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TECHNIQUES OF OPTICAL SPATIAL FILTERING APPLIED TO THE PROCESSING OF MOIRE FRINGE PATTERNS

by

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ABSTRACT

A coherent optical system which forms the image of an object in two stages is described. Spatial filtering is performed at the first image plane in such a way that the resulting image is altered according to a prespecified fashion.

Many filtering operations can be performed with such a system. Examples given in the paper include methods of moiré family separation (both before and after a cross moiré fringe pattern is formed), pattern contrast improvement, background grating elimination, and suppression of noises.

INTRODUCTION

Ever since the invention of the laser, the field of coherent optics has become the center of research interest for many physicists and engineers. With the help of the highly coherent and monochromatic light from a laser, many a phenomena which heretofore only existed in theory or poorly demonstrated in experiments can now be realized with strikingly good quality (e.g. holography). New concepts and ideas have been generated at such a rate that coherent optical technology has found applications in fields as diversified as antenna simulation [1]* and geophysical data processing [2]. A good account of the recent developments in coherent optics can be found in the survey paper by Cutrona [3].

The process of optical spatial filtering in a coherent light system is to alter the input signal (e.g. the light disturbance transmitted through a moire pattern) by placing filters along the optical path of the light in such a way that the output is changed in a prespecified way. Filtering can be categorized into amplitude, phase, and complex [4], depending upon whether the filtering is performed to the amplitude, phase or both of the light disturbances, respectively. For example, the celebrated** phase contrast microscopy of Zernike [5] is a method of phase filtering. Because of the similarity between optics and communication theory, many modern spatial filtering techniques have been developed by researchers in the field of electrical science, notably by Vander Lugt [4], Cutrona [6], Elias [7], and O'Neill [8], among others.

Since much of the information obtained in experimental stress analysis is done through optical means (e.g., photoelasticity and moiré method),

*Numbers in parentheses designate references at the end of the paper.

**Zernike won a Nobel Prize for his contribution in phase contrast microscopy.

it is only natural to extend the techniques of optical spatial filtering to the processing of experimental stress analysis data. While many methods presented here can be applied equally well to the photoelastic fringe pattern or other interference patterns, the paper will address itself only to the processing of moiré data.

Indeed, this exciting field is not ignored by experimental stress analysists. For example, the contrast improvement method of de Haas and Loof [9] and the fringe multiplication methods of Post [10, 11] and Sciammarella [12] are in fact methods of optical spatial filtering. In this paper a general amplitude filtering system will be described in such a way that many filtering operations can be performed. Operations presented in the paper are that of contrast improvement, background grating elimination, separation of moiré families, fringe multiplication and noise elimination. Possible filtering operations which are beneficial to the processing of moiré patterns are, of course, not limited to the ones presented herein.

OPTICAL SYSTEM FOR SPATIAL FILTERING

While there are many optical systems that can be used for spatial filtering, attention will be given to the system shown in Fig. 1. The system contains a laser (with its light expanded), a first imaging lens and a second imaging lens.Their relative position are as shown in the figure. A significant property of a coherent optical system is that the light disturbance at the front and back focal planes are related by a two-dimensional Fourier transform. Indeed, if f(x,y) is the complex amplitude of the light flux at plane P_1, the complex amplitude of the light flux at plane P_2 is given by

$$F(p,q) = \int_{-\infty}^{\infty}\int f(x,y)e^{i(Px + qy)} \, dxdy \qquad (1)$$

in which (x,y) and (p,q) are the coordinates of the front and back focal planes, respectively. F(p,q) is the so-called diffraction spectrum of the object f(x,y). This spectrum is then collected by the second imaging lens L_2 to form an image of the object at plane P_3. The image is inverted, because instead of inverse transform the lens L_2 performs another Fourier transform. With this system the image forming mechanism is accomplished in two stages, which allows room for manipulation. Filters can be inserted at plane P_2 to alter the information to be collected by lens L_2, and as a consequence, to change the image shown at plane P_3. The filter may be a mask to change the amplitude distribution, or a phase plate to vary the phase distribution, or a hologram complex-filter [4] to alter both the amplitude and the phase. In this paper only amplitude filters will be presented.

Moiré fringes are the interference pattern of gratings of which the diffraction phenomena are relatively simple and may be described in terms of geometric optics as the following. In Fig. 2, a parallel beam is seen propagating through a grating. Each slit of the grating acts, according to Huygen's Principle, as a sub-source which generates cylindrical wave-lets as shown. These wave-lets interfere one another constructively in a variety of ways. The zero order beam is composed of those wave fronts which are not disturbed by the presence of the grating. The first order beams deflected to both sides of the zero order, are composed of those wave fronts in which each one of them is one wave-length ahead or behind the neighboring one. The second order beams are those in which each one of them is two wave-lengths ahead or behind the neighboring one, and so on. If all these beams of different orders are collected by a lens, the image so formed is called the diffraction spectrum of the grating. In this case the spectrum is a series of dots located along a straight line perpendicular to the direction of the lines of the grating. The mechanism of the image forming is shown in Fig. 3. These dots (which are the images of the point source) have different intensities. The zero order image contains the largest energy and the intensity decreases as the number of the order increases. A photograph of the actual diffraction spectrum of a linear grating (300 lines per inch) is shown in Fig. 4.

The distance, d, between any two dots is a function of the focal length of the lens, f, and the pitch of the grating, P, and they are related through the following relations:

$$d = f \tan \theta_1 \qquad (1)$$

in which θ_1 is the diffraction angle of the first order beam. Since it is evident from Fig. 2 that

$$\theta_1 = \text{Sin}^{-1}\frac{\lambda}{p} \cong \frac{\lambda}{p} \tag{2}$$

it can be easily deduced that

$$d = \frac{f\,\lambda}{p} \tag{3}$$

which, for a certain lens and certain wave length of light, is a measure of the grating pitch. The analysis of the diffraction phenomenon of a cross grating will not be presented here but a photograph of the diffraction spectrum of a cross grating (1000 lines per linear inch) is shown in Fig. 5.

If all the diffraction orders are collected by the second imaging lens, the beams will interfere one another to form an image at the second image plane and it will be the exact reproduction of the original grating.* However, since a lens has a finite size, higher order diffractions are not collected by the second imaging lens. The image will, then, have some modification. The amount of modification depends upon the number of orders that are missed by the lens. For example, if a perfect bar-slip amplitude grating is placed at the object plane and only (0,±1) orders of the diffraction spectrum are collected by the second imaging lens, then the resulting grating seen at plane P_3 will be a purely sinusoidal grating. The more the number of orders are let through the second imaging lens, the more closely the resulting image resembles the original square-wave grating. In fact, a totally false image can be obtained if the diffraction spectrum is altered in certain ways as will be seen later on in the text. It has been established [13], however, that at least two of all the diffraction orders of a grating should be collected by the second imaging lens in order to form a gratinglike image.

*Ideal lenses free from aberrations are assumed.

Since every object has its own diffraction spectrum and the image formed at plane P_3 depends upon what is collected by the second imaging lens from the diffraction spectrum, it is then evident that one can choose what he wants to see at plane P_3 by proper filtering at the spectrum plane. Indeed, it is the whole principle of optical spatial filtering. While it is concievable that the number of possible filtering operations is unlimited, only some of those which the author thinks are useful to the processing of moiré fringe patterns will be presented in the following section.

DIFFERENT APPLICATIONS OF OPTICAL SPATIAL FILTERING

I. Contrast Improvement and Background Grating Illumination

It is not unusual that a moiré fringe pattern has a contrast less than desired. For example, in a high speed event with not enough illumination, the photographs will be underexposed and, as a consequence, low-contrasted. The negative of the photographs can then be placed at the object plane of the optical spatial filtering system shown in Fig. 1. The diffraction spectrum of the negative will be displayed at plane P_1. If a filter of a small circular mask (either opaque or with high density) is placed at P_1 at the optical axis to illuminate or greatly reduce the zero order light, the resulting image will have a better contrast than the original negative. If the negative is of such a quality that, although it is of poor contrast but the lines of the grating are resolved, the filtering technique is even simpler. It is only necessary then to place a filter of an opaque mask with a hole at plane P_1 in such a way that only one of the two first orders of the spectrum is passing through the optical system. Higher orders can also be used but with less intensity. An example is given in Fig. 6 where the photographs of a moiré pattern before and after filtering are shown. The difference in contrast is quite evident. It may be recalled that the intensity variation I of a moiré fringe pattern can be expressed as [14]

$$I = I_o + I_1 \cdot \text{Cos}\ \phi \qquad (5)$$

in which I_o, I_1 and ϕ are the background intensity, intensity amplitude change and phase change, respectively. And the contrast c of any light distribution pattern is given by

$$c = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{6}$$

In the case of moiré fringes, one has from eq. (4) that

$$I_{max} = I_o + I_1$$

$$I_{min} = I_o - I_1$$

It then follows that for a moiré pattern

$$c = \frac{I_1}{I_o} \; . \tag{7}$$

Since the zero order of a diffraction spectrum contains the undisturbed light from a diffraction grating, i.e. I_o, it is obvious that by eliminating or greatly reducing the zero order light the contrast of a fringe pattern will be improved.

One bonus of letting only half of any diffraction order (except zero) pass through the second imaging lens is that the background grating on which the moiré is formed is completely eliminated. Because as mentionned in the previous section, it takes at least two orders to form an (interference) image of the grating. The elimination of background grating is important if the density variation of the grating is to be traced by a microdensitometer and to be fed into a computer [15]. de Haas and Loof [9] in their paper used a similar technique to improve the contrast of moiré patterns obtained with the Lightenberg method. However, they claimed that in order to apply their technique to "plane" moiré fringe patterns, it is necessary to introduce a large amount of mismatch which, of course, is not the case shown in Fig. 6.

II. Separation of u- and v- families of Moiré Fringe Pattern

In using the moiré method for strain analysis both the u- and v-families are required for a complete determination of the state of strains of a plane model (u and v are the displacements in the direction of x and y, respectively). They are usually obtained by rotating the master grating through an angle of 90 degrees. However, as pointed out by Dr. Post [16], an inaccurate rotation will cause serious shear errors and it is advantageous to record u- and v- families simultaneously by using a cross master grating. It may be desirable sometimes to separate the two families for reducing the possible confusion caused by the tangling of two sets of fringes. Three methods are presented in the following for the separation of the two families.

Method A:

If both the master and model gratings (either in close contact or with a small gap [17]) are placed at the object plane of the optical system shown in Fig. 1, a cross moiré pattern is formed with both u- and v- fields of fringes. The diffraction spectrum of the grating displayed at P_2 will be similar to the one shown in Fig. 5 except that each dot is now composed of two beams and they interfere to form an interference pattern. A defocused picture of a diffraction spectrum is depicted in Fig. 7. It shows clearly what the information each diffraction order contains. The picture is the defocused spectrum of two cross gratings in contact with rotational mismatch between them. It is seen that u- field and v- field fringes are essentially separated at the orders along the central horizontal and vertical axes, while the zero order still contains the original cross moiré pattern. The orders along central horizontal axis contains only u- family and orders along vertical axis v- family only. The picture is taken with two exposures to bring out some of the weaker orders. Hence, for the separation, the filtering is a simple matter of blocking out all the orders except the one that contains the proper family of fringes.

Method B:

An alternative is to place the cross model grating along at plane P_1 and the cross master grating at plane P_3. A cross moiré fringe pattern will be formed at P_3 if the whole diffraction spectrum is collected by the second imaging lens. The configuration of the spectrum will be again similar to the one shown in Fig. 5 with variations due to the deformation of the model grating. If a filter is made in such a way that only two (or more) neighboring orders along the central vertical (horizontal) axis are collected, only the v- field (u- field) moiré fringes will appear at plane P_3. An example is given in Fig. 8 where the cross moiré pattern and the separated u- and v- field fringes are shown.

Method C:

There are cases where it may be inconvenient or impossible to place the model into the optical system. For example, when the model is not transparent and moiré pattern has to be formed by reflection, then the following technique may be used. First the cross moiré pattern and the background grating are recorded on a film. This film is then placed at plane P_1 of the optical system. The diffraction spectrum will again be similar to the one shown in Fig. 7 in that the two families of fringes are virtually separated. The technique described in Method A can then be used, namely to let only one order containing the proper family of fringes go through the second imaging lens. An example of this method is given in Fig. 9 where a rather confusing cross moiré pattern is separated into two individual families of fringes. Although the fringes shown have some discontinuities at places where the other family is missing, the essential feature of the individual family is preserved and it is definitely less confusing than the original cross moiré pattern.

III. Fringe Multiplication

As shown earlier in Eq. (3), the distance d between the dots of the diffraction spectrum of a grating is inversely proportional to the grating pitch p. This property renders itself as a means for fringe multiplication. Indeed, if one let pass only every other dot (e.g.: 0, ±2, ±4, ... orders) of the diffraction spectrum as shown in Fig. 4, the image formed at plane P_3 will have twice as many lines per inch as the original grating. And if every other two dots are let pass (e.g.: 0, ±3, ±6, ... orders) the image will have a line density three times as great as the original and so on. Since two diffraction orders are sufficient for forming a grating-like image, the maximum times of multiplication are limited only by the size of the lens to admit two separate orders. However, in practice, the intensity of the light also has to be considered because of the weakness in intensity of high diffraction orders. The method presented here is similar to that of Dr. Sciammarella [12] with his elegant mathematical analysis. Interested readers are referred to his paper for further details.

IV. Suppression of Unwanted Signals

If the diffraction spectrum of the unwanted signals is separable from those of the wanted at the spectrum plane, they can be barred from entering the second imaging lens, thus eliminating them from showing on the final picture. An example is given in Fig. 10 where a badly scratched picture is seen corrected by the processing of spatial filtering. The scars are not completely gone, but the quality of the picture is definitely improved.

CONCLUSION

It is seen that with an optical spatial filtering system described in the paper, many filtering operations can be performed to either improve the quality of an existing moiré fringe pattern (examples given are contrast improvement and noise suppression) or to facilitate the interpretations of moiré fringe patterns (examples given are three methods* or moiré fringe family separation and fringe multiplication). Of course the possible operations that can be performed to improve the moiré data analysis are not limited to the ones presented herein and the technique is not limited to the processing of moiré fringe data alone, but can be extended to other optically obtained information such as that from photoelasticity.

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* It should be mentioned that after the completion of the manuscript, the author became aware of the work by Holister [18] in which a method of fringe family separation was presented. The method is essentially the same as the method A of the three methods presented herein but looked at from different point of view.

ACKNOWLEDGMENT

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REFERENCES

1. Ingalls, A.L., "Optical Simulation of Microwave Antennas", presented at the 1964 PTGAP International Symposium, JFK International Airport, Long Island, N.Y., and published in the Program and Digest, Sept. 21-24, 1964.

2. Dobrin, M.B., Ingalls, A.L., and Long, J.A., "Velocity and Frequency Filtering of Seismic Data using Laser Light". Presented at the 34th Annual International Meeting of the Society of Exploration Geophysicists, Los Angeles, California.

3. Cutrona, L.J., "Recent Developments in Coherent Optical Technology", chapter 6 of Optical and Electro-Optical Information Processing, edited by Tippett, Clapp, Berkowitz, and Koester, M.I.T. Press, Cambridge, Massachusetts, 1965.

4. Vander Lugt, A., "Signal Detection for Complex Spatial Filtering", IEEE Trans. Inform. Theory, IT-10, No. 2, April, 1964.

5. See, for example, descriptions of this method in Chapter 8 of "Principles of Optics, by Born & Wolf. 3rd revised edition, Pergamon Press, 1965.

6. Cutrona, L.J., Leith, E.N., Palermo, C.J., and Porcello,L.J. "Optical Data Processing and Filtering Systems", IRE Trans. Inform. Theory, June, 1960.

7. Elias, P., "Optics and Communication Theory", J. of Opt. Soc. Am., 42, Feb., 1952.

8. O'Neill, E.,"Spatial Filtering in Optics", IRE Trans. Inform. Theory, IT-2, June, 1956.

9. de Haas, H.M., and Loof, H.W., "An Optical Method to Facilitate the Interpretation of Moire Pictures", VDI-Berichte, Nr. 102, 1966.

10. Post, D., "New Optical Methods of Moiré Fringe Multiplication", EXPERIMENTAL MECHANICS, Feb., 1968.

11. Post, D., "Analysis of Moiré Fringe Multiplication Phenomena", Applied Optics, 6, (11), 1967.

12. Sciammarella, C.A., "Moiré Fringe Multiplication by Wave Front Reconstruction", presented at SESA 1968 Spring Meeting, Albany, N.Y., May 7-10, 1968.

13. Guild, J., "The Interference Systems of Crosser Diffraction Gratings - Theory of Moiré Fringes". Oxford at the Clarendon Press, 1956.

14. Sciammarella, C.A., "Basic Optical Law in the Interpretation of Moiré Patterns Applied to the Analysis of Strains - Part I". EXPERIMENTAL MECHANICS, Vol. 5, No. 5, May, 1965.

15. Sciammarella, C.A., "Basic Optical Law in the Interpretation of Moiré Patterns Applied to the Analysis of Strains - Part II". EXPERIMENTAL MECHANICS, Vol. 5, No. 6, June, 1965.

16. Post, D., "The Moiré Grid-analyzer Method for Strain Analysis". EXPERIMENTAL MECHANICS, Vol. 5, No. 11, Nov., 1965.

17. Sciammarella, C.A., "Gap Effect in Moiré Fringes Observed with Coherent Monochromatic Collimated Light". EXPERIMENTAL MECHANICS, Vol. 8, No. 10, Oct.,1968.

18. Holister, G.S., "Moiré Method of Surface Strain Measurement". ENGINEER, Jan. 27, 1967.

FIGURE CAPTIONS

1. Arrangement of the components of an optical spatial filtering system
2. Diffraction Phenomenon of a plane wave passing through a grating
3. Image formation mechanism of the diffraction spectrum of a grating
4. Photograph of the diffraction spectrum of a linear grating (300 lines per inch)
5. Photograph of the diffraction spectrum of a cross grating (1000 lines per inch)
6. Photographs showing contrast improvement of a moire fringe pattern
   - (a) before filtering
   - (b) after filtering
7. Defocused diffraction spectrum of two cross gratings in contact with rotational mismatch showing the separation of u-field and v-field moire fringes
8. Separation of moire fringe families with master grating at second image plane:
   - (a) cross moire fringe pattern
   - (b) u-family
   - (c) v-family
9. Separation of moire fringe families after the cross fringe pattern is recorded on film
   - (a) cross fringe pattern
   - (b) u-family
   - (c) v-family
10. Noise suppression by optical spatial filtering
    - (a) before
    - (b) after

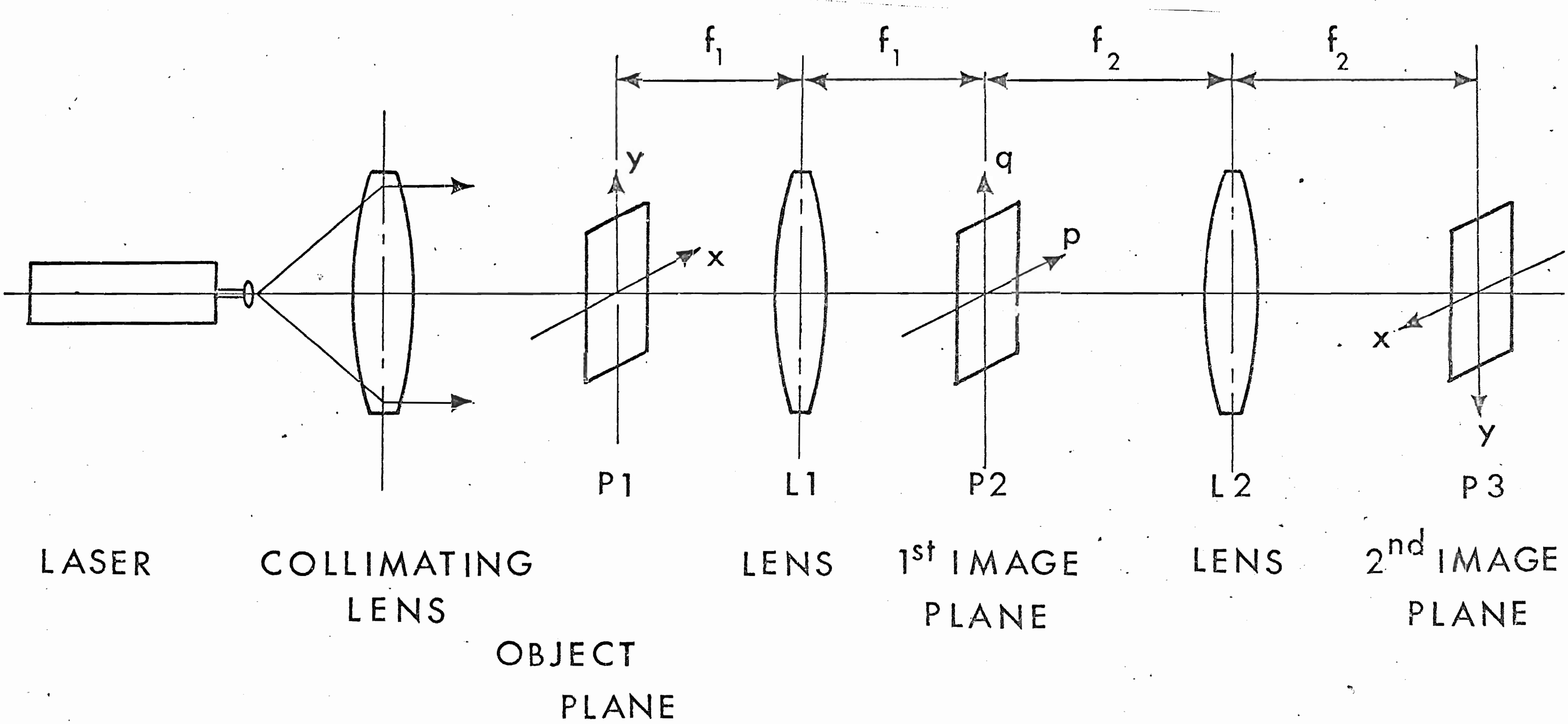


FIG. 1 ARRANGEMENT OF THE COMPONENTS OF AN OPTICAL SPATIAL FILTERING SYSTEM

$$\Theta_n = \sin^{-1} \frac{n \lambda}{p}$$

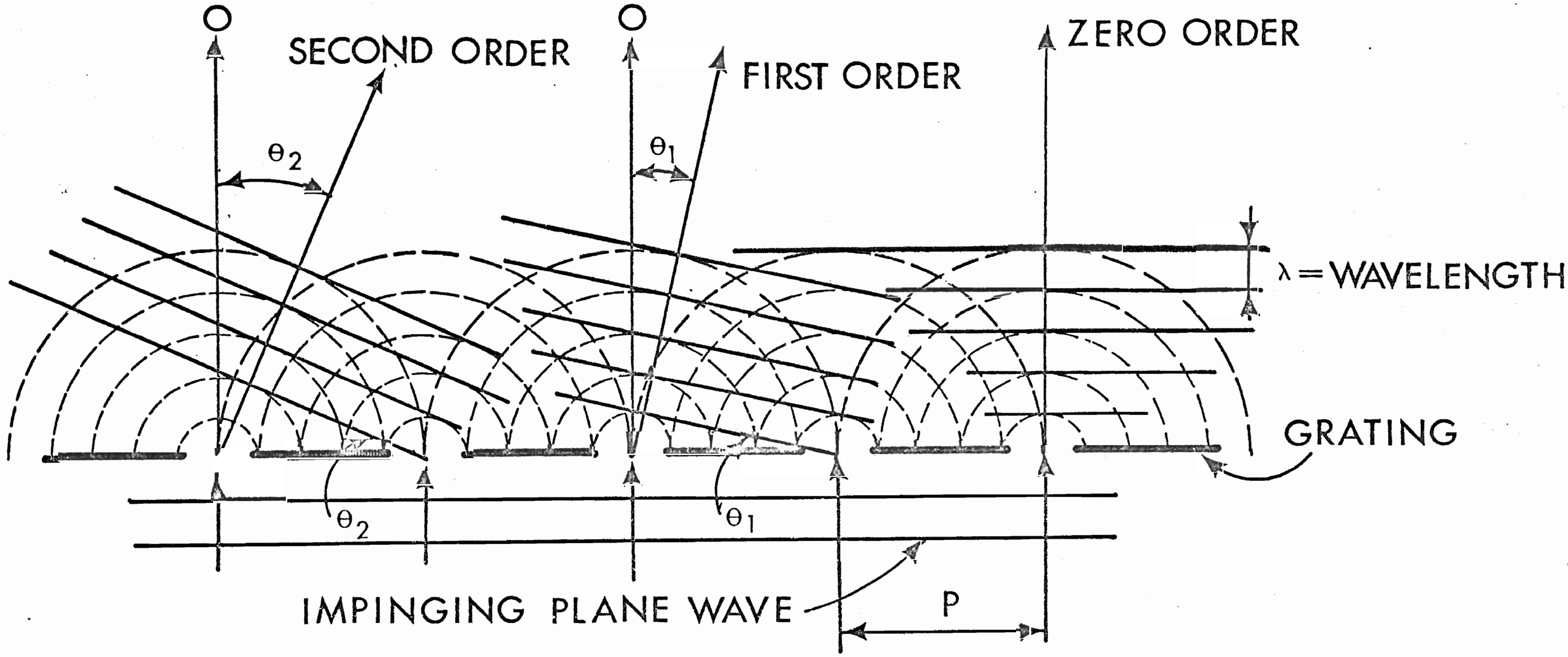


FIG. 2 DIFFRACTION PHENOMENON OF A PLANE WAVE PASSING THROUGH A GRATING

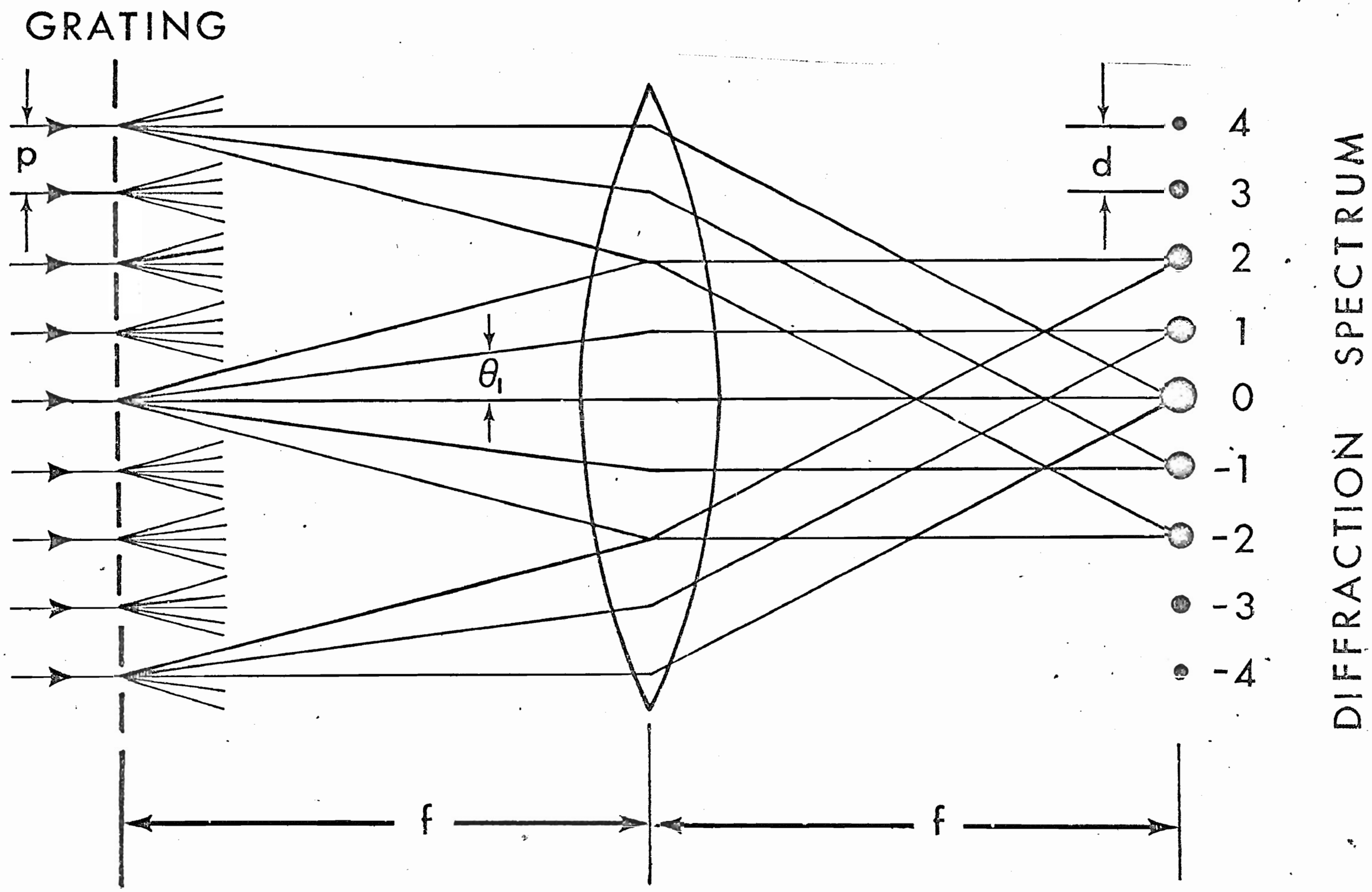


FIG. 3 IMAGE FORMATION MECHANISM OF THE DIFFRACTION SPECTRUM OF A GRATING

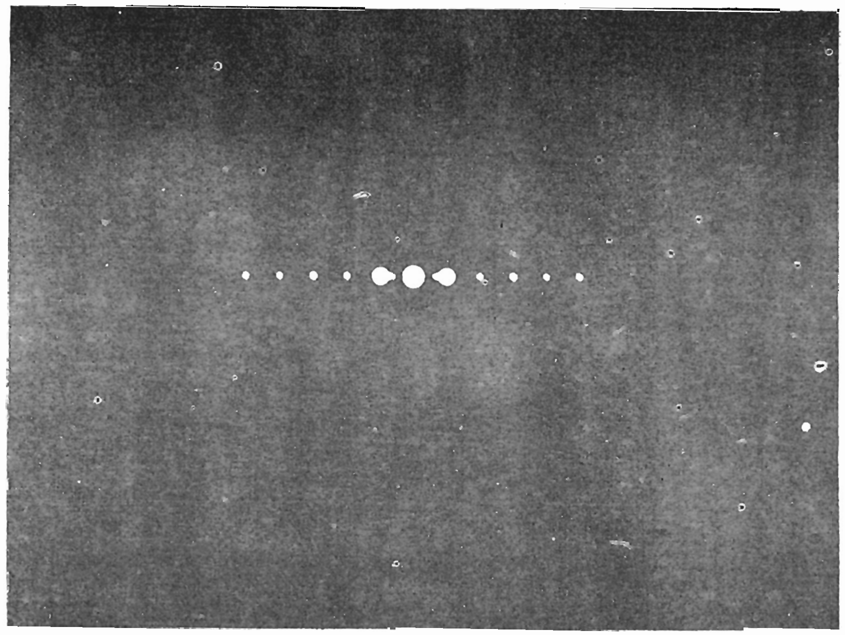


FIG. 4 PHOTOGRAPH OF THE DIFFRACTION SPECTRUM OF A LINEAR GRATING (300 LINES PER INCH)

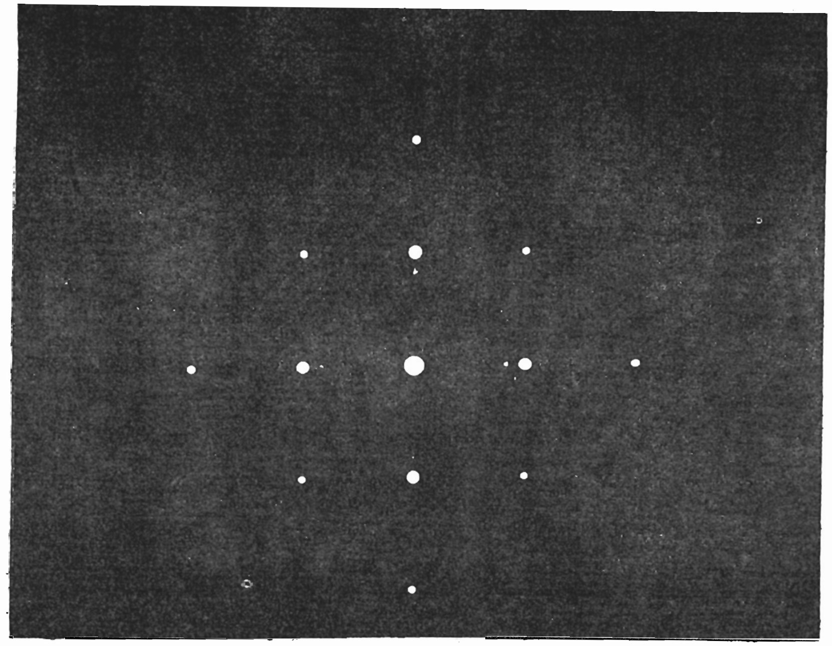
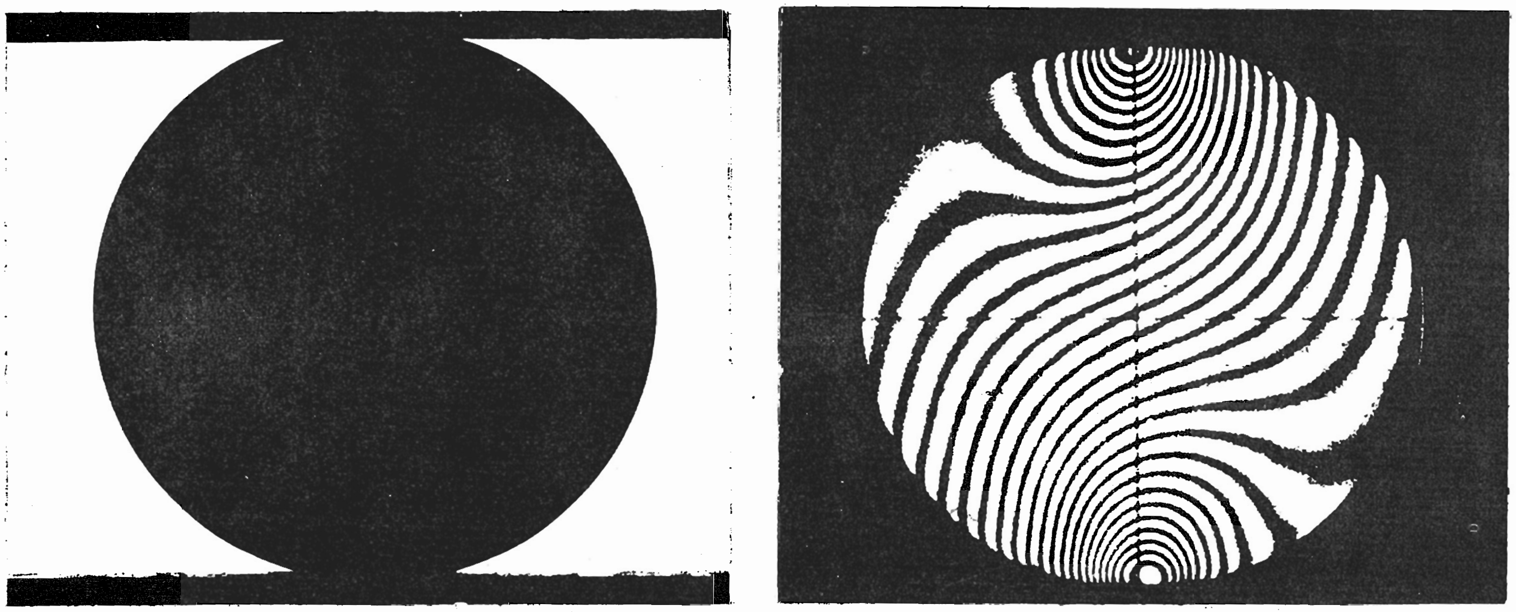


FIG. 5 PHOTOGRAPH OF THE DIFFRACTION SPECTRUM OF A CROSS GRATING (1000 LINES PER INCH)



A B

FIG. 6 PHOTOGRAPHS SHOWING CONTRAST IMPROVEMENTS OF A MOIRE FRINGE PATTERN
(A) BEFORE FILTERING
(B) AFTER FILTERING

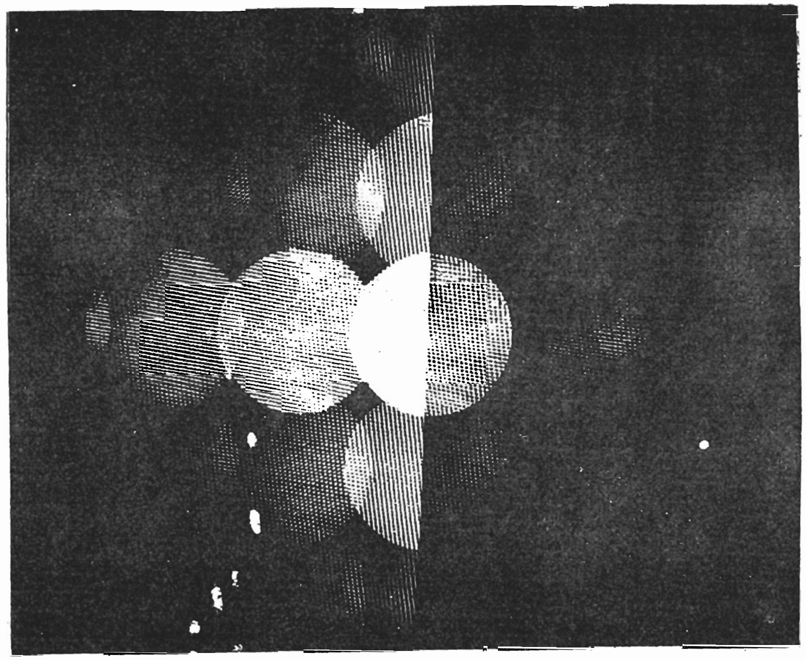
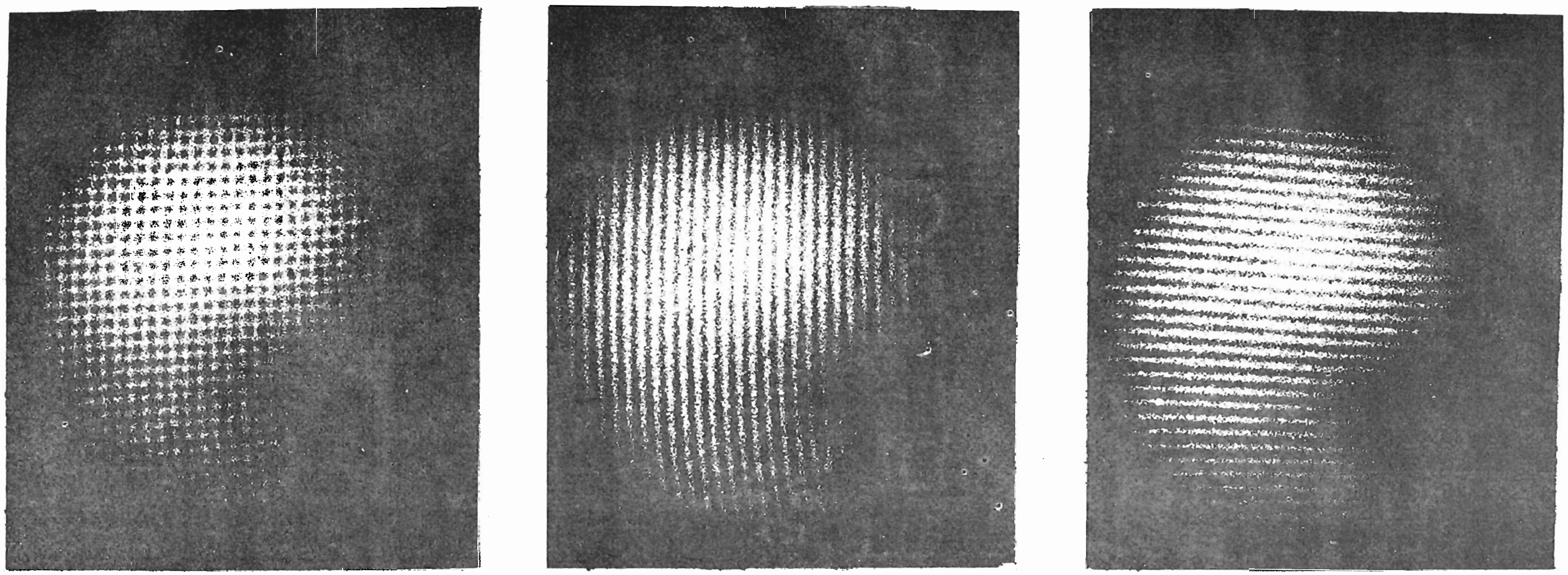


FIG. 7 DEFOCUSED DIFFRACTION SPECTRUM OF TWO CROSS GRATINGS IN CONTACT
WITH ROTATIONAL MISMATCH SHOWING
THE SEPARATION OF U-FIELD AND V-FIELD MOIRÉ FRINGES



A B C

FIG. 8 SEPARATION OF MOIRE FRINGE FAMILIES WITH MASTER GRATING AT SECOND IMAGE PLANE
(A) CROSS MOIRE FRINGE PATTERN
(B) U-FAMILY
(C) V-FAMILY

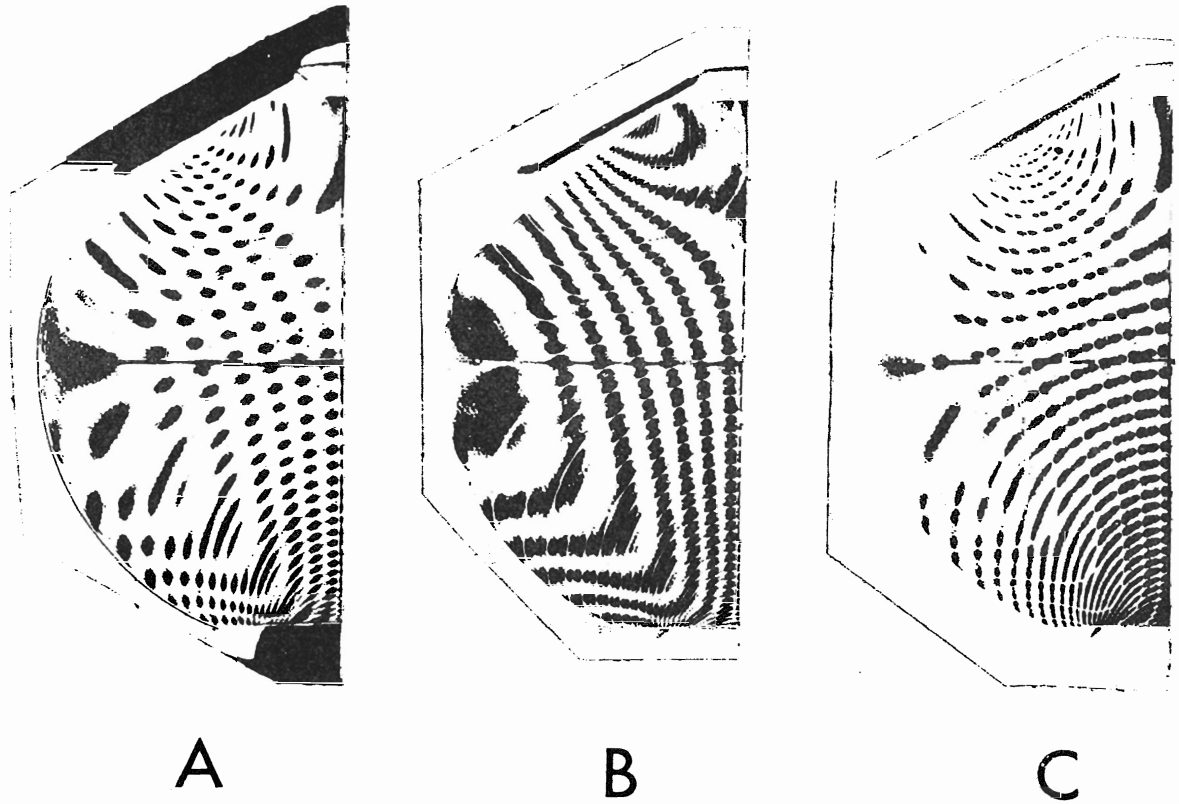


FIG. 9 SEPARATION OF MOIRE FRINGE FAMILIES AFTER THE CROSS FRINGE PATTERN IS RECORDED ON FILM

(A) CROSS FRINGE PATTERN
(B) U-FAMILY
(C) V-FAMILY

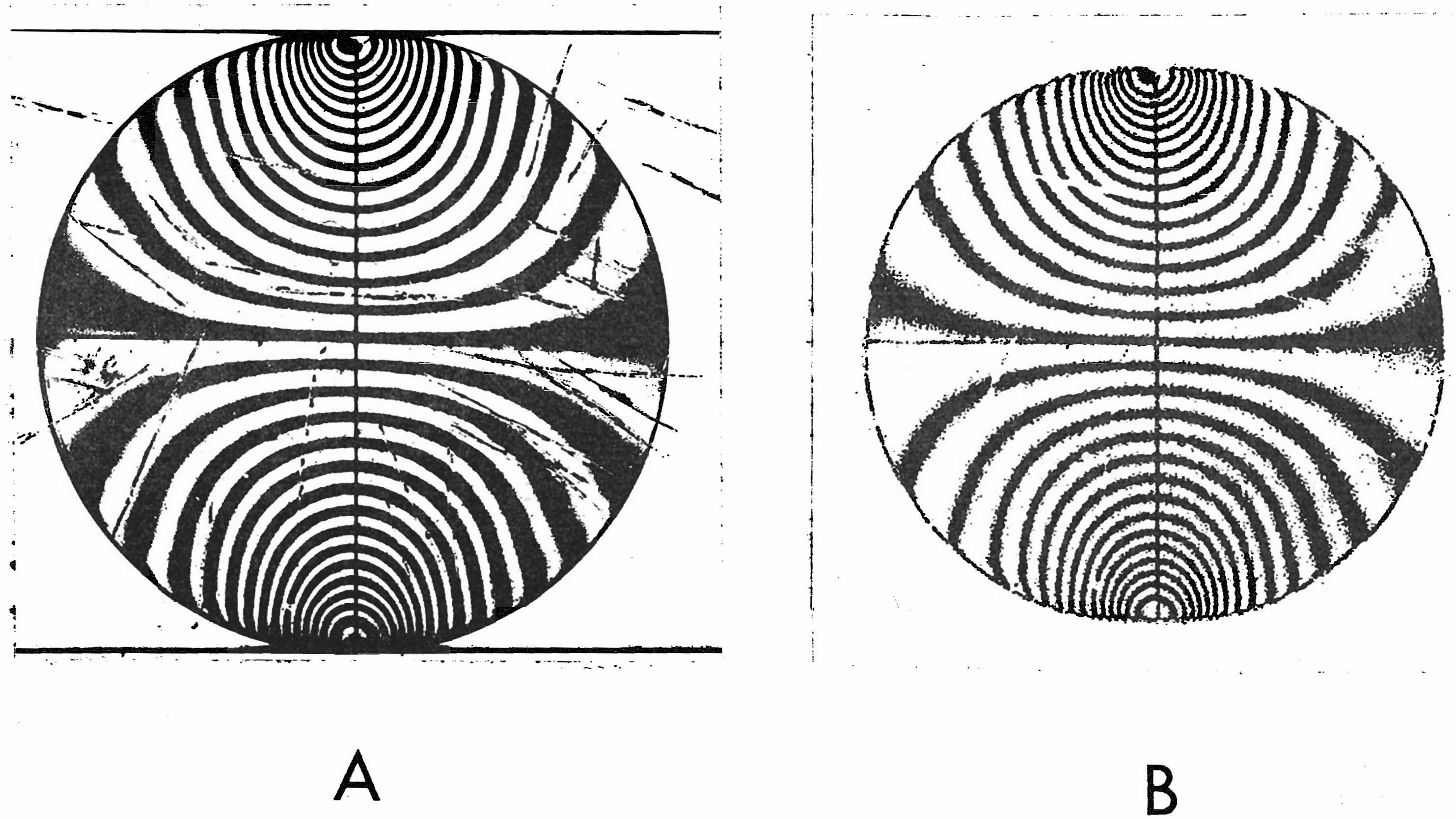


FIG. 10 NOISE SUPPRESSION BY OPTICAL SPATIAL FILTERING
(A) BEFORE
(B) AFTER