-cell high-capacity micro-cellular configurations. Because of overlap, users in at least some parts of the service area may have access to channels at more than one base station. With an appropriate control strategy, this enhanced access can be used to improve teletraffic performance characteristics.

We consider a Generalized Fixed Channel Assignment Scheme with priority for hand-off calls. The scheme systematically accounts for and exploits coverage overlap in its channel assignment strategy. An analytically tractable model is developed for a linear array of cells, such as those that might be used along a highway.

A suitable state characterization is identified which allows the problem to be treated in the framework of multi-dimensional birth-death processes. The effects of shadow fading are included in the model. Overall performance characteristics that are computed account for both signal availability and channel availability. Theoretical performance characteristics show Blocking Probability, Forced Termination Probability, Carried Traffic and Hand-Off Activity.

The research reported in this paper was supported in part by the U.S. National Science Foundation under Grant No. NCR-9025131 and in part by IST/SDIO under Grant No. N00014-91-J-4063 administered by the U.S. Office of Naval Research. Some of the results were obtained using the Cornell National Supercomputer Facility.

1 Introduction

In cellular communication systems, channels are allocated to base stations by various channel assignment schemes. There are three categories: fixed channel assignment (FCA), dynamic channel assignment (DCA) and hybrid channel assignment (HCA). Basically they assume that a call is served by a base station that provides best signal quality in some sense. In practice a mobile station may be able to establish a communication link of acceptable quality with more than one base. This results in overlapping coverage (usually) by nearby base stations [1]. This overlap can be used to advantage. Firstly, if a base is fully occupied or even fails, a call that originates in a region of overlapping coverage will still have a chance for service. Secondly, if there is a hot spot, the neighboring base stations can share the loading. With appropriate system control strategies the overlapping coverage can be used to improve traffic performance characteristics, such as blocking probability and carried traffic. Several schemes that utilize the overlapping coverage have been suggested [2]-[5]. In [2] a Generalized Fixed Channel Assignment (GFCA) scheme was discussed. GFCA allows a call to be served by any of the nearest k base stations. Directed retry and load sharing was analyzed in [3] and [4]. In [5] highway microcells with an overlayed macrocell was analyzed.

Here we consider exploiting overlapping coverage areas using GFCA in a micro-cellular environment when hand-off is important. A linear array of cells along a highway is an example. Other work dealing with this configuration appears in [6],[7]. It may be advantageous to give priority access to hand-off calls even at the cost of increased blocking of new calls. This is because hand-off needs that fail result in interruptions of calls. Some work [8]-[11] has focused on communication traffic performance and hand-off issues, but significant overlapping coverage areas such as those that can arise in GFCA or micro-cellular layouts were not considered.

Another important characteristic of the mobile environment is fading. To receive service through a base station it is necessary for a mobile to be able to establish a link of acceptable quality (to that base). In addition a channel at that base must be available to accommodate the call when needed. Thus, signal availability as well as channel availability should be considered.

We focus our attention on performance analysis of cellular systems using generalized fixed channel assignment with hand-off priority in a shadow fading environment. A suitable state representation is developed to treat the problem using multi-dimensional birth-death processes. An iterative method is used to solve a set of implicit nonlinear equations to find system state probabilities and theoretical performance characteristics. Performance measures of interest include Blocking Probability, Forced Termination Probability, Hand-Off Activity and Carried Traffic. Example numerical results for different system parameters are displayed and discussed.

2 Model Description

2.1 Signal Model

The transmission path between base station and mobile station is generally characterized by rapid Rayleigh fading on a slowly varying mean signal strength. The fluctuation of the mean is called shadow fading and typically follows a log-normal distribution [12],[13],[14]. Since space diversity is quite effective in mitigating Rayleigh fading, we focus on the log-normal shadowing effect for signal availability consideration. Signal to Interference Ratio (SIR) is usually used to characterize signal quality. There are two kinds of SIR in cellular systems: down-link SIR (from base to mobile) and up-link SIR (from mobile to base). Even they are not exactly the same, they are related. For convenience we assume that if a mobile can establish an acceptable down-link to a base, it may also establish an acceptable up-link to that base. Therefore, we consider only down-link SIR. Thus mobile stations receive interference from all the base stations which use the same base-to-mobile channel(co-channel base stations). Co-channel base stations that have about the same distance from the serving base can be considered as one tier of interferers. Since interference is dominated by the interferers closest to the mobile, for computational purposes consideration of two or three tiers is sufficient. In this paper we consider three tiers. Thus we consider the nearest six interferers because one tier interference has two co-channel base stations for one-dimensional highway case. Fig.1 shows this relationship.

We take acceptable signal quality to mean that SIR is above the required threshold, R. Since SIR is a normal random variable, the fraction of time for which SIR is below R at some particular position with distance x from the serving base was defined as the link outage probability $P_o(x)$ [14].

$$P_o(x) = \Phi(\frac{R - m_{sir}(x)}{\sigma_{sir}(x)}) \tag{1}$$

where the threshold R is measured in dB; m_{sir} and σ_{sir} are mean and standard deviation of SIR and Φ is the cumulative distribution function of a normal variate with zero mean and unit variance. That is, $\Phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} exp(-z^2/2)dz$. This link outage probability only characterizes the link to *one* base station. In our model, since we consider overlapping coverage, outage of one link does not necessarily mean that a call can not gain access to a channel. Note that $P_o(x)$ is a function of distance x of the mobile from the base that we consider. This is because m_{sir} and σ_{sir} are dependent on x. Details are given in Appendix A.

2.2 System Layout

Consider a highway with vehicular traffic streams heading in each of two directions (say, left and right). Base Stations are installed along the highway and separated by a distance L. The configuration is shown in Fig.2a. In Fig.2a, base l and base r are the left and right base stations; base ll is the base station on the left of base l and base rr is the base station on the right of base r. The total number of channels allocated to the system are equally divided into several groups. The number of groups is taken as the cluster size, N, which is a system parameter that is determined largely by the co-channel interference that can be tolerated. Each group of channels is assigned to one base station in such a way that minimizes co-channel interference. As a result, each base station has C channels. In addition, at each base station C_h channels are reserved to serve hand-off calls exclusively. Specific channels are not reserved, only a certain number.

For a highway cellular system consisting of a linear array of cells it is reasonable to organize the system so that calls from a significant part of the coverage area can be served by up to two base stations. That is, $k = 2$ in the usage presented in [2] for GFCA. Since distance between serving base and mobile station is a major factor in signal strength and quality, the overlapping coverage is primarily in some central area between two adjacent base stations. Thus as shown in Fig.2a, there are overlapping and non-overlapping areas. The coverage area of a base station is the area in which calls can be served by that base station. As shown in Fig.2a, it consists of one non-overlapping area and two overlapping areas.

Since the system is limited by co-channel interference, the *absolute* distance is not important in evaluating SIR. The distance between two adjacent base stations, L, is normalized to be 2. In addition we put the origin at base l. Consequently, all base stations are located at even numbered positions. The overlapping width is also normalized to be $2w$. Due to symmetry the overlapping area between 0 and 2 is located from $1 - w$ to $1 + w$. Thus w is the ratio of overlapping width, $2w$, to the distance between two adjacent bases, 2. The layout is shown in Fig.2b.

It is convenient to use some notion of a boundary between overlapping and non-overlapping areas. This is only for modeling and calculation purposes. In practice the boundaries are not rigid. Both in practice and in the model presented here, mobile users need not know their position with respect to the base stations. All actions are based on measurements of signal quality. The boundary between overlapping and non-overlapping areas is determined by the probability distribution of the *difference* in SIR *from two adjacent base stations.* Since the overlapping area is in the central area between two adjacent bases, there are two boundaries between base l and base r. Let SIR_l be the SIR of the link to base l and SIR_r be the SIR of the link to base r. The difference in SIR between base l and base r is $SIR_l - SIR_r$ or $SIR_r - SIR_l$. In Appendix A it is shown that this difference is a normal random variable. We define the boundaries in terms of a threshold D and a probability H_D, both of which are system parameters. Thus the boundary near base l is the position where the following equation holds:

$$Prob\{SIR_l - SIR_r > D\} = H_D \tag{2}$$

And the boundary near base r is the position where the following equation holds:

$$Prob\{SIR_r - SIR_l > D\} = H_D \tag{3}$$

For example, if $D = 10$ dB and $H_D = 80\%$, the boundary near base r is the position where $Prob\{SIR_r - SIR_l > 10\text{dB}\} = 80\%$. Fig 3.1 shows the probability for which $SIR_r - SIR_l$ is greater than 10 dB as a function of the mobile position between base l and base r. The effect of different threshold D is also shown in Fig 3.2.

We use the link outage probability to characterize the signal quality. Since the link outage probability depends on mobile position, for convenience we divide the coverage area of a base station into several segments. Within each segment, the average signal quality is characterized. That is we use the average link outage probability to approximate the continuous outage probability curve. This approximation is shown in Fig.4. This will be further discussed in next section.

The non-overlapping area is taken as one segment and area of overlap is divided into m segments ($m = 1, 2, 3 \cdots$) because the outage probability changes faster (with respect to distance) in the area of overlap. Therefore the coverage area of a base station can have 3, 5, $7 \cdots 2m{+}1$ segments. These segments are labeled from $-m$ to m. The base station is located in seg_0. For convenience we consider only the three-segment case, i.e. $m = 1$. For $m = 1$, the coverage area of a base station is divided into three segments: seg_{-1}, seg_0 and seg_1. Both seg_{-1} and seg_1 are overlapping segments and seg_0 is a non-overlapping segment. Because of overlap seg_1 of base l and seg_{-1} of base r actually correspond to the same physical area. It is shown in Fig.2b. A call arises in this area may establish an acceptable link to either base.

2.3 Signal Availability

It is important to characterize signal availability, which is different for new calls and hand-off calls. First consider new calls. There are two kinds of new calls distinguished by where the calls arise. A new call that arises in an overlapping area may have signal access to either the left or right base station. Because of shadow fading one of the four following conditions will prevail at the time of new call arrival: 1) Signal quality of both links are acceptable. 2) Signal quality of the link to right base is acceptable but that for left base is not. 3) Signal quality of the link to left base is acceptable but that for right base is not. 4) Signal quality of both links are unacceptable. We define the events Q_i corresponding to these conditions. The symbol Q_i denotes the event condition labeled i as shown above. It is assumed that the signal quality of both links are independent. Then refer to Fig.2b, for a mobile at some position x from the left base station the probability of Q_1 is $[1 - P_o(x)][1 - P_o(2 - x)]$. This applies for a mobile at some position x in the region of overlap. Similar expressions can be obtained for the probability of Q_2, Q_3 and Q_4. These probabilities are averaged over the area of overlap. Thus we can calculate the average probabilities of Q_1, Q_2, Q_3 and Q_4, P_{Q_1}, P_{Q_2}, P_{Q_3} and P_{Q_4} as follows:

$$P_{Q_1} = \frac{1}{2w} \int_{1-w}^{1+w} [1 - P_o(x)][1 - P_o(2 - x)]dx \tag{4}$$

$$P_{Q_2} = \frac{1}{2w} \int_{1-w}^{1+w} P_o(x)[1 - P_o(2 - x)]dx \tag{5}$$

$$P_{Q_3} = \frac{1}{2w} \int_{1-w}^{1+w} [1 - P_o(x)]P_o(2 - x)dx \tag{6}$$

$$P_{Q_4} = \frac{1}{2w} \int_{1-w}^{1+w} P_o(x)P_o(2 - x)dx \tag{7}$$

Due to symmetry P_{Q_2} equals to P_{Q_3} and of course, $P_{Q_1} + P_{Q_2} + P_{Q_3} + P_{Q_4} = 1$.

For a new call that arises in a non-overlapping area we have two events to consider. Specifically, Z_1: signal quality of the link is acceptable; and Z_2: signal quality of the link is unacceptable. For a mobile at position x in the non-overlapping area, the probability of Z_2 is just the link outage probability $P_o(x)$. This must be averaged over the non-overlapping area to get the average probability of Z_2, P_{Z_2}:

$$P_{Z_2} = \frac{1}{1-w} \int_{0}^{1-w} P_o(x)dx \tag{8}$$

Consequently the average probability of Z_1 is $1 - P_{Z_2}$.

Now consider signal availability for hand-off calls. When a mobile leaves the coverage area of the serving base station, a hand-off need occurs. At this time the signal quality of the link to the target base may be acceptable or not. The probability of not having an acceptable target link is just the link outage probability for target base at the boundary of coverage area of current base. From (1) this is given by $P_o(1-w)$ since the distance between the boundary of current base and the target base is $1-w$. Of course, the probability of having acceptable target link is $1-P_o(1-w)$. When the SIR falls below the threshold for a call served in the area of overlap, a hand-off to a neighboring base will also be needed. There are also two conditions for signal quality consideration. Y_1: Signal quality of the target link is acceptable. Y_2: Signal quality of the target link is unacceptable. In a manner similar to that described above for new calls, we can calculate the average probability of Y_2, P_{Y_2} using

$$P_{Y_2} = \frac{1}{2w}\int_{1-w}^{1+w} P_o(x)dx \tag{9}$$

Similarly, the average probability of Y_1 is 1-P_{Y_2}. If SIR falls below the threshold for a call served in non-overlapping area, it will be terminated since there is only one base station available. Thus it is not necessary to characterize the signal availability for this kind of situation.

To evaluate the average probabilities defined in (4)-(9), we used Gaussian Quadrature numerical integration [15].

2.4 Traffic Model

The demands on base station resources are a result of new call arrivals, call completions, hand-off call arrivals and hand-off call departures. It should be noted that new calls include not only those that arise at the mobile but those for which a mobile is the called party. Our model assumes that a small fraction of the population of mobile stations need service at any time. As a result we have a constant new call arrival rate. We define a sector as the area between two adjacent base stations. The new call arrival rate per *sector* is denoted by Λ_S and Poisson arrivals are assumed. The unencumbered call duration has a negative exponential density function with mean $1/\mu_c$.

There are two events that result in hand-off departures: 1) The mobile station leaves the coverage area of the current base and 2) SIR of current link fluctuates below the threshold. The time duration a mobile station stays in a segment is defined as residing time. It is assumed to have a negative exponential distributed function with mean $1/\mu_{do}$ and $1/\mu_{dn}$ for overlapping and non-overlapping segments respectively. Here we define another system parameter V_m as mean speed of mobile stations. Then the mean residing time for an overlapping segment, $1/\mu_{do}$, is calculated as follows:

$$1/\mu_{do} = w \cdot L/V_m \tag{10}$$

Similarly the mean residing time for a non-overlapping segment, $1/\mu_{dn}$, is calculated by

$$1/\mu_{dn} = (1-w) \cdot L/V_m \tag{11}$$

We also assume that the duration from a call gains access to a channel to it experiences insufficient SIR has negative exponential density function with mean $1/\mu_{lo}$ or $1/\mu_{ln}$ for overlapping and non-overlapping segments respectively. It is important to determine appropriate value for $1/\mu_{lo}$ and $1/\mu_{ln}$. We focus on $1/\mu_{lo}$ first. This is the mean time that a non-blocked call in overlapping segments will lose its acceptable link. Consider Fig.5a which is a SIR profile experienced by mobile at some particular position. From this profile we can calculate the average duration between two consecutive upcrossing (the threshold R) points. Actually we can calculate it as the reciprocal of the average level crossing rate (lcr). From [16], we know that if α is SIR in dB and l is a given level in dB, the rate of crossing level l in segment k (k=-1,0,1) is

$$lcr(l,k) = \sigma_{\dot{\alpha}}/(2\pi\sigma_{\alpha(k)}) \cdot exp(\frac{-(l-m_{\alpha(k)})^2}{2\sigma^2_{\alpha(k)}}) \tag{12}$$

where $\sigma_{\alpha(k)}$ and $\sigma_{\dot{\alpha}}$ are the standard deviation of $\alpha(k)$ and $\dot{\alpha}$(derivative respect to time); and $m_{\alpha(k)}$ is the mean of $\alpha(k)$ ($m_{\dot{\alpha}} = 0$). Note that actually the lcr depends on the mobile position. Since we use the average signal quality within a segment, the lcr now depends on the segment.

In Fig.5b we consider one cycle of this average duration. It consists of two parts: one is above the threshold and another is below it. The ratio of insufficient SIR duration to the whole average cycle duration is just the average link outage probability of the serving base. Thus the acceptable signal duration is (1-$\overline{P_o}(k)$)$(1/lcr(R,k))$, where $\overline{P_o}(k)$ is the average link outage probability for seg_k. Due to symmetry $\overline{P_o}(-1)$ equals to $\overline{P_o}(1)$ because both of them are for overlapping segments. Since the time instant when a non-blocked call access to a channel is uniformly distributed within the sufficient SIR duration, we can determine $1/\mu_{lo}$ as follows:

$$1/\mu_{lo} = \frac{1}{2} \cdot \frac{1}{lcr(R,1)} \cdot (1 - P_{Y_2}) \tag{13}$$

Similarly,

$$1/\mu_{ln} = \frac{1}{2} \cdot \frac{1}{lcr(R,0)} \cdot (1 - P_{Z_2}) \tag{14}$$

For hand-off arrivals we can use hand-off departures to characterize them since any hand-off arrival is from another hand-off departure in the system.

3 State Characterization

Because of the homogeneous property of the system, the statistical behavior in equilibrium of all base stations are identical, though their behavior are not independent. We define a Basic Element (BE) as the area served by two adjacent base stations. It consists of three overlapping areas and two non-overlapping areas. This is shown in Fig.2b.

Since we focus on a basic element, the state s is characterized by six non-negative integers for three-segment case:

$$l_{-1}(s), l_0(s), l_1(s); r_{-1}(s), r_0(s), r_1(s) \tag{15}$$

Where $l_k(s)$ is the number of calls served by base l in segment k $(k = -1, 0, 1)$ for state s, and $r_k(s)$ is the number of calls served by base r in segment k for state s. When the system is in state s, the total number of calls carried by base l is

$$n_l(s) = \sum_{k=-1}^{1} l_k(s) \tag{16}$$

A similar expression determines $n_r(s)$. Permissible states are those six-tuples which meet the constraints: $n_l(s) \leq C$ and $n_r(s) \leq C$. The total number of (permissible) states of a basic element increases rapidly with increasing C. Table 1 shows the relationship between state number and channel number for three configurations with different number of segments.

4 Transition Rate

There are six kinds of transition rates: 1) new call arrivals 2) call completions 3) hand-off call arrivals- without affecting both base stations 4) hand-off call departures- without affecting both base stations 5) hand-off call departure and arrival- within the same basic element 6) transitions between segments- within the same base station. Expressions for the state transition rates and the relationship between current states and successor states are given in Appendix B. Below we outline the issues developed there.

4.1 New Call Arrivals

Not every new call arrival contributes to the transition rate. Only those which can establish an acceptable link to some base contribute to it. It is because if a new call can not establish an acceptable link, the system is impossible to go to new states even there is a channel available for new calls. Combining the new call arrival rate per *sector*, Λ_S, and signal availability, we can calculate the transition rate due to new call arrivals.

4.2 Call Completions

When a call completes the system will change state. The unencumbered session duration has a negative exponential density function with mean $1/\mu_c$. The total rate flow out from state s due to call completion is $[n_l(s) + n_r(s)] \cdot \mu_c$. Upon a call completion the system will go to some other state depending on which base is serving the call and which segment the mobile is in.

4.3 Hand-Off Call Arrivals- without affecting both base stations

Any hand-off call arrival corresponds to some hand-off call departure in the system. Therefore the hand-off call arrival rate can be calculated by using hand-off call departure rate. Due to signal availability not every hand-off departure contributes to the hand-off call arrival rate. In order to calculate this rate, we need not only include the signal availability but also know some conditional probabilities to do the average for characterizing the hand-off arrivals from base ll and rr. These conditional probabilities can be calculated from the state probabilities. Since the rates depend on the state probabilities and the state probabilities depend on the rates, the hand-off call arrival rates are implicit and decided by the dynamics of the whole system.

4.4 Hand-Off Call Departures- without affecting both base stations

There are two reasons that a call may need a hand-off: 1) A mobile station moves away from the coverage area of the current base; 2) SIR falls below the threshold due to log-normal shadowing. In section 2.4 Traffic Model we defined and calculated μ_{do}, μ_{lo} and μ_{ln} as rates for hand-off departure per call in overlapping and non-overlapping segments. Combining them, we can get the hand-off call departure rates.

4.5 Hand-Off Call Departure and Arrival- within the same basic element

Since we consider one basic element, which includes two base stations. There are state transitions due to hand-off departure and arrival within the same basic element: hand-off departure from base l and arrival to base r or vice versa. They result in change of two state variables simultaneously. The way of calculating these rates is the same as the way for calculating hand-off departure rates.

4.6 Transitions Between Segments- within the same base station

In this section we consider those cases for which a communicating mobile moves to another segment but the call is still served by the same base station. This kind of transitions due to residing time completion in a segment also causes change in two state variables simultaneously. The completion of last segment(seg_1 or seg_{-1}, depending on the direction of mobile) results in a hand-off need and has been discussed in section 4.4 and 4.5. The rate of completing the residing time in overlapping segment per call is μ_{do}; in non-overlapping segment it is μ_{dn}.

5 Performance Measures

With all the transition rates of the system, the state probabilities of equilibrium can be calculated by solving probability flow balance equations. Since it is a set of non-linear implicit equations, we use iterative method to solve them. Firstly, we give initial guesses to state probabilities, then use these probabilities to compute the transition rates and solve for the state probabilities by Gauss-Seidel method. When state probabilities converge, we use these new state probabilities to update the transition rates and solve the equations again. This process continues until the state probabilities converge for two successive sets of transition rates. Therefore we have two level convergence.

After obtaining the state probabilities $P(s)$, we are ready to compute the performance measures. There are four performance measures of interest: 1) blocking probability 2) forced termination probability 3) hand-off activity 4) carried traffic.

5.1 Blocking Probability

If a new call can not gain access to a channel, it is blocked. Lack of acceptable link or available channel will result in failure of a new call arrival. Since both signal availability and channel availability are different for a new call in overlapping and non-overlapping segments, the blocking probability is averaged over these two kinds of segments. The detail formulation of the blocking probability is presented in Appendix C.

5.2 Forced Termination Probability

Forced termination probability P_{FT} is the probability for which a non-blocked call is interrupted due to hand-off failure during its lifetime. If the SIR of current link falls below threshold or the mobile leaves the coverage area of the current base, a hand-off need occurs. It may or may not succeed due to signal quality and channel availability of the target base station. If the hand-off need fails, the call is terminated.

Since the system is assumed to be homogeneous, P_{FT} is the same regardless the direction of mobile and which base serves it at the beginning. Therefore we only consider a non-blocked east bound call(moving to the right) served by a typical base station (say base l) at the beginning. We trace the life experience of such a call in Fig.6. We define the set H_i as the east bound call which is terminated at the i^{th} hand-off need. That is:

$$H_i = \{\text{east bound call} \mid \text{hand-off fails at the } i^{th} \text{ need}\}$$

The probability of H_i is P_{H_i}. Details for calculation of P_{H_i} are shown in Appendix D. Thus the forced termination probability is

$$P_{FT} = \sum_{i=1}^{\infty} P_{H_i} \tag{17}$$

We can add P_{H_i} from $i = 1$ until it converges.

5.3 Hand-Off Activity

Hand-off activity is the expected number of hand-off needs which a non-blocked call will have. Having exactly i hand-off needs means that 1) this call fails at the i^{th} hand-off need or 2) it succeeds at the i^{th} hand-off need but completes before having the $(i+1)^{th}$ hand-off need. The first kind of calls is just exactly the set H_i defined in the previous section. For the second kind of calls we define the set C_i as follows:

$$\begin{aligned} C_i = \{ & \text{ east bound call} \mid \text{after succeeding at } i^{th} \text{ hand-off} \\ & \text{need completes before } (i+1)^{th} \text{ hand-off need}\} \end{aligned} \tag{18}$$

The probability of C_i is P_{C_i} and calculation of P_{C_i} is similar to that of calculating P_{H_i}. The calculation is shown in Appendix E. Therefore, the hand-off activity H_A can be calculated using:

$$H_A = \sum_{i=1}^{\infty} i \cdot (P_{H_i} + P_{C_i}) \tag{19}$$

5.4 Carried Traffic

The carried traffic A_C per base station is the average number of channels occupied by calls. It can show us how efficient base stations are utilized. Under some criterion of blocking and forced termination probability, we require the carried traffic as much as possible. A_C is defined as follows:

$$A_C = \sum_{all\ s} n_l(s) \cdot P(s) \tag{20}$$

Because of symmetry, the following equation also holds.

$$A_C = \sum_{all\ s} n_r(s) \cdot P(s) \tag{21}$$

6 Numerical Results And Discussion

For the purpose of generating numerical results of this paper, the following system parameters are used for all the figures from Fig 7.X to Fig 12.X: L = 2000 ft (0.3788 mile); V_m = 55 miles/hour; $\gamma = 3.7$; σ = 3.0 dB; R = 18 dB; $\sigma_{\dot{\alpha}}$ = 0.1 dB/sec; $H_D = 0.8$; $\mu_c = 0.01$ calls/sec.

From Fig 7.X to Fig 12.X, there are four performance measures shown for various system parameters. Fig 7.X are for different boundary parameter, D. Fig 8.X are for different cluster size, N. Fig 9.X are for different channel numbers C per one base station. Fig 10.X are for different C_h, the number of reserved channels for hand-off calls. Fig 11.X compare the system performance under the same number of total channels in the whole system. Fig 12.X move away the signal availability part; only keep the channel availability part and we can compare them to Fig 9.X to see the difference.

For Fig 7.X, the effect of different boundary positions is examined. Since the parameter D is used to define boundary of the coverage area of a base station, changing D will change the boundary position. In these figures, N is 4, C is 6 and C_h is 1. From these figures, it is observed that performance measures are insensitive to changes of D. The reason can be seen from Fig 3.2. In Fig 3.2 from D = 5 dB to D = 15 dB, the boundary does not change too much.

For Fig 8.X, effect of different cluster size is examined. The larger cluster size, the better signal quality because co-channel base stations will be separated farer away. Therefore different cluster size really has big impact on performance measures. As we see on Fig 8.1 and 8.2, $N = 5$ is always better than others. Note that when demand is light, blocking and forced termination Probability converge to a fixed level. It is because in Fig 8.1 and 8.2 we consider signal availability. The fixed level is just the signal quality limitation. Thus, even though we decrease the demand, P_B and P_{FT} will not go down any more. For Fig 8.3, when demand is light, the call in the system of larger cluster size will continue longer than in the system of small cluster size. It is because the demand has not fully utilized the channels. At this time, signal quality has more impact on the Hand-Off Activity. When demand increases, the result changes because at this time the channel utilization of larger cluster size is higher than small cluster size and a call is harder to hand-off successfully due to lack of available channels.

In Fig 9.X, different channel numbers per base station are considered. In these figures, N is 4, C_h is 1 and D is 10 dB. Of course, more channels will provide better performance, but as stated above, P_B and P_{FT} will converge to a fixed level due to signal availability when demand decreases. Note that in Fig 9.1 the fixed level does not appear until the new call arrival rate is less than 10^{-3} calls/sec/sector. In these figures N is always 4. Thus for different channel numbers, the fixed limitation levels are the same. In Fig 8.1 and 8.2 they are not the same. And we can compare to Fig 12.X to see the difference of considering and not considering signal availability.

For Fig 10.X, the effect of different number of reserved channels are examined. When we give priority to hand-off calls, P_{FT} will decrease by sacrificing P_B. They are exchangeable. We can see the exchange from Fig 10.1 and 10.2. When demand is light, the exchange is not worth because P_{FT} is already limited by signal availability. Channel resources are not the problem in this situation. If we still give priority to hand-off calls, we will not improve P_{FT}, but at the same time P_B will increase a lot. When the demand is heavy, we can sacrifice a little bit P_B to get better P_{FT}. In Fig 10.3 and 10.4, more reserved channels will result in larger hand-off activity and smaller carried traffic. It is what we expect.

In Fig 8.X and 9.X, we compare different cluster size and channel number. Note that the total number of channels needed for the whole system is $N \cdot C$. Therefore if we fix one parameter and change another one, actually the total number of channels are also changed. In Fig 11.X, we fix the total number of channels and change cluster size and channel number simultaneously. From Fig 11.1 and 11.2, we know that for heavy demand, more channel is better. On the other hand, for light demand, larger cluster size is better.

In Fig 12.X, we present the results for considering only channel availability. In these figures, they have the same parameters as in Fig 9.X: N is 4, C_h is 1 and D is 10 dB. Thus they can be used to compare with Fig 9.X.

There is still one thing need to be mentioned that better signal quality means worse channel availability for the same demand. Therefore the system performance measures are really the race result of signal and channel availability. When reading the performance figures, we need to keep this in mind.

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Appendix A

Since we only use log-normal shadowing effect for signal availability consideration, the signal power, $A_s^2(x)$, received by mobile stations with the distance x from the serving base can be treated as a log-normal distributed random variable. Furthermore the signal power in dB, $P_s(x)$ ($= 10\log_{10} A_s^2(x)$), is normal distributed with mean $m_s(x)$ and standard deviation σ_s. The mean value $m_s(x)$ is a function of distance, x, from the serving base and it can be characterized by

$$m_s(x) = k \cdot 10\log_{10}\left|\frac{1}{x^{\gamma}}\right| \tag{A.1}$$

where k is a constant and γ is the propagation parameter which varies between 2 and 5. The standard deviation σ_s indicates the severity of shadowing. In urban environment value of σ_s are typically between 6 and 13 dB. For highway situation it should be smaller than 6 dB. To calculate SIR we need to get total interference power first. In this paper we consider three tiers (total six interferers). Each interferer has some contribution to the total interference power, $A_i^2(x)$. Unfortunately, there is no close form formula to combine individual log-normal interference. But since the sum of two log-normal variates can be approximated by another log-normal variate, an iterative procedure was developed to sum a finite number of log-normal variates in [17]. We use this algorithm to get the total interference power $P_i(x)$ (in dB), which is also normally distributed with mean $m_i(x)$ and standard deviation $\sigma_i(x)$. The signal to interference ratio in dB is defined by

$$SIR(x) = 10\log_{10}\frac{A_s^2(x)}{A_i^2(x)} = P_s(x) - P_i(x) \tag{A.2}$$

Since $P_s(x)$ and $P_i(x)$ are normal distributed, $SIR(x)$ is also a normal distributed random variable with mean value and standard deviation as following:

$$m_{sir}(x) = m_s(x) - m_i(x) \tag{A.3}$$

$$\sigma_{sir}(x) = \sqrt{\sigma_s^2 + \sigma_i^2(x)} \tag{A.4}$$

Then we can calculate the link outage probability $P_o(x)$ of equation (1). Since SIR is a normal random variable, the difference in SIR between two adjacent base stations is also a normal random variable.

Appendix B

Since any transition rate is *flow out* from current state, s, and *flow in* to next state, s_n, consideration of either one is enough to identify all the rates in the system. Here *flow out* is considered because it is easier to think of successors than predecessors. Note that in the following transition rate expressions, $n_l(s)$, $n_r(s)$, $l_k(s)$ and $r_k(s)$ are simply written as n_l, n_r, l_k and r_k. Thus the current state, s, is represented by $(l_{-1}, l_0, l_1, r_{-1}, r_0, r_1)$.

1 New Call Arrivals

The transition rate due to new call arrival is denoted by $r_n(s, s_n)$. When a new call arises in a non-overlapping segment, if signal quality of the link is unacceptable it does not contribute to the transition rate; otherwise it does. When a new call arises in an overlapping segment, there are three cases: 1)If both links are unacceptable, it does not contribute to the transition rate; 2)If signal quality of only left(right) link is acceptable, it contributes to the transition rate of left(right) base; 3)If signal quality of both links are acceptable, there are two more cases: a)If only one base has available channels for new calls, it will contribute to the transition rate of that base, b)Otherwise, it will choose left or right base with equal probability. Λ_S is the new call arrival rate per *sector*. $P(n_{ll} \geq C - C_h | l_{-1}, l_0, l_1)$ is the conditional probability for which the number of calls carried by base ll is greater than or equals to $C - C_h$ provided that the number of calls carried by base l in each segment are l_{-1}, l_0 and l_1. In addition notation $\lfloor a \rfloor$ is the largest integer not exceeds a. Therefore $r_n(s, s_n)$ is:

If $n_l < C - C_h$

$$r_n(s, s_n) = \Lambda_S \cdot w \cdot [(1 - P(n_{ll} \geq C - C_h | l_{-1}, l_0, l_1)) (\frac{1}{2} P_{Q_1} + P_{Q_2}) + P(n_{ll} \geq C - C_h | l_{-1}, l_0, l_1)(P_{Q_1} + P_{Q_2})] \tag{B.1}$$

$$where \;\; s_n = (l_{-1} + 1, l_0, l_1; r_{-1}, r_0, r_1)$$

$$r_n(s, s_n) = \Lambda_S (1 - w)(1 - P_{Z_2}) \tag{B.2}$$

$$where \;\; s_n = (l_{-1}, l_0 + 1, l_1; r_{-1}, r_0, r_1)$$

$$r_n(s, s_n) = \Lambda_S \cdot w \cdot (\frac{1}{2} P_{Q_1} + P_{Q_2} + \lfloor \frac{n_r}{C} \rfloor \cdot \frac{1}{2} P_{Q_1}) \tag{B.3}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1 + 1; r_{-1}, r_0, r_1)$$

If $n_r < C - C_h$

$$r_n(s, s_n) = \Lambda_S \cdot w \cdot (\frac{1}{2}P_{Q_1} + P_{Q_2} + \lfloor\frac{n_l}{C}\rfloor \cdot \frac{1}{2}P_{Q_1}) \tag{B.4}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1} + 1, r_0, r_1)$$

$$r_n(s, s_n) = \Lambda_S(1 - w)(1 - P_{Z_2}) \tag{B.5}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0 + 1, r_1)$$

$$r_n(s, s_n) = \Lambda_S \cdot w \cdot [(1 - P(n_{rr} \geq C - C_h | r_{-1}, r_0, r_1))$$

$$(\frac{1}{2}P_{Q_1} + P_{Q_2}) + P(n_{rr} \geq C - C_h | r_{-1}, r_0, r_1)(P_{Q_1} + P_{Q_2})] \tag{B.6}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0, r_1 + 1)$$

2 Call Completions

Upon a call completion the system will go to some other state depending on which base is serving the call and which segment the mobile is in. If this call is served by base l and completes in its seg_1, l_1 is decreased by one. The unencumbered session duration has a negative exponential density function with mean $1/\mu_c$, the transition rate due to call completion, $r_c(s, s_n)$, is:

If $l_{-1} > 0$

$$r_c(s, s_n) = l_{-1} \cdot \mu_c \tag{B.7}$$

$$where \;\; s_n = (l_{-1} - 1, l_0, l_1; r_{-1}, r_0, r_1)$$

If $l_0 > 0$

$$r_c(s, s_n) = l_0 \cdot \mu_c \tag{B.8}$$

$$where \;\; s_n = (l_{-1}, l_0 - 1, l_1; r_{-1}, r_0, r_1)$$

If $l_1 > 0$

$$r_c(s, s_n) = l_1 \cdot \mu_c \tag{B.9}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1 - 1; r_{-1}, r_0, r_1)$$

If $r_{-1} > 0$

$$r_c(s, s_n) = r_{-1} \cdot \mu_c \tag{B.10}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1} - 1, r_0, r_1)$$

If $r_0 > 0$

$$r_c(s, s_n) = r_0 \cdot \mu_c \tag{B.11}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0 - 1, r_1)$$

If $r_1 > 0$

$$r_c(s, s_n) = r_1 \cdot \mu_c \tag{B.12}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0, r_1 - 1)$$

3 Hand-Off Call Arrivals- without affecting both base stations

A rate due to hand-off call arrival is denoted by $r_h(s, s_n)$. Since any hand-off call arrival corresponds to some hand-off call departure in the system, the hand-off call arrival rate can be calculated by using hand-off call departure rate. For each base station hand-off call arrivals come from two neighboring bases: for base l they are base ll and r; for base r they are base l and rr. In either case there is one base(base ll or rr) outside the basic element we consider. Thus we also need to know the probabilities for every possible status of this neighboring base under the current state. These probabilities are the conditional probabilities given the current status of the base station at which hand-off calls arrive. They can be calculated from the state probabilities. Thus hand-off call arrival rates depend on the state probabilities; on the other hand, state probabilities depend on the rates. Therefore the hand-off call arrival rates are implicit and decided by the dynamics of the whole system. In the rate expressions, $P(ll_1 = i | l_{-1}, l_0, l_1)$ is the conditional probability for which the number of calls carried by base ll in its seg_1 is equal to i provided that the number of calls carried by base l in each segment are l_{-1}, l_0 and l_1. Other conditional probabilities have a similar meaning. The rate $r_h(s, s_n)$ is as follows:

If $n_l < C$

$$r_h(s, s_n) = \sum_{i=1}^{C} P(ll_1 = i | l_{-1}, l_0, l_1) \cdot i \cdot \mu_{lo}(1 - P_{Y_2}) \tag{B.13}$$

$$where \;\; s_n = (l_{-1} + 1, l_0, l_1; r_{-1}, r_0, r_1)$$

$$r_h(s, s_n) = \sum_{i=1}^{C} P(ll_1 = i | l_{-1}, l_0, l_1) \cdot$$

$$\frac{1}{2} \cdot i \cdot \mu_{do} \cdot [1 - P_o(1 - w)] \tag{B.14}$$

$$where \;\; s_n = (l_{-1}, l_0 + 1, l_1; r_{-1}, r_0, r_1)$$

If $n_r < C$

$$r_h(s, s_n) = \sum_{i=1}^{C} P(rr_{-1} = i | r_{-1}, r_0, r_1) \cdot \frac{1}{2} \cdot i \cdot \mu_{do} \cdot [1 - P_o(1 - w)] \quad \text{(B.15)}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0 + 1, r_1)$$

$$r_h(s, s_n) = \sum_{i=1}^{C} P(rr_{-1} = i | r_{-1}, r_0, r_1) \cdot i \cdot \mu_{lo}(1 - P_{Y_2}) \quad \text{(B.16)}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0, r_1 + 1)$$

4 Hand-Off Call Departures- without affecting both base stations

In section 2.4 Traffic Model we defined and calculated μ_{do}, μ_{lo} and μ_{ln} as rates for hand-off departure in overlapping and non-overlapping segments. We also assume that in a homogeneous system mobile traffic toward left and right are the same. Therefore half μ_{do} goes toward left and another half goes toward right. The flow out of state, s, due to hand-off departure is denoted by $r_d(s, s_n)$. Summarizing, it is:

If $l_{-1} > 0$

$$r_d(s, s_n) = \frac{1}{2} \cdot l_{-1} \cdot \mu_{do} + l_{-1} \cdot \mu_{lo} \quad \text{(B.17)}$$

$$where \;\; s_n = (l_{-1} - 1, l_0, l_1; r_{-1}, r_0, r_1)$$

If $l_0 > 0$

$$r_d(s, s_n) = l_0 \cdot \mu_{ln} \quad \text{(B.18)}$$

$$where \;\; s_n = (l_{-1}, l_0 - 1, l_1; r_{-1}, r_0, r_1)$$

If $l_1 > 0$

$$r_d(s, s_n) = \frac{1}{2} \cdot l_1 \cdot \mu_{do}[P_o(1 - w) + \lfloor \frac{n_r}{C} \rfloor \cdot (1 - P_o(1 - w))] + l_1 \cdot \mu_{lo}[P_{Y_2} + \lfloor \frac{n_r}{C} \rfloor \cdot (1 - P_{Y_2})] \quad \text{(B.19)}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1 - 1; r_{-1}, r_0, r_1)$$

If $r_{-1} > 0$

$$r_d(s, s_n) = \frac{1}{2} \cdot r_{-1} \cdot \mu_{do}[P_o(1 - w) + \lfloor \frac{n_l}{C} \rfloor \cdot (1 - P_o(1 - w))] + r_{-1} \cdot \mu_{lo}[P_{Y_2} + \lfloor \frac{n_l}{C} \rfloor \cdot (1 - P_{Y_2})] \quad \text{(B.20)}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1} - 1, r_0, r_1)$$

If $r_0 > 0$

$$r_d(s, s_n) = r_0 \cdot \mu_{ln} \quad \text{(B.21)}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0 - 1, r_1)$$

If $r_1 > 0$ $$r_d(s, s_n) = \frac{1}{2} \cdot r_1 \cdot \mu_{do} + r_1 \cdot \mu_{lo} \tag{B.22}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0, r_1 - 1)$$

5 Hand-Off Call Departure and Arrival- within the same basic element

Since one basic element includes two base stations, there are cases that a call hand-off from one base station to another in the same basic element we consider. Therefore, two state variables in the state representation change simultaneously. This component of flow is denoted by $r_{ad}(s, s_n)$. It is:

If $l_1 > 0 \;\; \cap \;\; n_r < C$

$$r_{ad}(s, s_n) = l_1 \cdot \mu_{lo} \cdot (1 - P_{Y_2}) \tag{B.23}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1 - 1; r_{-1} + 1, r_0, r_1)$$

$$r_{ad}(s, s_n) = \frac{1}{2} \cdot l_1 \cdot \mu_{do} \cdot [1 - P_o(1 - w)] \tag{B.24}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1 - 1; r_{-1}, r_0 + 1, r_1)$$

If $r_{-1} > 0 \;\; \cap \;\; n_l < C$

$$r_{ad}(s, s_n) = \frac{1}{2} \cdot r_{-1} \cdot \mu_{do} \cdot [1 - P_o(1 - w)] \tag{B.25}$$

$$where \;\; s_n = (l_{-1}, l_0 + 1, l_1; r_{-1} - 1, r_0, r_1)$$

$$r_{ad}(s, s_n) = r_{-1} \cdot \mu_{lo} \cdot (1 - P_{Y_2}) \tag{B.26}$$

$$where \;\; s_n = (l_{-1}, l_0, l_1 + 1; r_{-1} - 1, r_0, r_1)$$

6 Transitions Between Segments- within the same base station

There are also transitions between segments within the same base station: from seg_{-1} to seg_0, seg_0 to seg_1, seg_1 to seg_0 or seg_0 to seg_{-1}. This kind of transitions also causes change in two state variables simultaneously. One state variable is decreased by 1 and another is increased by 1. The transition rate due to segment change within the same base station is denoted by $r_s(s, s_n)$. Summarizing as follows:

If $l_{-1} > 0$ $$r_s(s, s_n) = \frac{1}{2} \cdot l_{-1} \cdot \mu_{do} \tag{B.27}$$

$$where \;\; s_n = (l_{-1} - 1, l_0 + 1, l_1; r_{-1}, r_0, r_1)$$

If $l_0 > 0$
$$r_s(s, s_n) = \frac{1}{2} \cdot l_0 \cdot \mu_{dn} \tag{B.28}$$
$$where \quad s_n = (l_{-1}, l_0 - 1, l_1 + 1; r_{-1}, r_0, r_1)$$
$$r_s(s, s_n) = \frac{1}{2} \cdot l_0 \cdot \mu_{dn} \tag{B.29}$$
$$where \quad s_n = (l_{-1} + 1, l_0 - 1, l_1; r_{-1}, r_0, r_1)$$

If $l_1 > 0$
$$r_s(s, s_n) = \frac{1}{2} \cdot l_1 \cdot \mu_{do} \tag{B.30}$$
$$where \quad s_n = (l_{-1}, l_0 + 1, l_1 - 1; r_{-1}, r_0, r_1)$$

If $r_{-1} > 0$
$$r_s(s, s_n) = \frac{1}{2} \cdot r_{-1} \cdot \mu_{do} \tag{B.31}$$
$$where \quad s_n = (l_{-1}, l_0, l_1; r_{-1} - 1, r_0 + 1, r_1)$$

If $r_0 > 0$
$$r_s(s, s_n) = \frac{1}{2} \cdot r_0 \cdot \mu_{dn} \tag{B.32}$$
$$where \quad s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0 - 1, r_1 + 1)$$
$$r_s(s, s_n) = \frac{1}{2} \cdot r_0 \cdot \mu_{dn} \tag{B.33}$$
$$where \quad s_n = (l_{-1}, l_0, l_1; r_{-1} + 1, r_0 - 1, r_1)$$

If $r_1 > 0$
$$r_s(s, s_n) = \frac{1}{2} \cdot r_1 \cdot \mu_{do} \tag{B.34}$$
$$where \quad s_n = (l_{-1}, l_0, l_1; r_{-1}, r_0 + 1, r_1 - 1)$$

Appendix C

When new calls arise in an overlapping area, there are four situations for channel availability consideration: 1) Each base has at least one channel available for new calls. 2) Right base has at least one channel available for new calls but not for left base. 3) Left base has at least one channel available for new calls but not for right base. 4) Both bases do not have any channel available for new calls. In order to calculate the probabilities for each of them we define the following four disjoint sets of states:

$$U_1 = \{s|n_l(s) < C - C_h \text{ and } n_r(s) < C - C_h\} \tag{C.1}$$

$$U_2 = \{s|n_l(s) \geq C - C_h \text{ and } n_r(s) < C - C_h\} \tag{C.2}$$

$$U_3 = \{s|n_l(s) < C - C_h \text{ and } n_r(s) \geq C - C_h\} \tag{C.3}$$

$$U_4 = \{s|n_l(s) \geq C - C_h \text{ and } n_r(s) \geq C - C_h\} \tag{C.4}$$

Then the probabilities of U_1, U_2, U_3 and U_4, P_{U_1}, P_{U_2}, P_{U_3} and P_{U_4}, can be calculated as following:

$$P_{U_1} = \sum_{s \in U_1} P(s) \qquad P_{U_2} = \sum_{s \in U_2} P(s) \tag{C.5}$$

$$P_{U_3} = \sum_{s \in U_3} P(s) \qquad P_{U_4} = \sum_{s \in U_4} P(s) \tag{C.6}$$

Due to symmetry P_{U_2} equals to P_{U_3}. Considering both U_i of channel availability and Q_i of signal availability (i =1,2,3,4), there are sixteen cases when a new call originates in an area of overlap. Of course there are four situations for which a new call will be blocked: 1) If signal quality of both links are unacceptable. 2) If both base stations carry more than or equal to $C - C_h$ calls. 3) If signal quality is only acceptable for base l but base l carries more than or equal to $C - C_h$ calls. 4) If signal quality is only acceptable for base r but base r carries more than or equal to $C - C_h$ calls. Note that 1) and 2) are not mutually exclusive. Table 2a shows all the conditions for which a new call is blocked in an area of overlap.

Similarly, in non-overlapping area there are only two conditions for channel availability considerations, 1) The base station has at least one channel available for new calls. 2)

The base station does not have any channel available for new calls. We define V_1 and V_2 corresponding to these two conditions:

$$V_1 = \{s | n_l(s) < C - C_h\} \tag{C.7}$$

$$V_2 = \{s | n_l(s) \geq C - C_h\} \tag{C.8}$$

Then the probability of V_1 and V_2, P_{V_1} and P_{V_2}, can be calculated using:

$$P_{V_1} = \sum_{s \in V_1} P(s) \qquad P_{V_2} = \sum_{s \in V_2} P(s) \tag{C.9}$$

Similarly combining both V_i of channel availability and Z_i of signal availability (i =1,2), there are four cases when a new call originates in an non-overlapping area. The blocking conditions for non-overlapping area are shown in Table 2b. Therefore the blocking probability P_B for the whole system is:

$$P_B = (1-w)(P_{V_2} + P_{Z_2} - P_{V_2} \cdot P_{Z_2}) + w \cdot (P_{U_4} + P_{Q_4} - P_{U_4} \cdot P_{Q_4} + 2 \cdot P_{U_2} \cdot P_{Q_2}) \tag{C.10}$$

Appendix D

In Fig.6 f_{-1} is the fraction of new calls that are not blocked and originate in seg_{-1}.

$$f_{-1} = \frac{\text{the number of non-blocked new calls originate in } seg_{-1}}{\text{the number of non-blocked new calls}} \tag{D.1}$$

Similarly, f_0 and f_1 are the fraction of new calls which get service and originate in seg_0 and seg_1.

There are only three outcomes for a non-blocked call in a segment. Firstly, it may complete in that segment. Secondly, it may lose the acceptable link due to shadow fading. Thirdly, if it does not complete and lose communication link, it will leave that segment after the mobile station completes the residing time of that segment. From μ_c, μ_{lo} and μ_{do} (or μ_{ln} and μ_{dn}), we know the probability for which any one of the above three outcomes will occur first. Due to symmetry, the signal condition in seg_{-1} and seg_1 are the same. Therefore actually we have two kinds of segments: overlapping and non-overlapping segments. P_{co} is the probability for which a call in overlapping segment will complete.

$$P_{co} = \mu_c/(\mu_c + \mu_{lo} + \mu_{do}) \tag{D.2}$$

Similarly, P_{lo} and P_{do} are defined as following:

$$P_{lo} = \mu_{lo}/(\mu_c + \mu_{lo} + \mu_{do}) \tag{D.3}$$

$$P_{do} = \mu_{do}/(\mu_c + \mu_{lo} + \mu_{do}) \tag{D.4}$$

Similarly are P_{cn}, P_{ln} and P_{dn}. The only difference is that they are for a call in non-overlapping segment.

$$P_{cn} = \mu_c/(\mu_c + \mu_{ln} + \mu_{dn}) \tag{D.5}$$

$$P_{ln} = \mu_{ln}/(\mu_c + \mu_{ln} + \mu_{dn}) \tag{D.6}$$

$$P_{dn} = \mu_{dn}/(\mu_c + \mu_{ln} + \mu_{dn}) \tag{D.7}$$

From the system's point of view, the property of seg_{-1} and seg_1 are the same. But if we consider only a east bound call, there is a difference between them. When a east bound call is in seg_{-1} of base l, if it loses its acceptable link it needs a hand-off to base ll. If it succeeds, it will belong to seg_1 of base ll. If it completes the residing time before completion of the call and losing its link, it will still be served by base l; but it will belong to seg_0 of the base l. When a east bound call is in seq_1 of base l, if it loses the link it needs a hand-off to base r. If it accesses to channel, it will belong to seg_{-1} of base r. But if it completes the residing time first, since seg_1 is the last segment (for east bound call), it will still result in a hand-off need to base r and if it succeeds, it will belong to seg_0 of base r. When a east bound call is in seg_0 and loses its link, since this is a non-overlapping segment, there is no alternative base and the call will be terminated due to hand-off failure. If it completes the residing time first, it will still be served by the same base; but now, it belongs to seg_1.

When a hand-off need occurs, this need may fail with hand-off failure probability. This probability is also account for both signal part and channel part and it is different in different segments. In seg_{-1} and seg_1, they are the same. They are denoted by P_{ho} because they are the <u>h</u>and-off failure probability in <u>o</u>verlapping segment. It can be obtained by

$$P_{ho} = P_{Y_2} + P_G - P_{Y_2} \cdot P_G \tag{D.8}$$

Where P_G is the probability for which all channels of a base station are occupied. The hand-off failure probability in seg_0 is always 1 because it is non-overlapping segment. There is another hand-off failure probability for the east bound call which completes the residing time of seg_1 and produces a hand-off need to base r. It is denoted by P_{hn}:

$$P_{hn} = P_o(1-w) + P_G - P_o(1-w) \cdot P_G \tag{D.9}$$

If, at the beginning, the total number of new calls which access to channel from base l is normalized to 1. Then there is f_{-1} in seg_{-1}, f_0 in seg_0 and f_1 in seg_1. In seg_{-1}, P_{lo} is the fraction of new calls which will lose quality link first. And the hand-off failure probability is P_{ho}. Therefore, $f_{-1} \cdot P_{lo} \cdot P_{ho}$ is the fraction of non-blocked east bound new calls which will experience hand-off failure at the first hand-off need in seg_{-1}. In seg_0, P_{ln} is the fraction of new calls which will lose quality link first and the hand-off failure probability is always 1 there. But except the new calls originated in seg_0, calls originated in seg_{-1} also come into

seg_0 after completing the residing time. Therefore, $(f_{-1} \cdot P_{do} + f_0) \cdot P_{ln}$ is the fraction of non-blocked east bound new calls which will experience hand-off failure at the first hand-off need in seg_0. Similarly, in seg_1, we need to include the calls that are originated in seg_{-1} and seg_0 and come into seg_1 without hand-off. Therefore, $(f_{-1} \cdot P_{do} \cdot P_{dn} + f_0 \cdot P_{dn} + f_1)(P_{lo} \cdot P_{ho} + P_{do} \cdot P_{hn})$ is the fraction of non-blocked east bound new calls which will experience hand-off failure at the first hand-off need in seg_1. Then summarizing from above, we can get P_{H_1} which is:

$$\begin{aligned} P_{H_1} = f_{-1} \cdot P_{lo} \cdot P_{ho} + (f_{-1} \cdot P_{do} + f_0) \cdot P_{ln} + \\ [(f_{-1} \cdot P_{do} + f_0) \cdot P_{dn} + f_1] \cdot (P_{lo} \cdot P_{ho} + P_{do} \cdot P_{hn}) \end{aligned} \quad \text{(D.10)}$$

This is the fraction of non-blocked east bound new calls which will fail at the first hand-off need. Up to now, we only consider the hand-off failure at the first need. After the first success of hand-off need, the calls will be served by a new base station and belong to one of the three segments as stated above. Even though the base station is not the same as the original one, but all same kind of segments have the same statistic behavior. For example, a call served by base l in seg_1 hand-off successfully will belong to seg_{-1} of base r. The seg_{-1} of base r has the same property as the seg_{-1} of the base l. Therefore the above equation still can be used except we need to update the f_{-1}, f_0 and f_1. Let's define $F_{-1,i}, F_{0,i}$ and $F_{1,i}$ as the fraction of calls which is still alive after the i^{th} hand-off need in each segment. Of course, $F_{-1,0} = f_{-1}$, $F_{0,0} = f_0$ and $F_{1,0} = f_1$. For $i = 1,2,3\cdots$,

$$F_{-1,i} = [(F_{-1,i-1} \cdot P_{do} + F_{0,i-1}) \cdot P_{dn} + F_{1,i-1}] \cdot P_{lo} \cdot (1 - P_{ho}) \quad \text{(D.11)}$$

$$F_{0,i} = [(F_{-1,i-1} \cdot P_{do} + F_{0,i-1}) \cdot P_{dn} + F_{1,i-1}] \cdot P_{do} \cdot (1 - P_{hn}) \quad \text{(D.12)}$$

$$F_{1,i} = F_{-1,i-1} \cdot P_{lo} \cdot (1 - P_{ho}) \quad \text{(D.13)}$$

Therefore, P_{H_i} is as following:

$$\begin{aligned} P_{H_i} = F_{-1,i-1} \cdot P_{lo} \cdot P_{ho} + (F_{-1,i-1} \cdot P_{do} + F_{0,i-1}) \cdot P_{ln} + \\ [(F_{-1,i-1} \cdot P_{do} + F_{0,i-1}) \cdot P_{dn} + F_{1,i-1}] \cdot (P_{lo} \cdot P_{ho} + P_{do} \cdot P_{hn}) \end{aligned} \quad \text{(D.14)}$$

Appendix E

Calculation of P_{C_i} is similar to that of calculating P_{H_i}. Note that we need to use $F_{k,i}$ ($k = -1, 0, 1$) in this calculation because this part of calls already successfully handed-off i times. As a result,

$$P_{C_i} = F_{-1,i} \cdot P_{co} + (F_{-1,i} \cdot P_{do} + F_{0,i}) \cdot P_{cn} + \\ [(F_{-1,i} \cdot P_{do} + F_{0,i}) \cdot P_{dn} + F_{1,i}] \cdot P_{co} \tag{E.1}$$

C	Segments 1	3	5
1	4	16	36
2	9	100	441
3	16	400	3136
4	25	1225	15876
5	36	3136	63504
6	49	7056	213444
7	64	14400	627264
8	81	27225	1656369
9	100	48400	4008004
10	121	81796	9018009

Table 1. Number of states for one Basic Element

	$Q_1(P_{Q_1})$	$Q_2(P_{Q_2})$	$Q_3(P_{Q_3})$	$Q_4(P_{Q_4})$
$U_1(P_{U_1})$				×
$U_2(P_{U_2})$			×	×
$U_3(P_{U_3})$		×		×
$U_4(P_{U_4})$	×	×	×	×

(a)

	$Z_1(1-P_{Z_2})$	$Z_2(P_{Z_2})$
$V_1(1-P_{V_2})$		×
$V_2(P_{V_2})$	×	×

(b)

Table 2. Blocking conditions under consideration of both channel and signal availability for (a) overlapping area (b) non-overlapping area

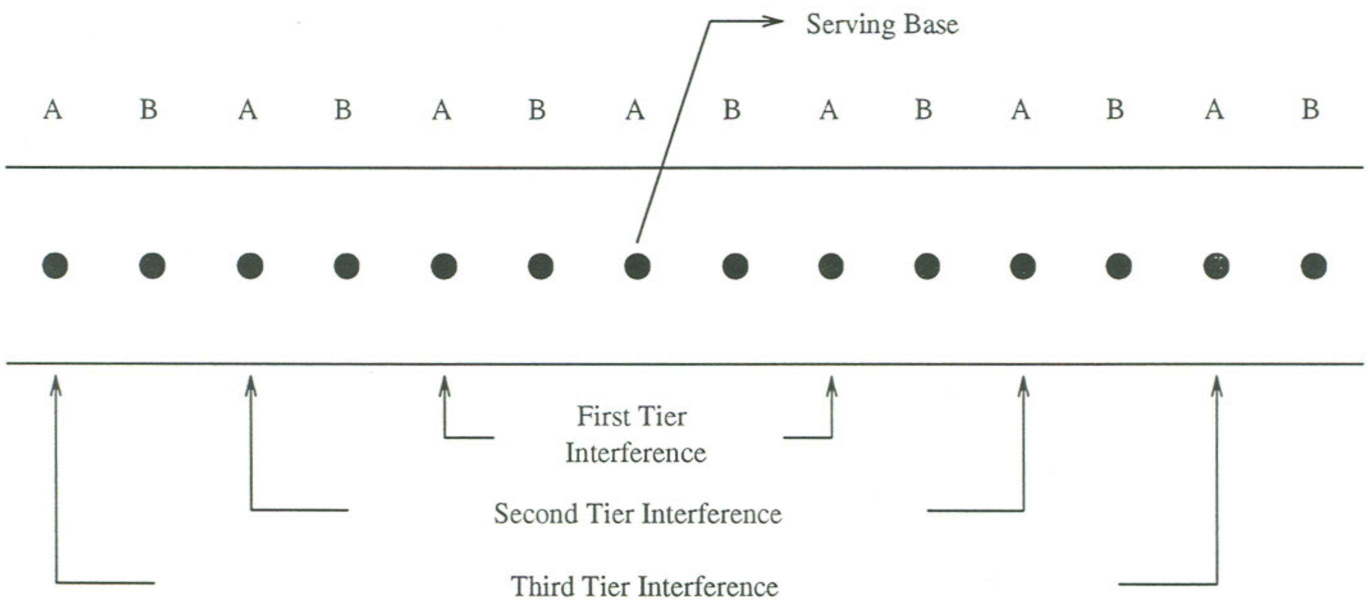


Fig.1 Diagram of three tier interference for the system with two channel groups (A and B)

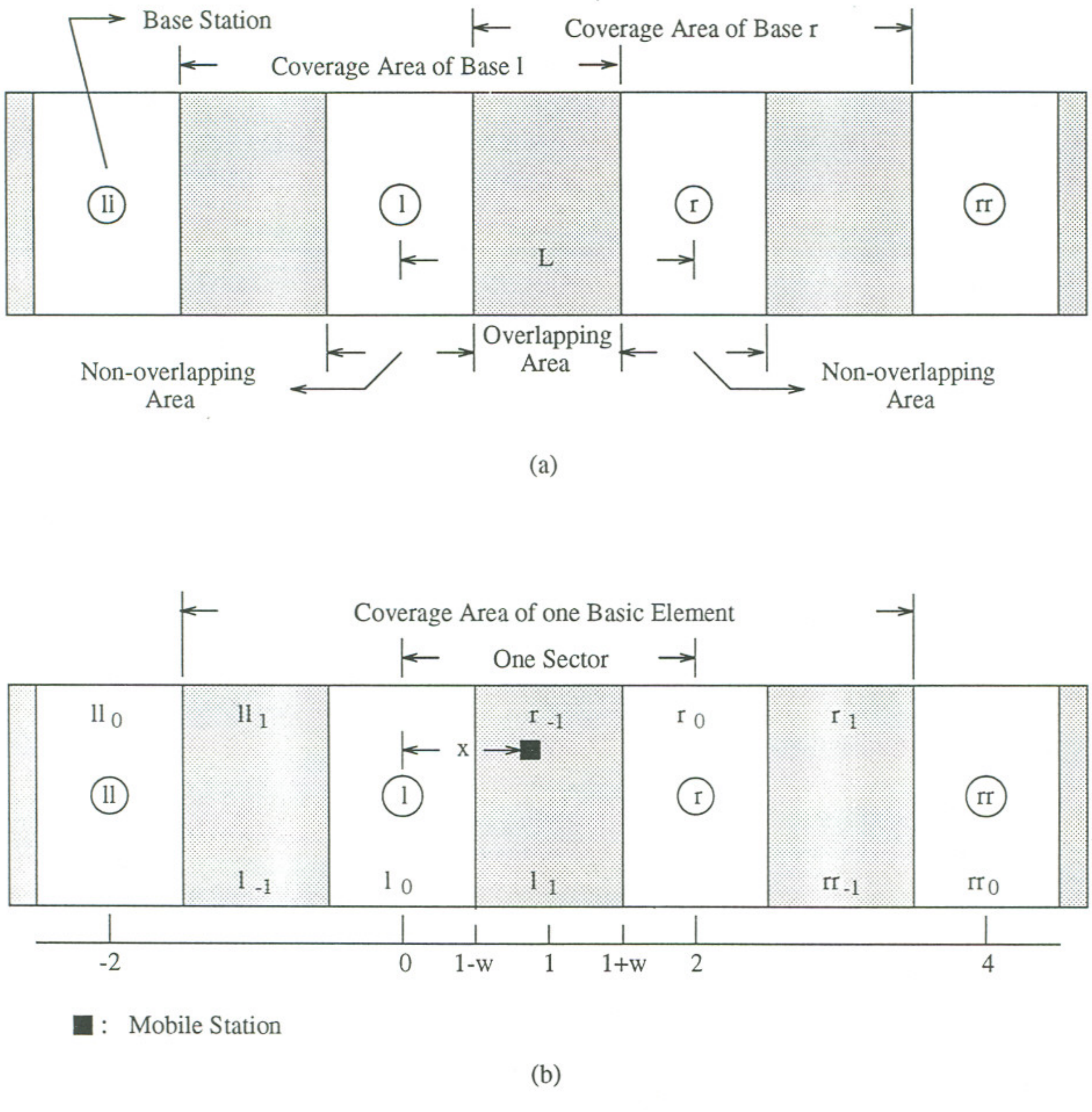


Fig.2 System layout for linear microcells

Fig 3.1 Probability of SIR Difference greater than 10 dB

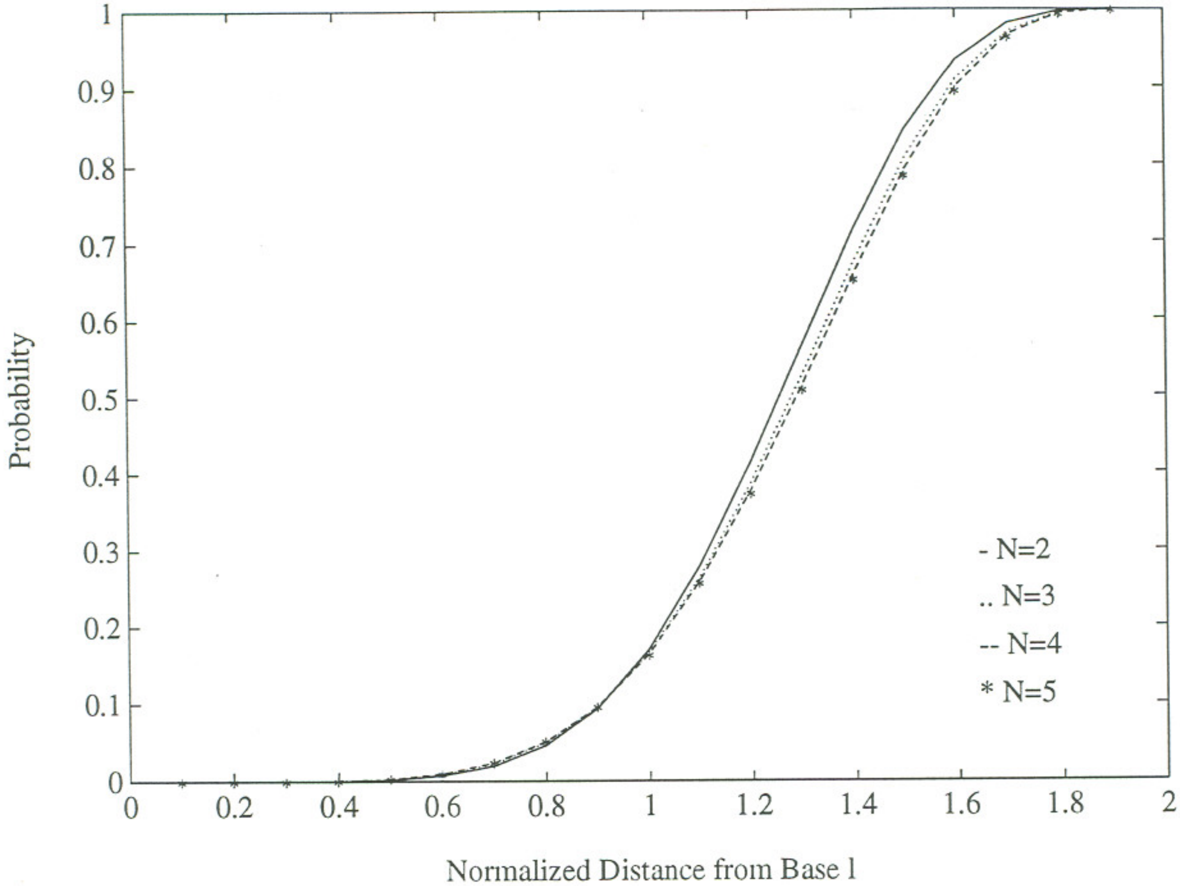


Fig 3.2 Probability of SIR Difference for Cluster Size = 4

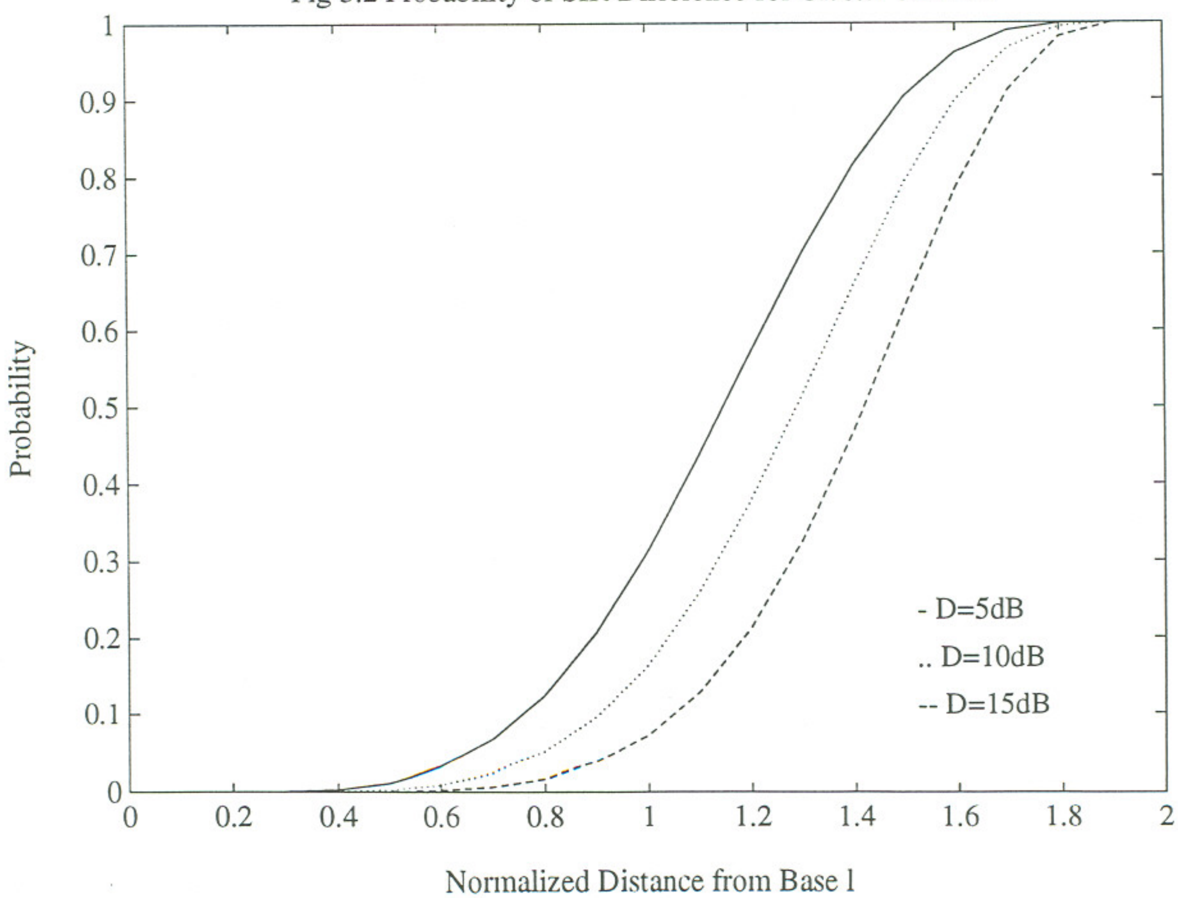
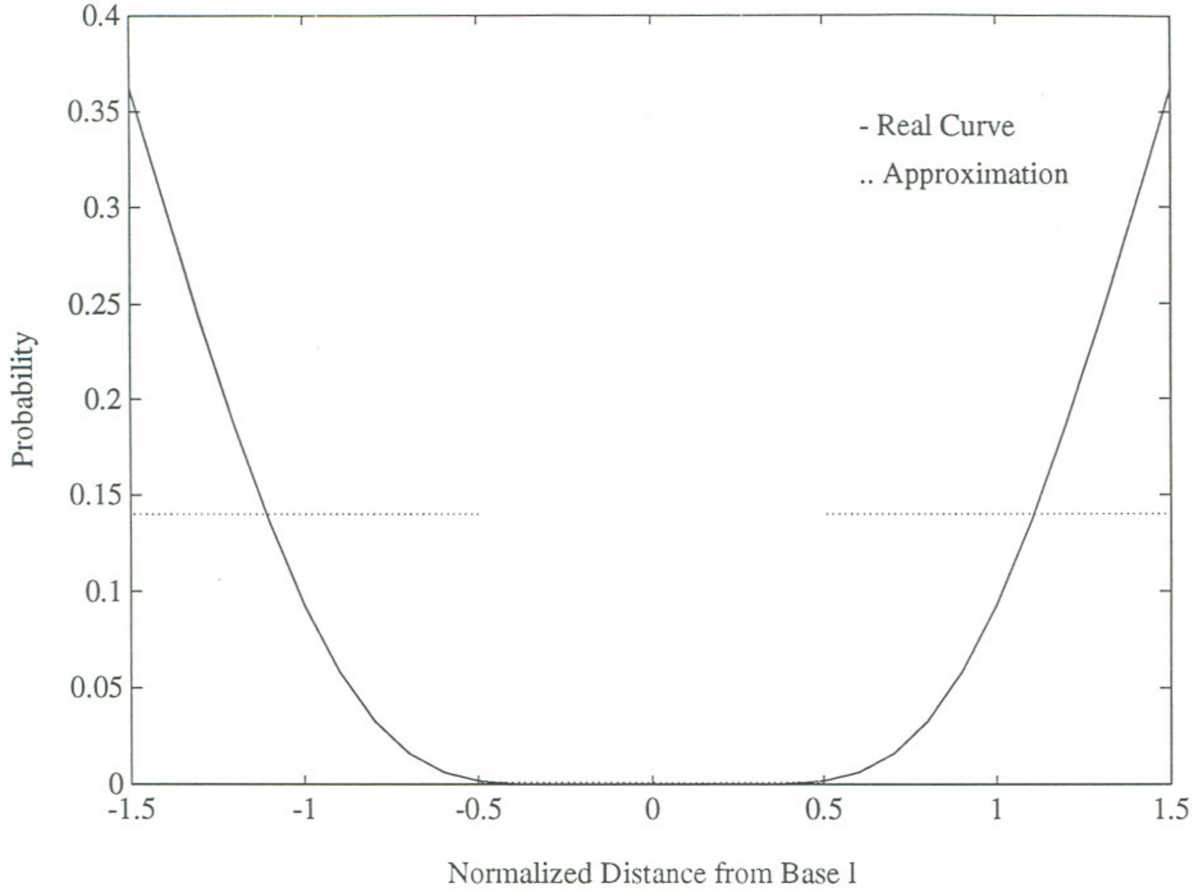
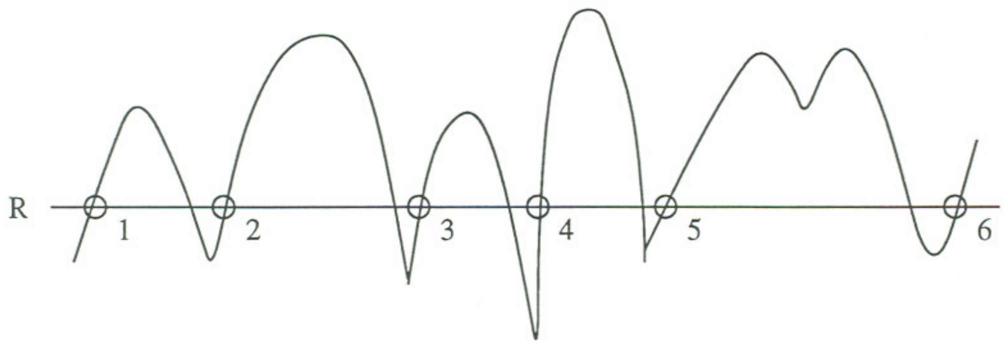
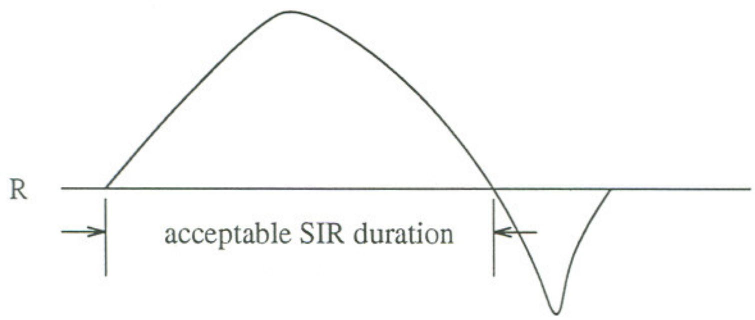


Fig 4 Outage Probability and Approximation





(a)



(b)

Fig.5 (a) Typical SIR profile a mobile will experience

(b) One cycle of average SIR variation

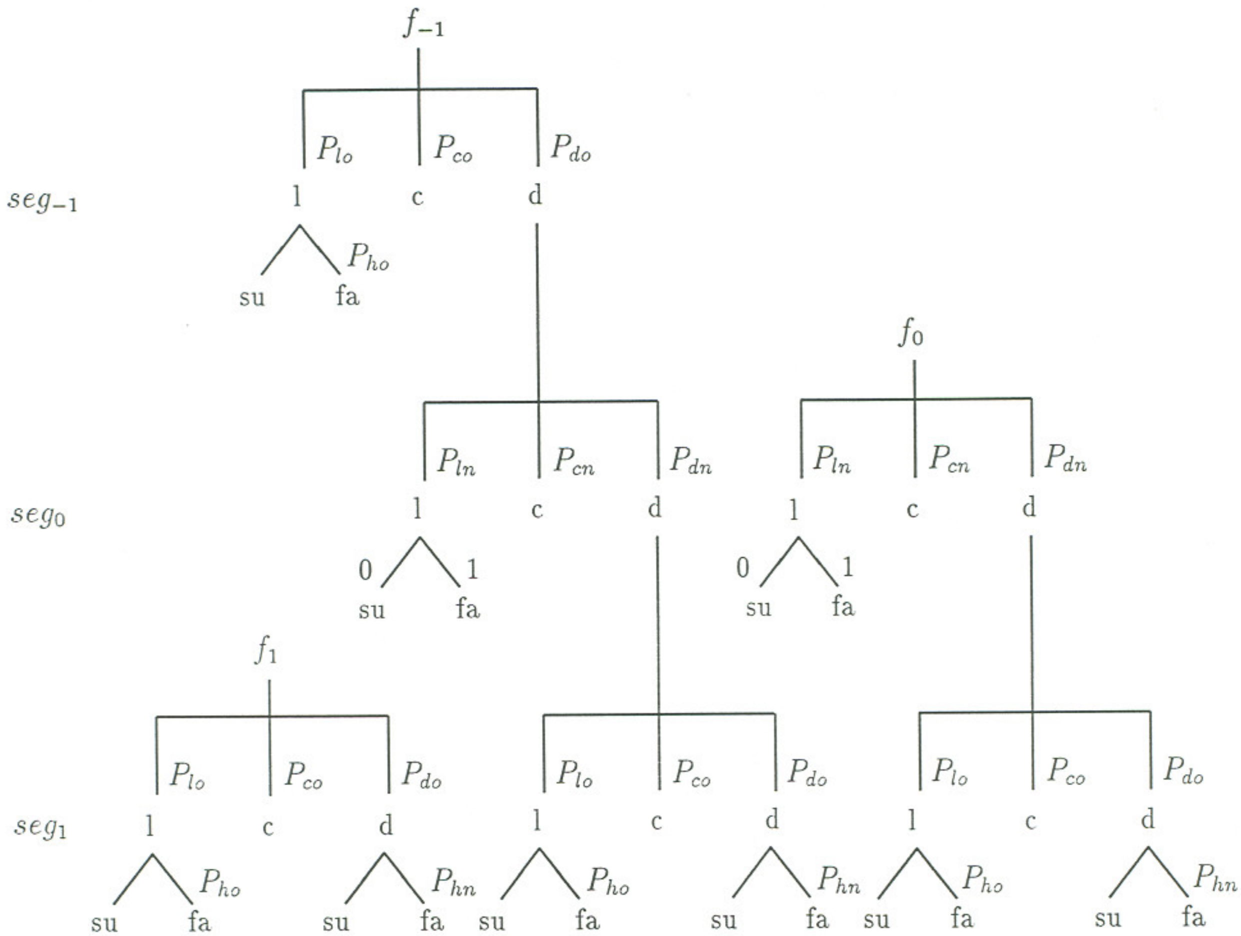


Fig.6 The life experience of a east bound call within the coverage area of a base station
l: lose quality link, c: complete call, d: complete reside time
su: success, fa: fail

Fig 7.1 Blocking Probability

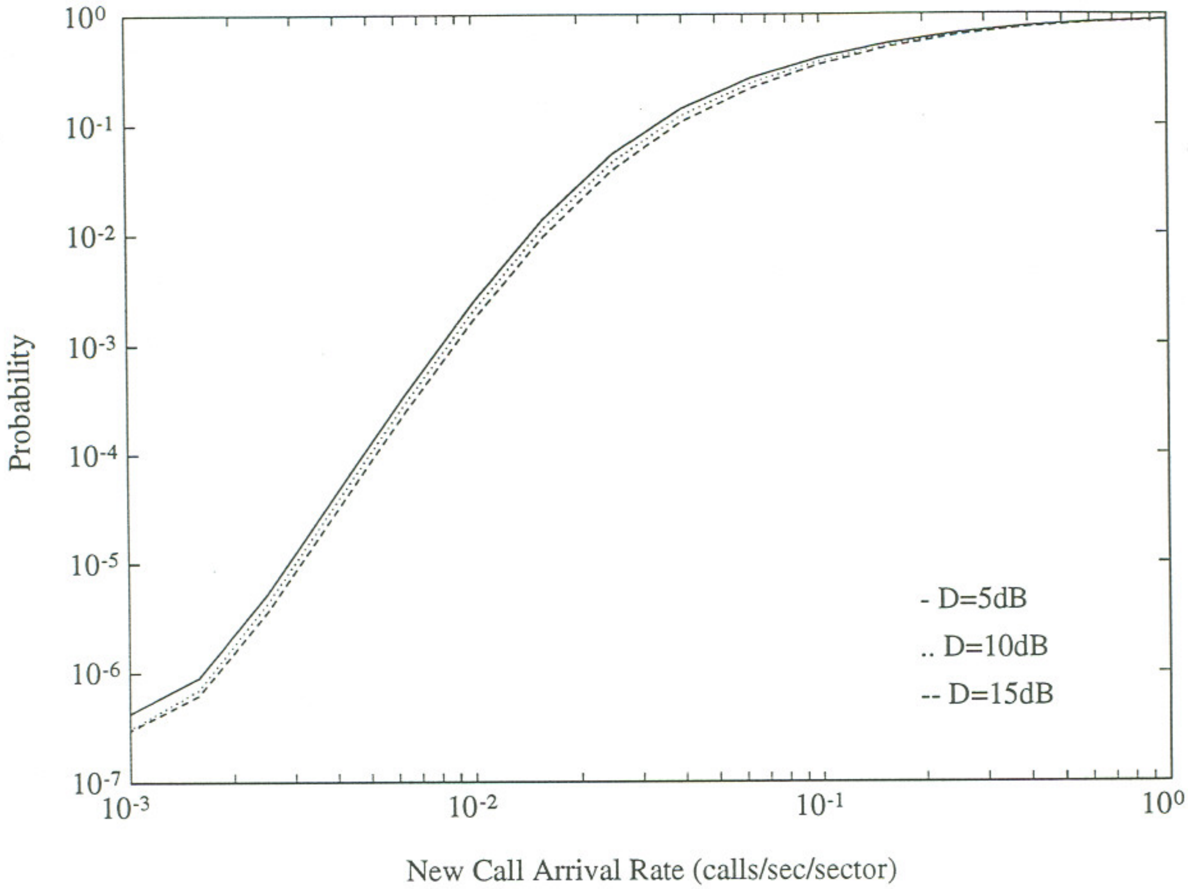


Fig 7.2 Forced Termination Probability

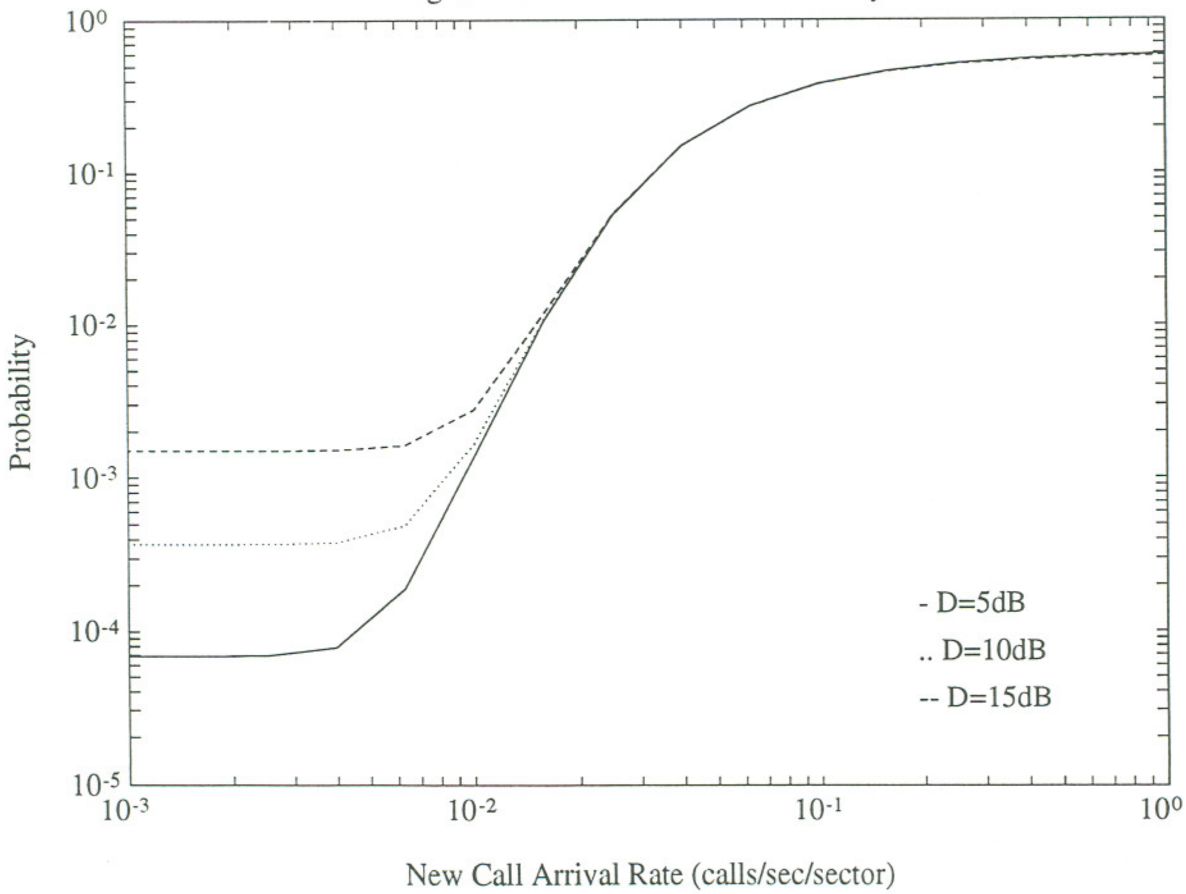


Fig 7.3 Handoff Activity

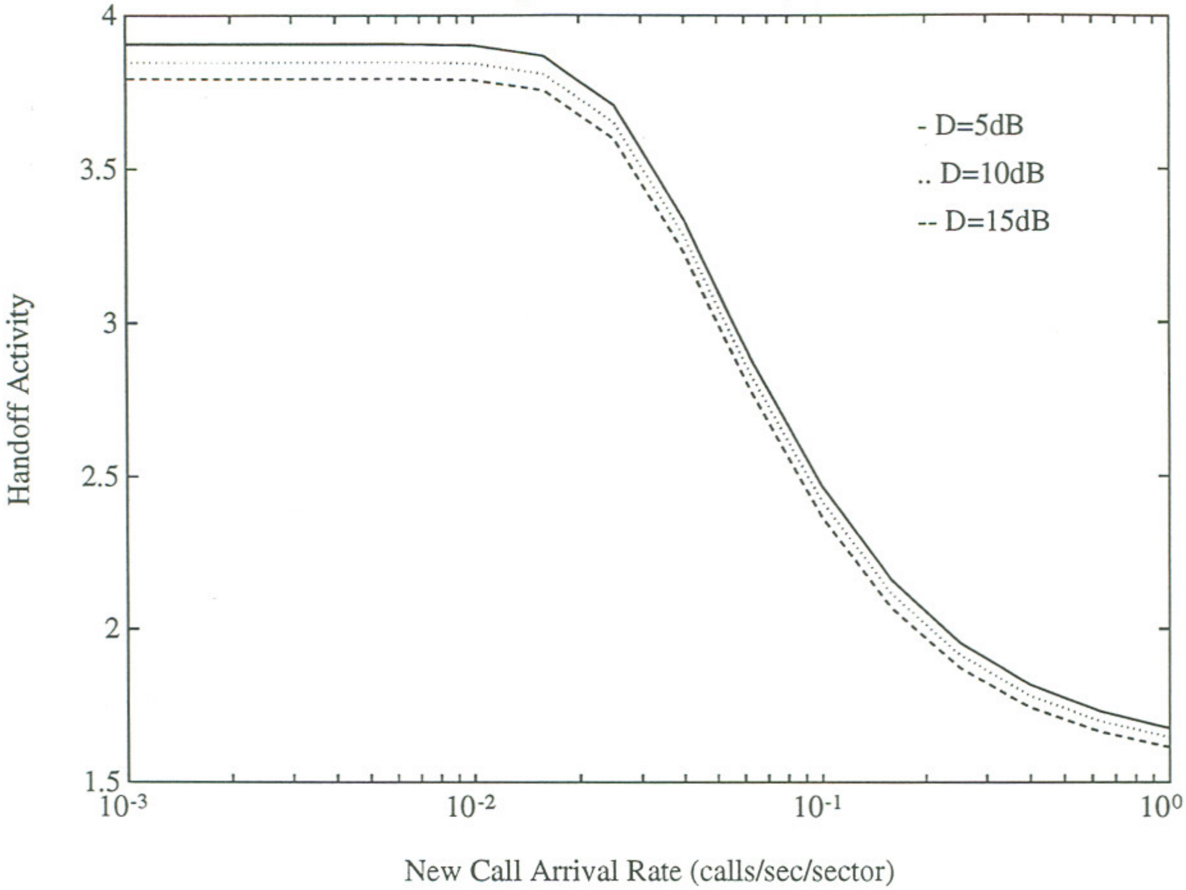


Fig 7.4 Carried Traffic

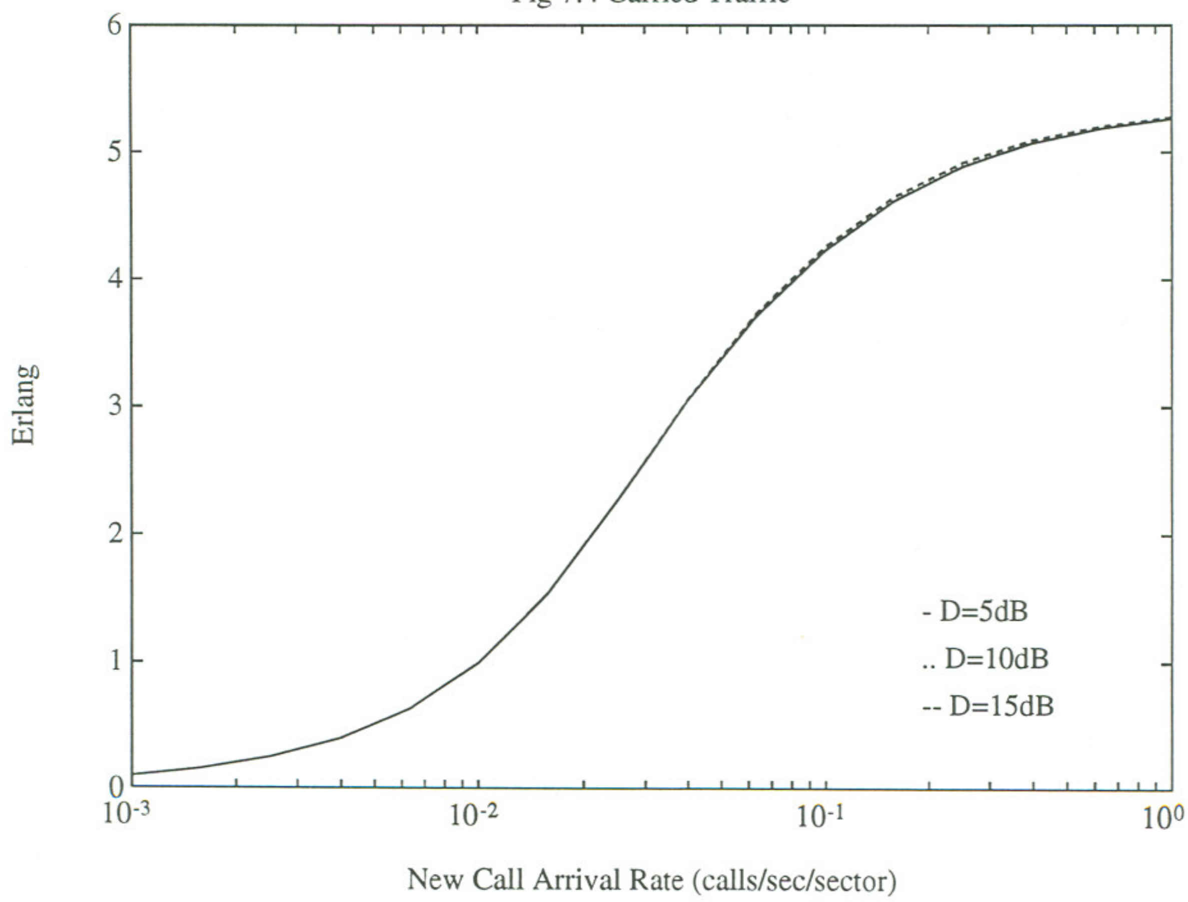


Fig 8.1 Blocking Probability

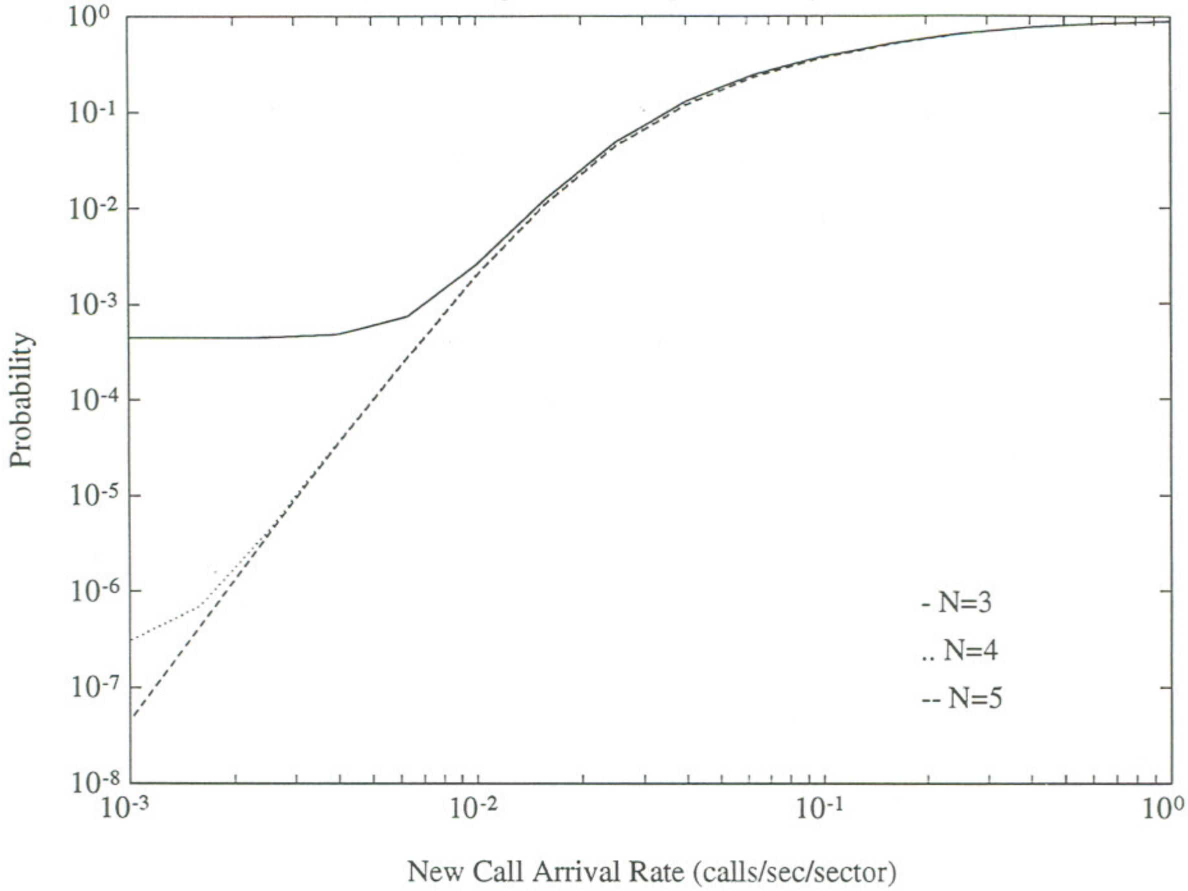


Fig 8.2 Forced Termination Probability

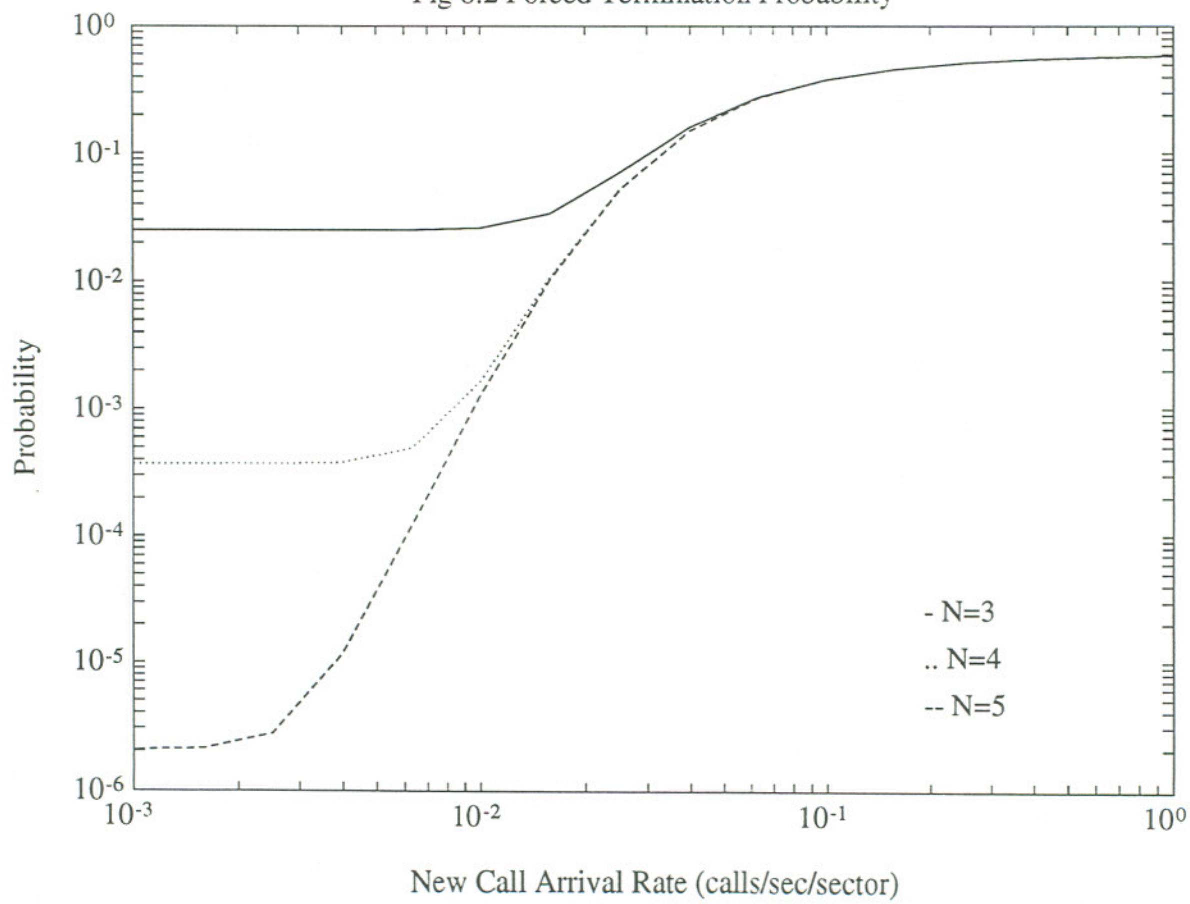


Fig 8.3 Handoff Activity

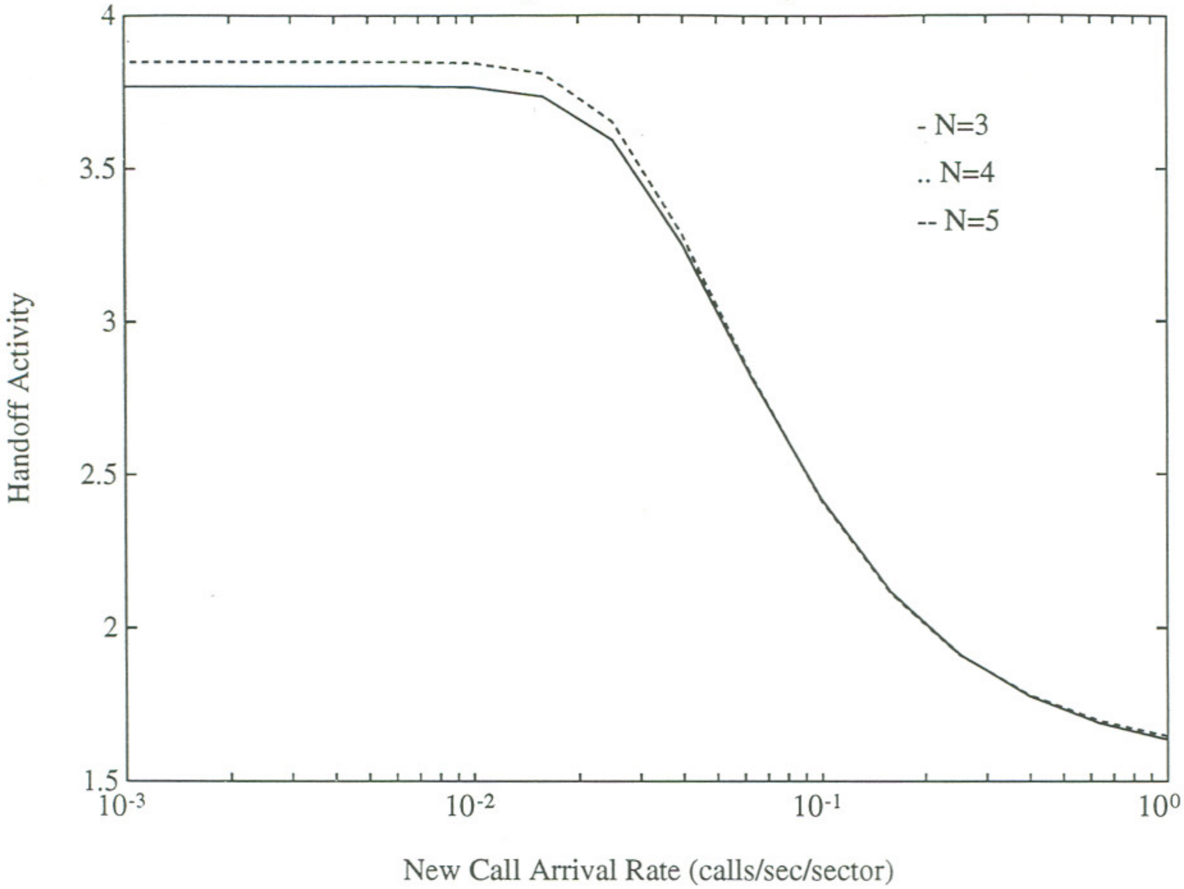


Fig 8.4 Carried Traffic

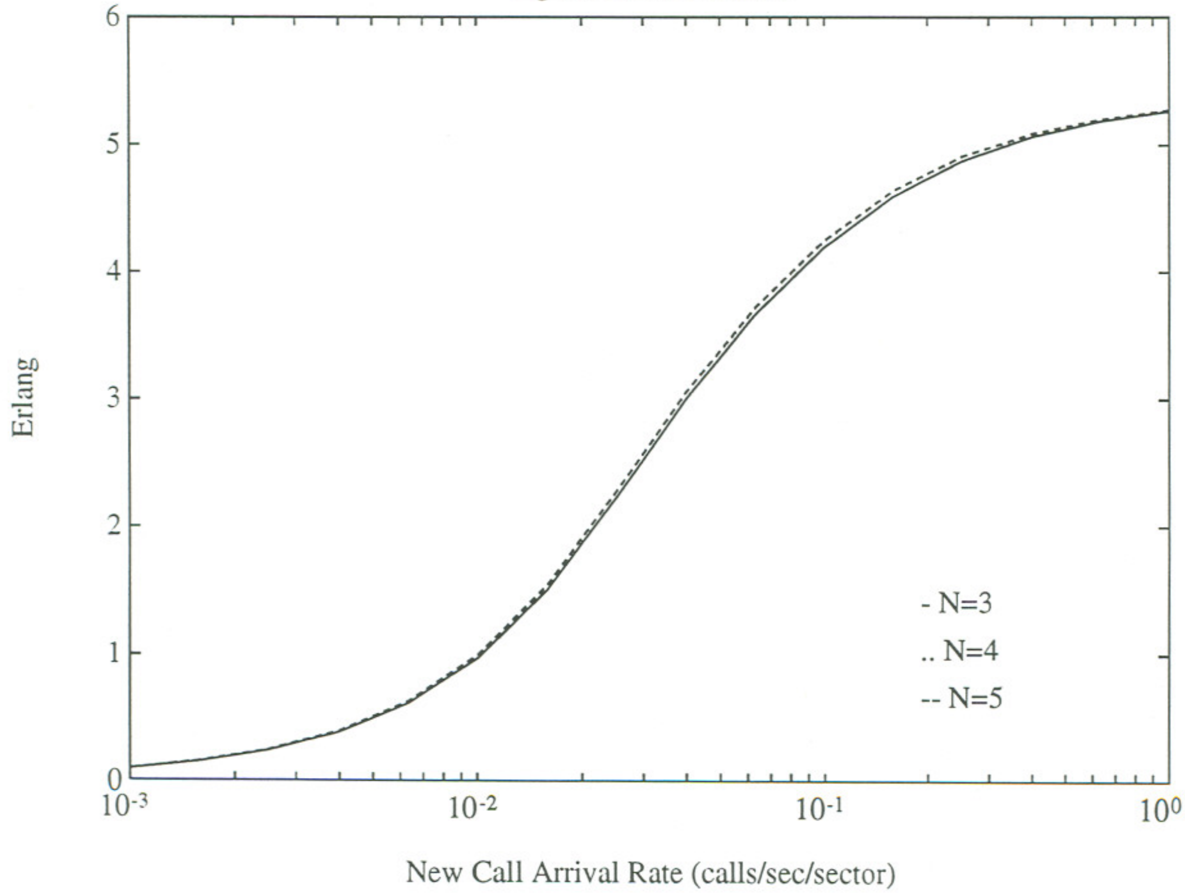


Fig 9.1 Blocking Probability

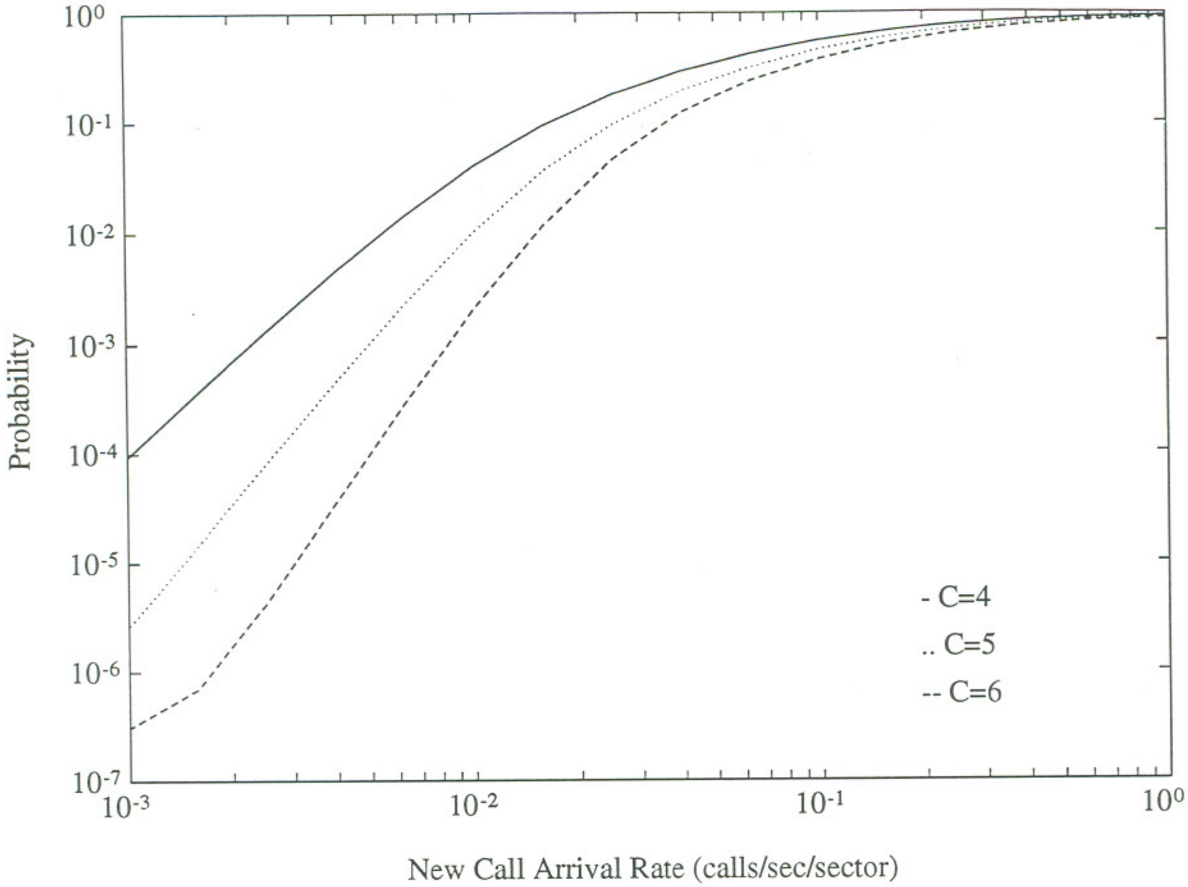


Fig 9.2 Forced Termination Probability

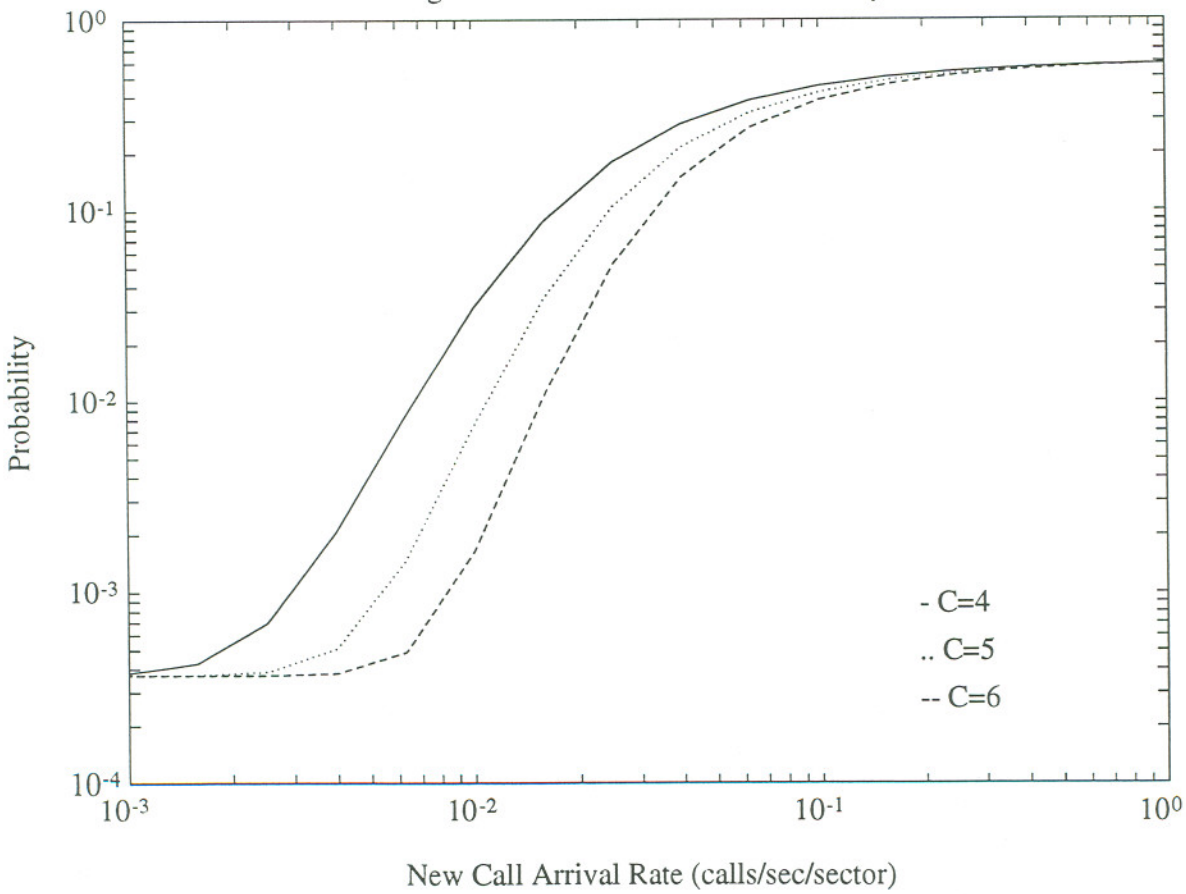
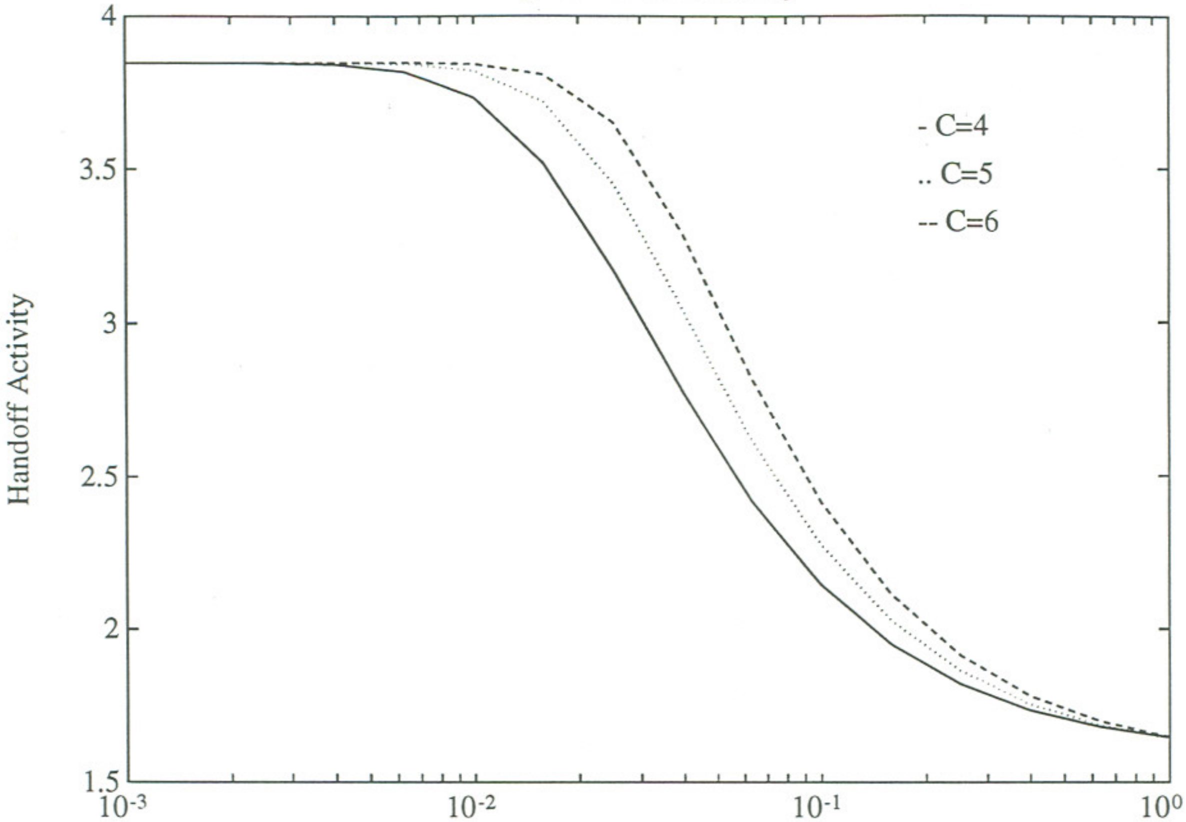
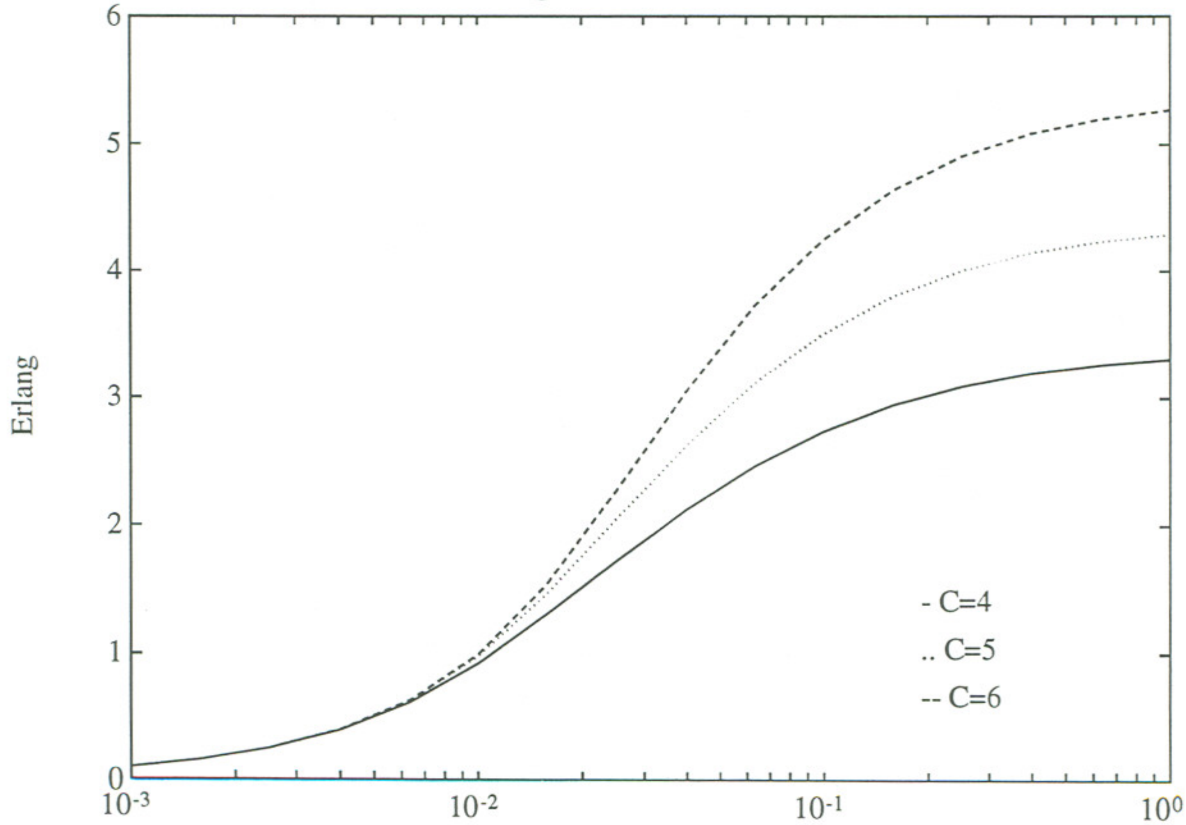


Fig 9.3 Handoff Activity



New Call Arrival Rate (calls/sec/sector)

Fig 9.4 Carried Traffic



New Call Arrival Rate (calls/sec/sector)

Fig 10.1 Blocking Probability

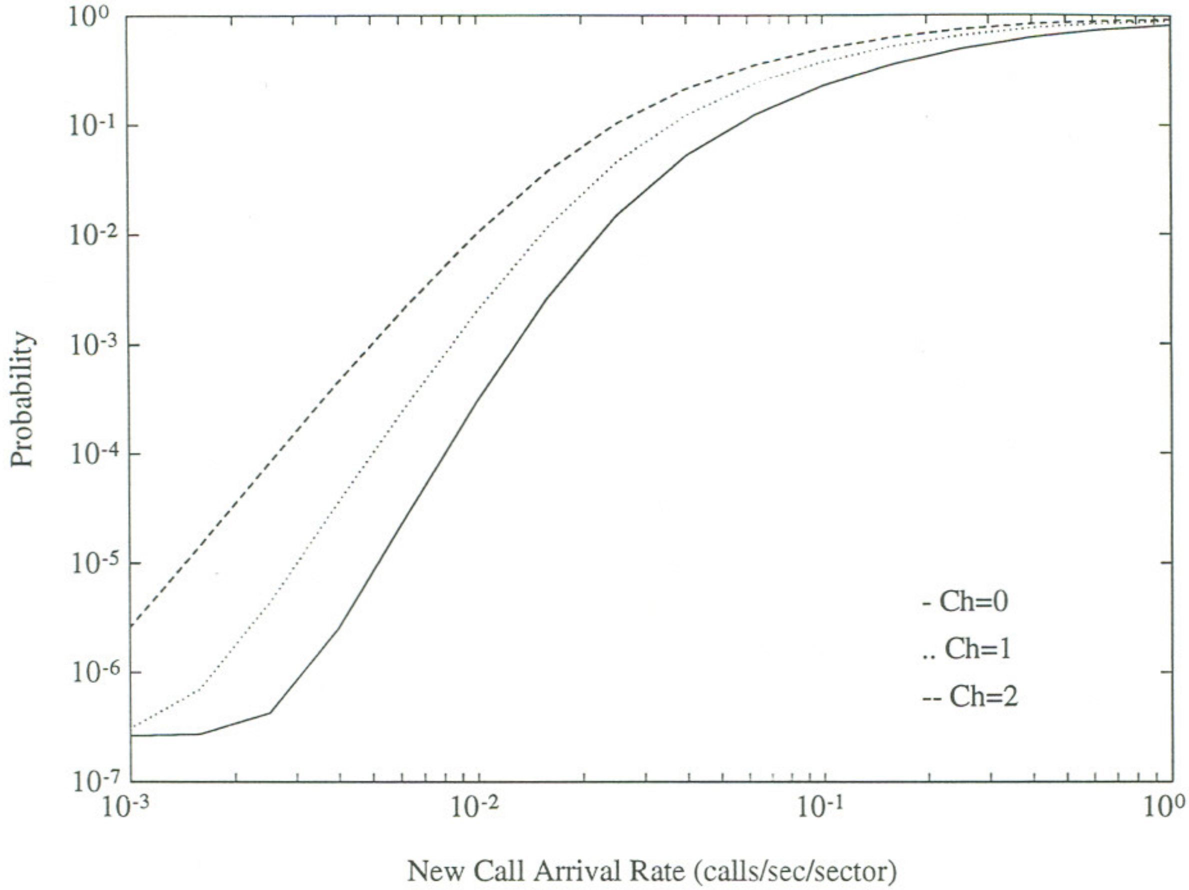


Fig 10.2 Forced Termination Probability

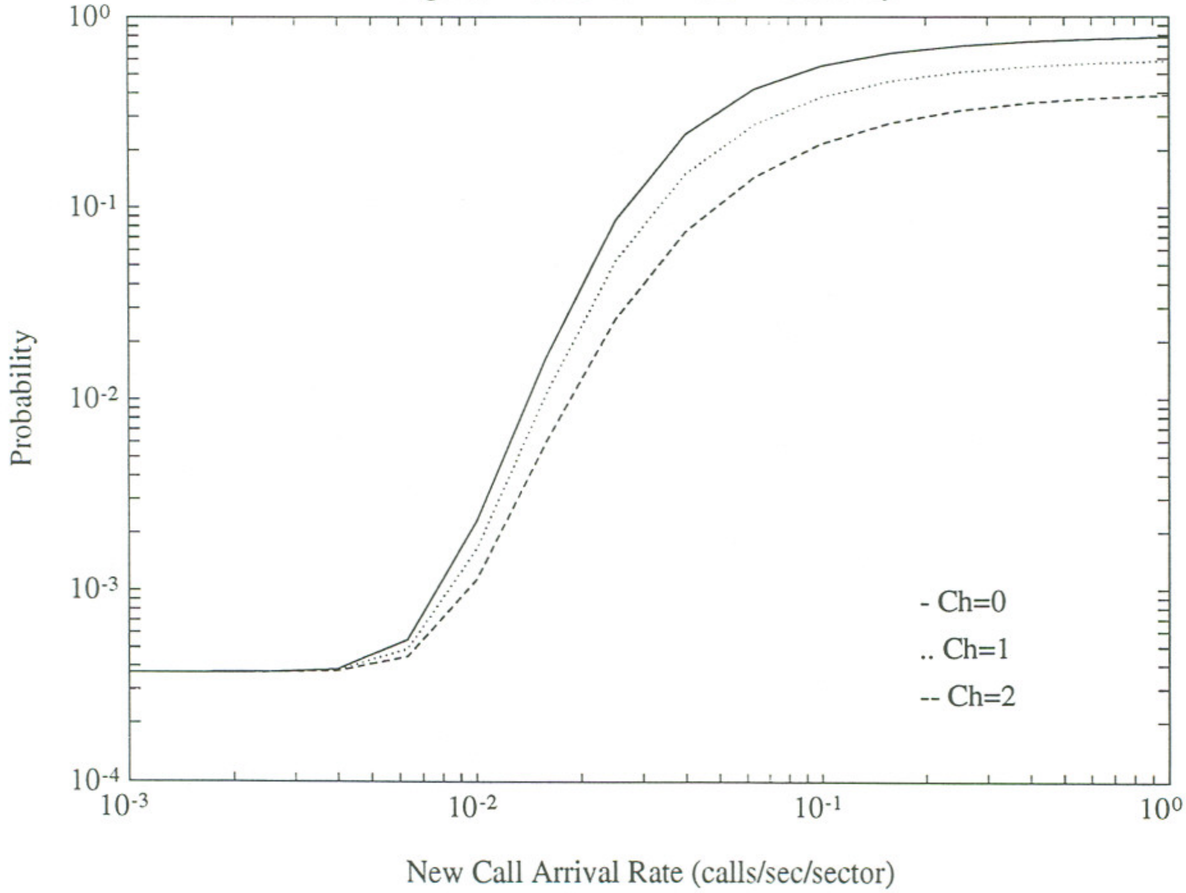


Fig 10.3 Handoff Activity

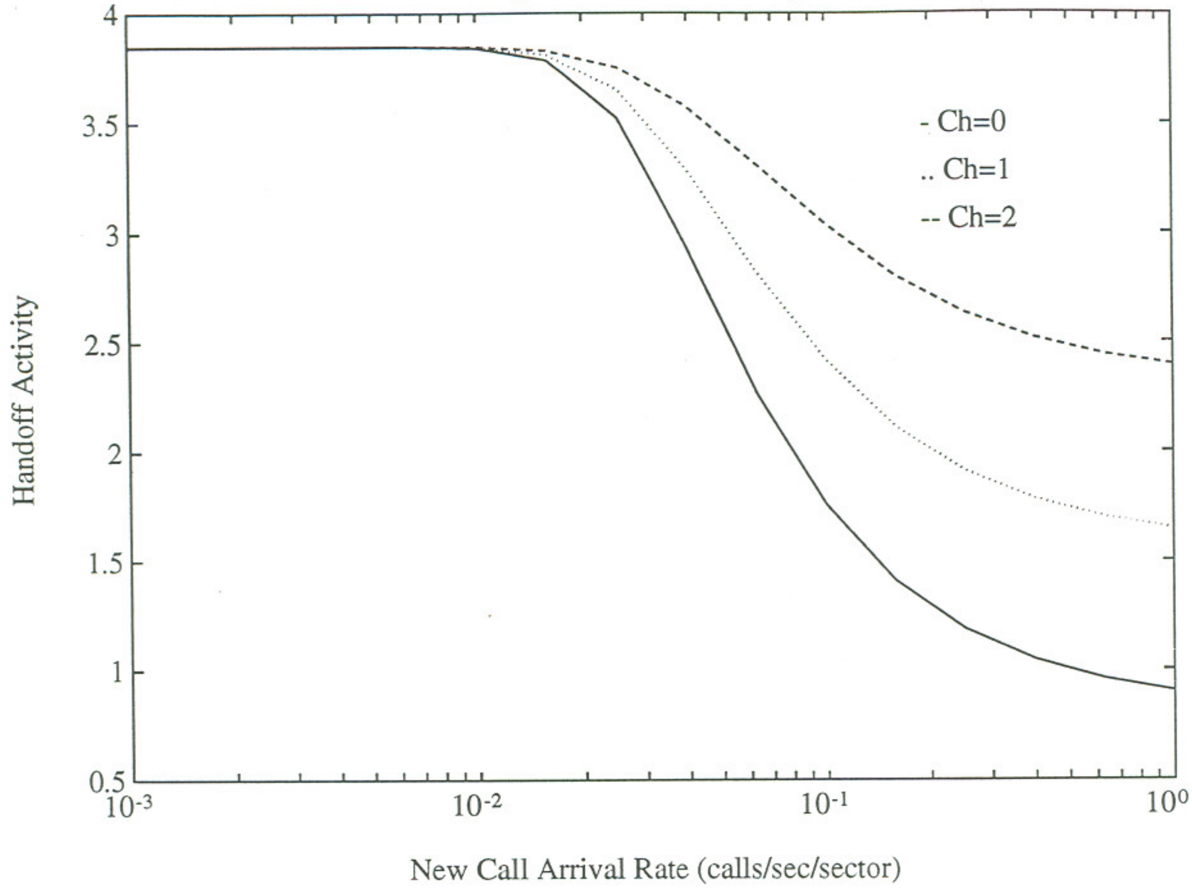


Fig 10.4 Carried Traffic

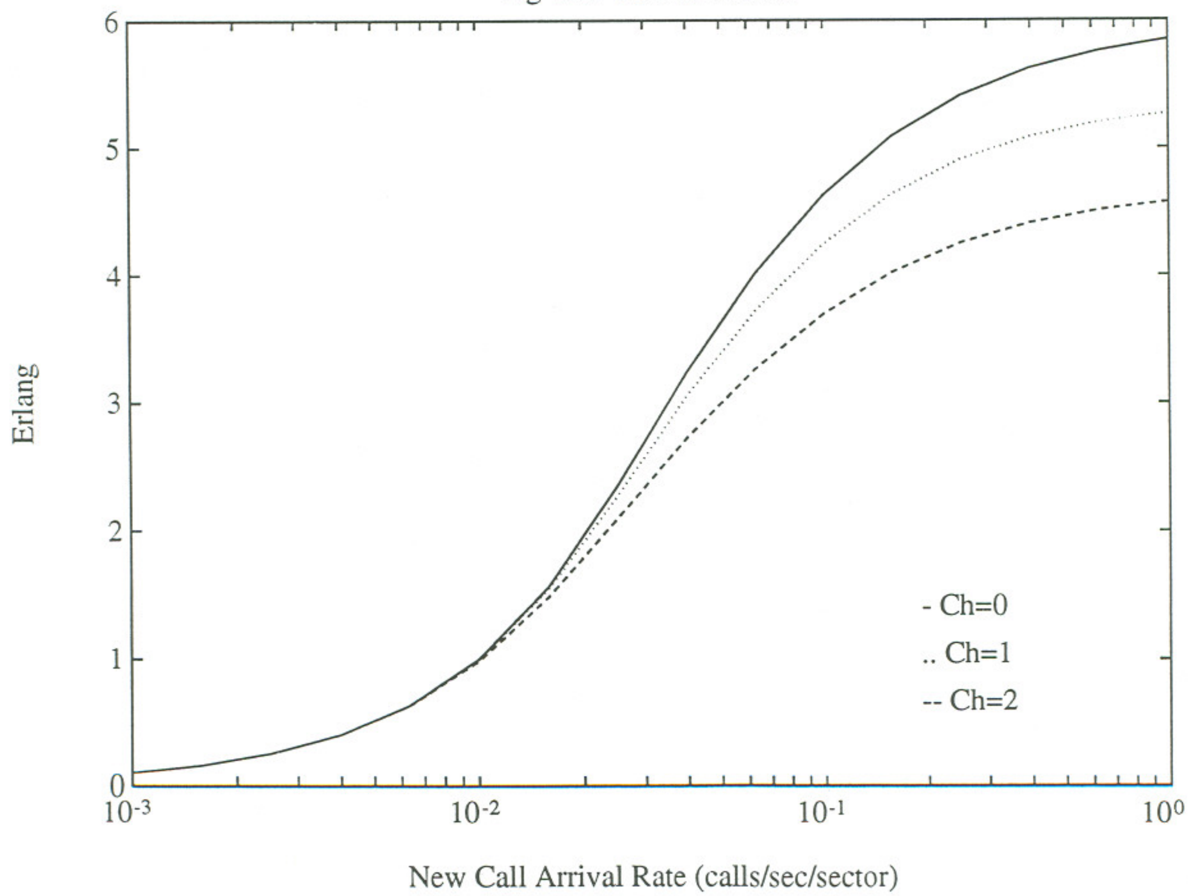


Fig 11.1 Blocking Probability

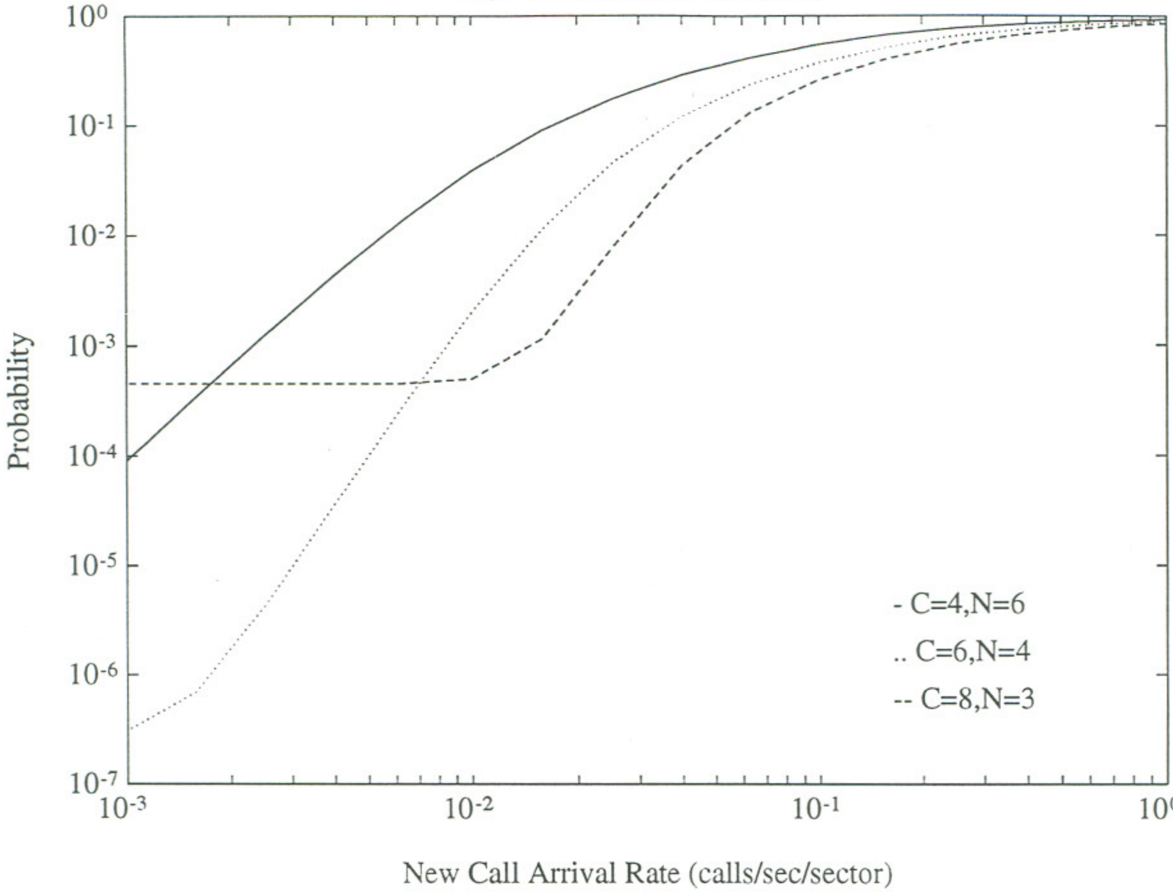


Fig 11.2 Forced Termination Probability

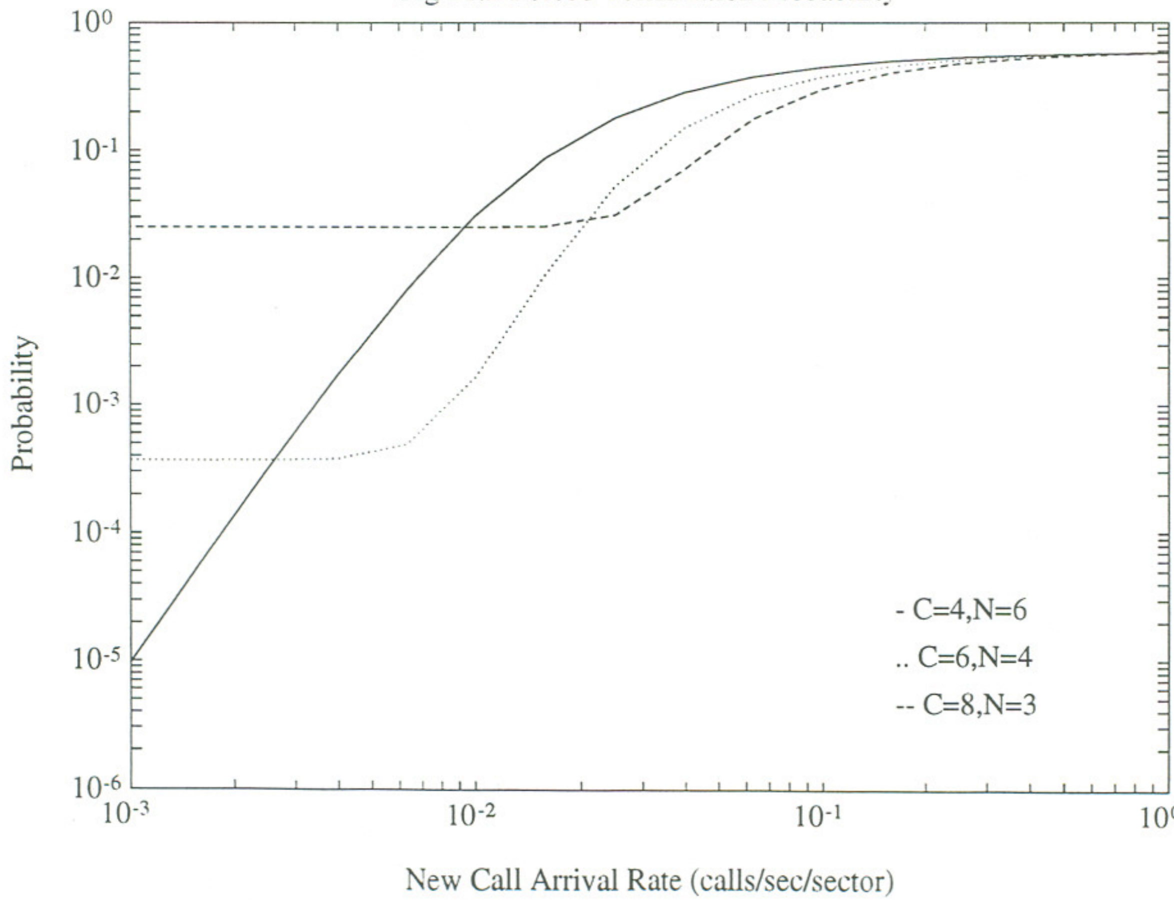


Fig 11.3 Handoff Activity

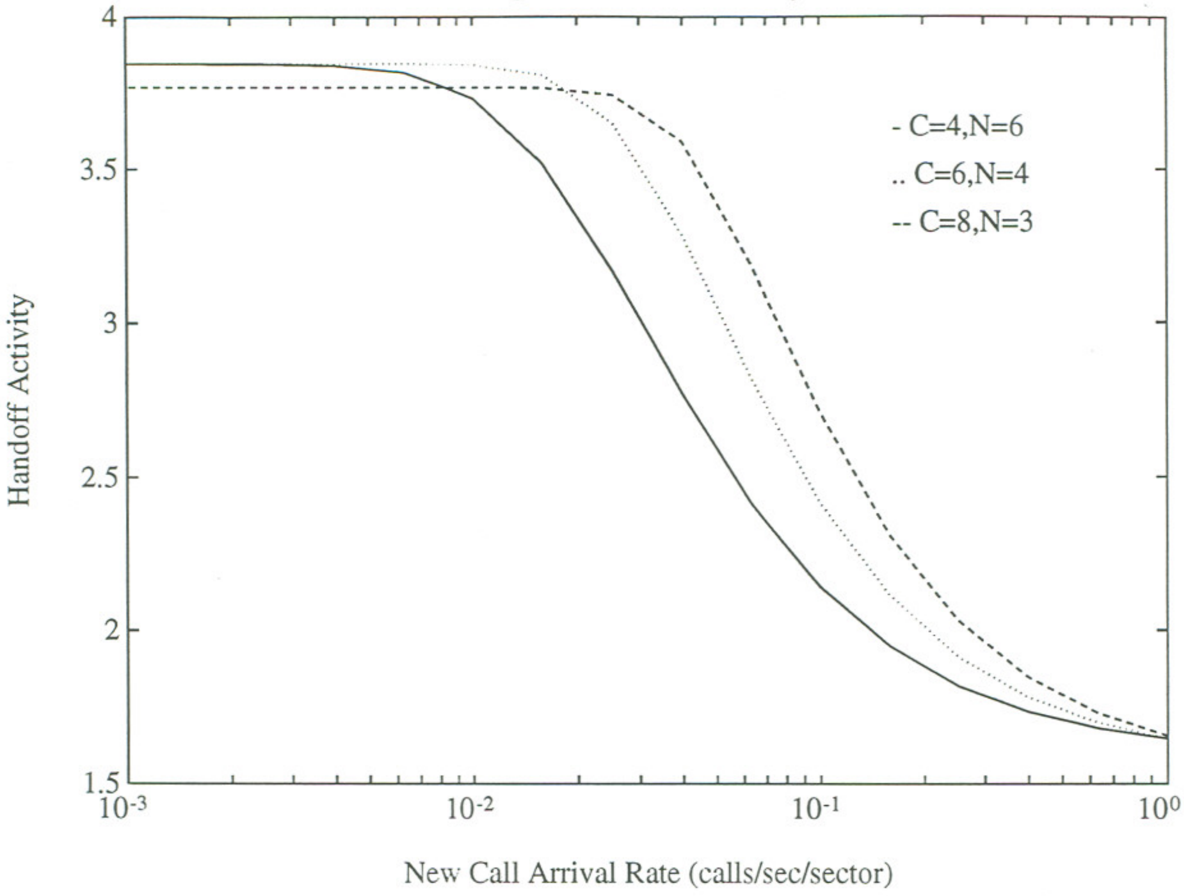


Fig 11.4 Carried Traffic

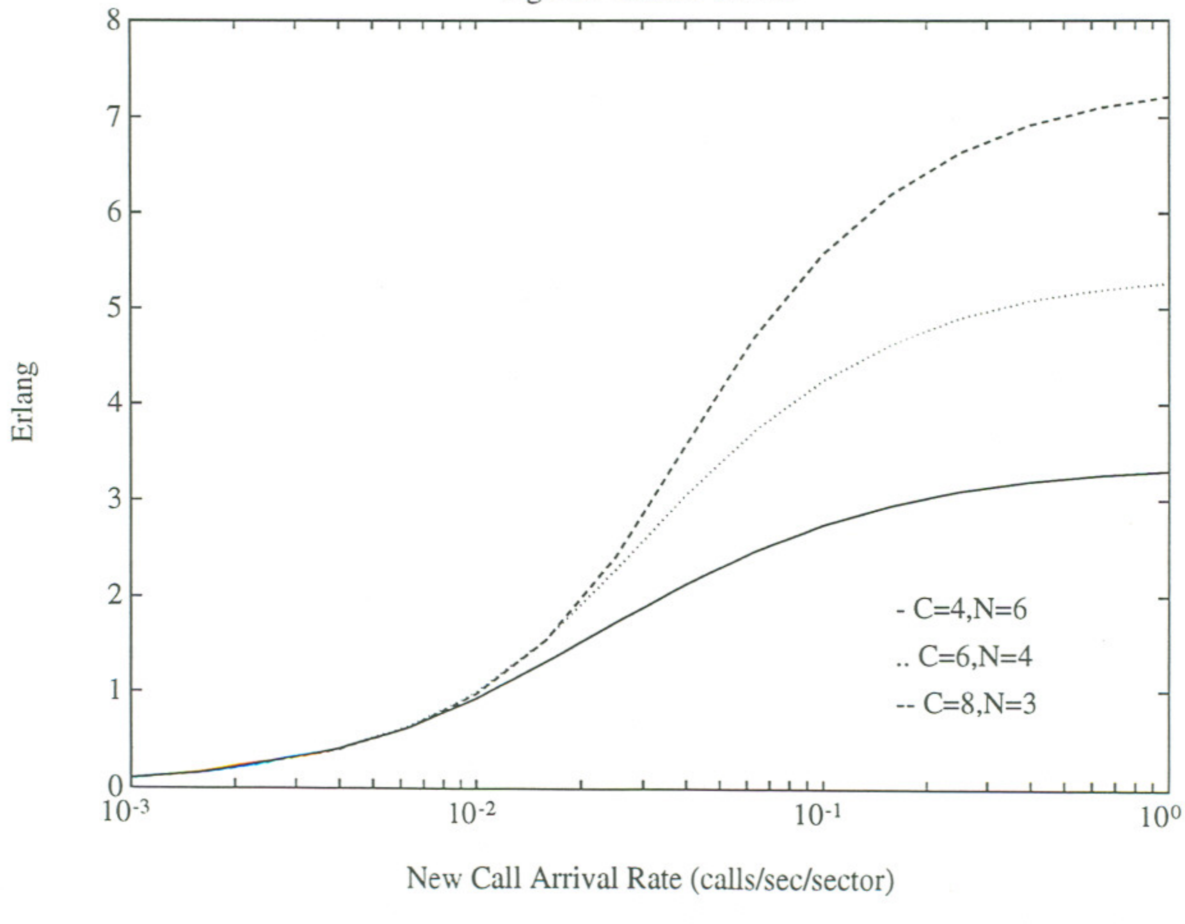


Fig 12.1 Blocking Probability without fading

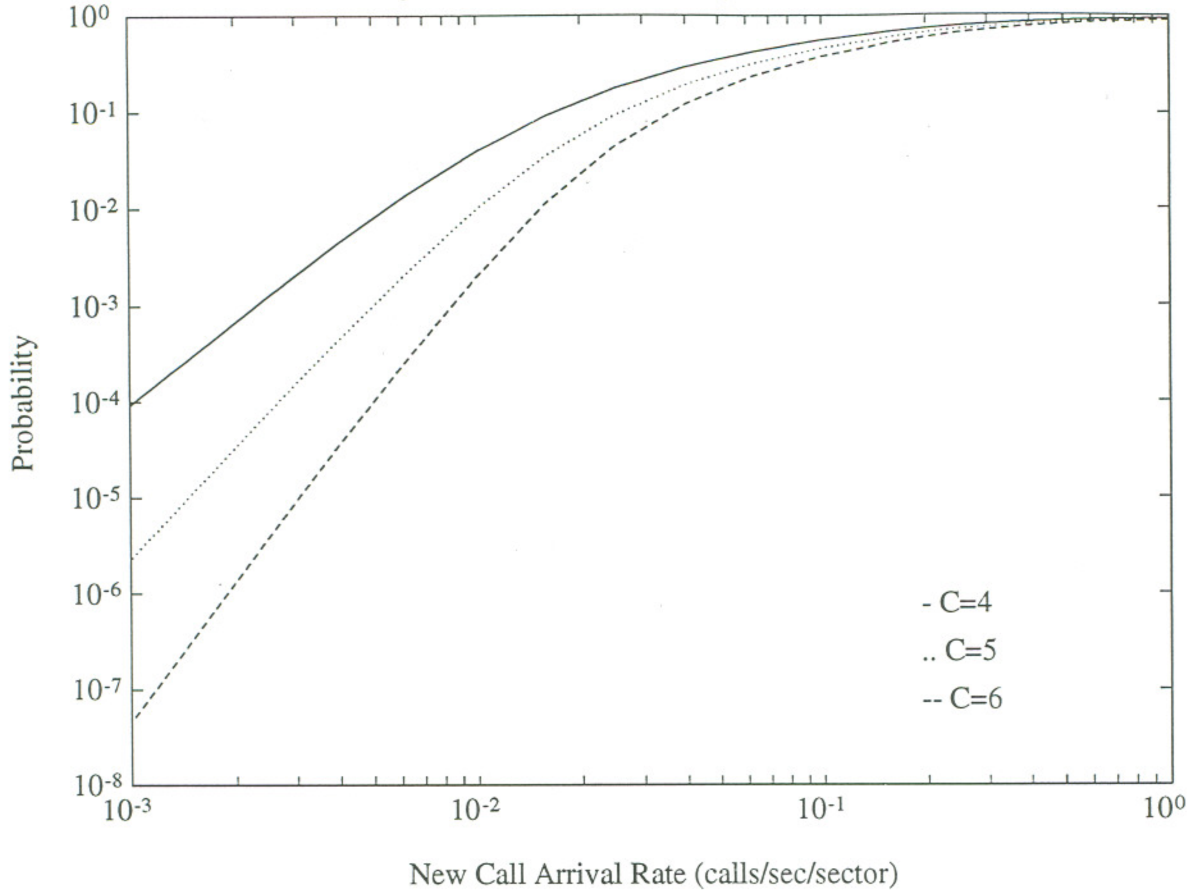
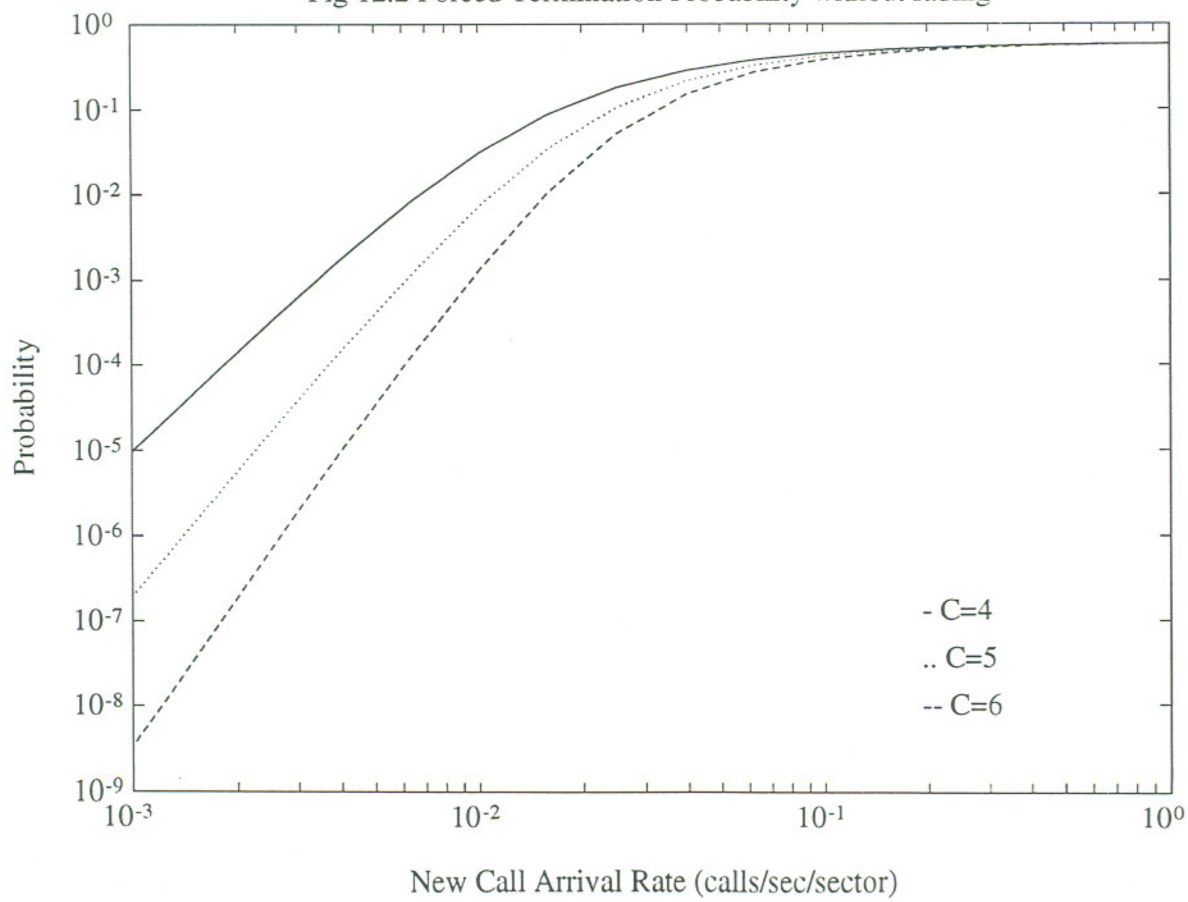


Fig 12.2 Forced Termination Probability without fading



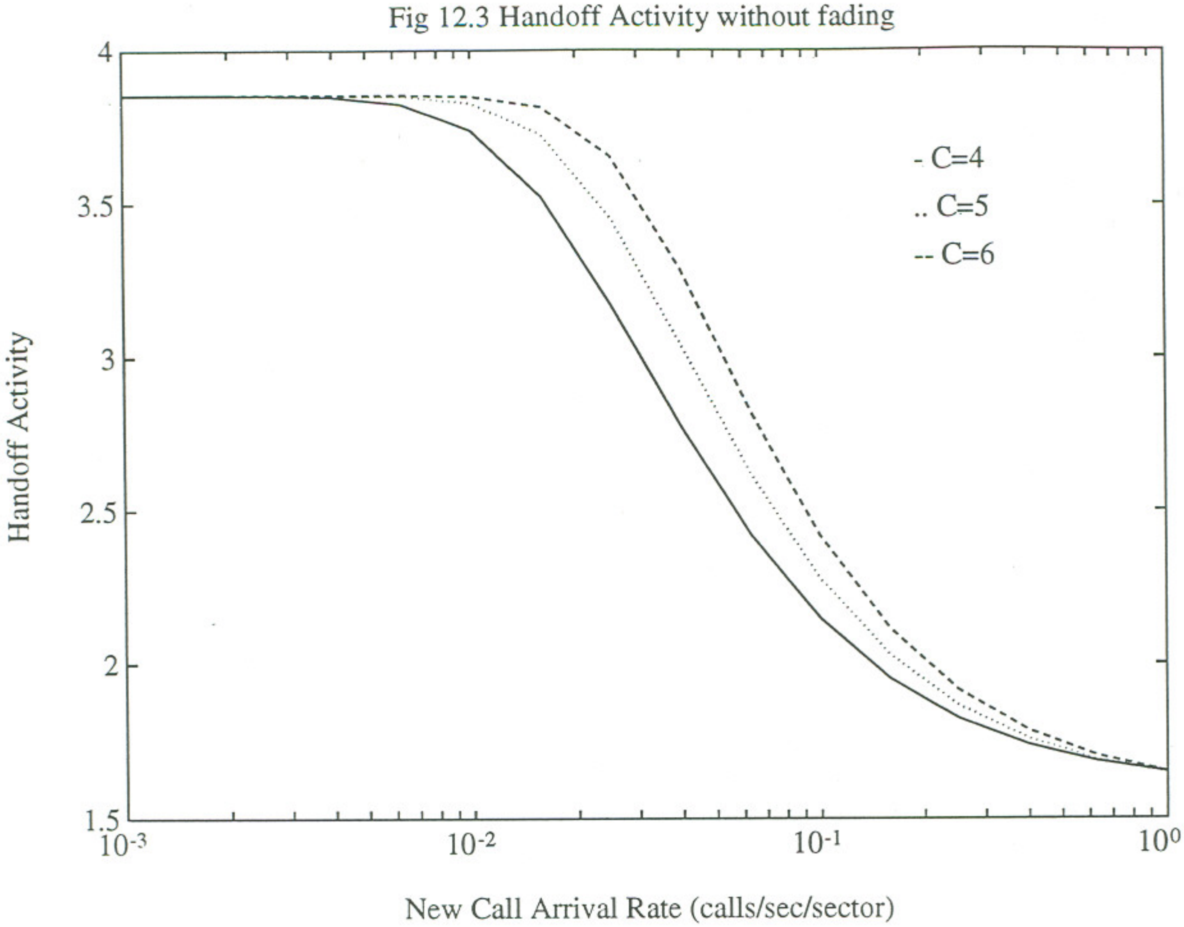


Fig 12.3 Handoff Activity without fading
Handoff Activity
4
3.5
3
2.5
2
1.5
10-3
10-2
10-1
100
- C=4
.. C=5
-- C=6
New Call Arrival Rate (calls/sec/sector)

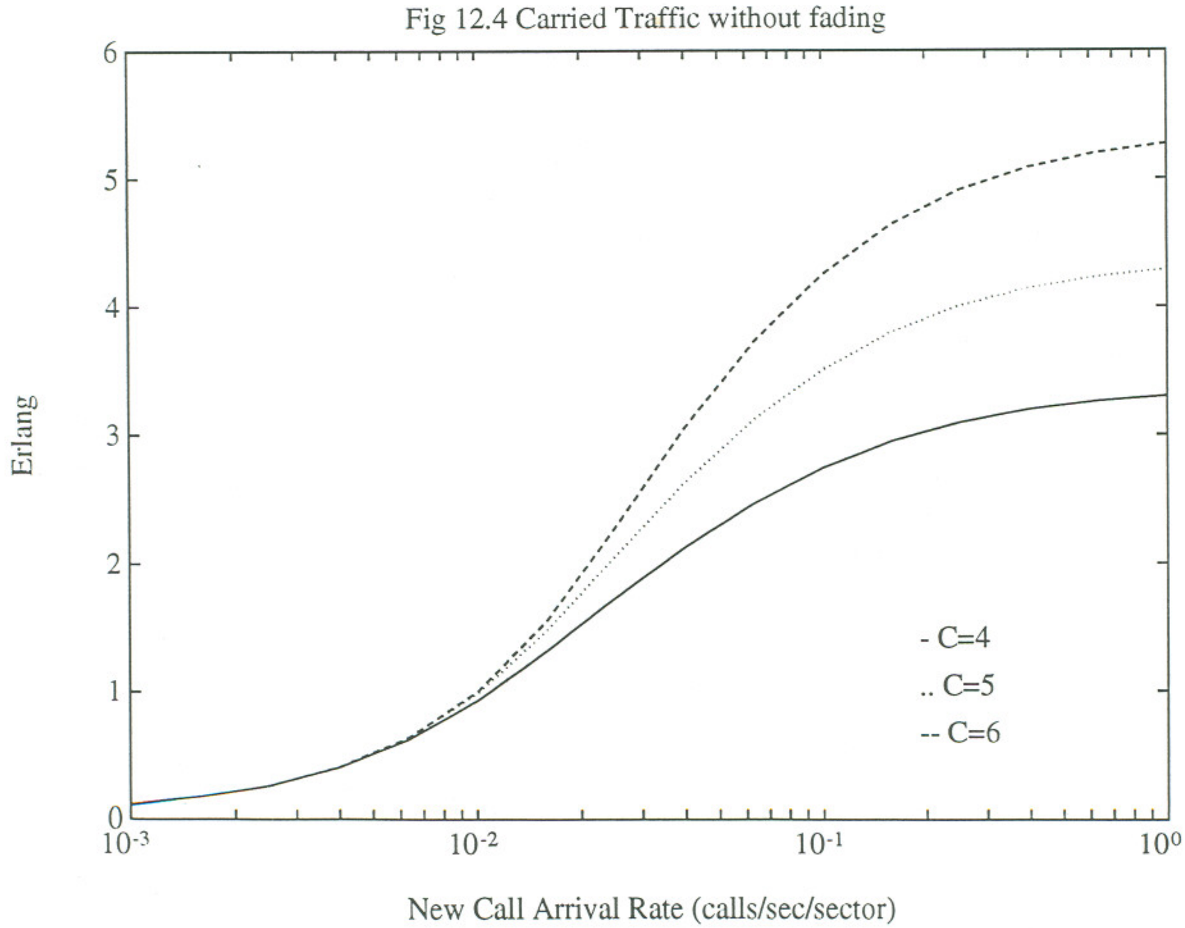


Fig 12.4 Carried Traffic without fading
Erlang
6
5
4
3
2
1
0
10-3
10-2
10-1
100
- C=4
.. C=5
-- C=6
New Call Arrival Rate (calls/sec/sector)