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Prioritized Resource Assignment for Cellular Communication Systems with Queued Hand-Offs

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ABSTRACT

Emerging trends in cellular communications will require systems that must support a number of different call types. These may include voice only, mixed voice and data, high speed data and intelligent network services. Each of the call types requires a distinct combination of several network resources. In addition, these calls are supported on platforms of different mobilities. As a result of these factors calls will experience differences in call blocking and/or hand-off failure probabilities.

A cellular system with mixed platforms supporting calls with differing resource requirements and hand-off queuing is considered. A suitable framework for performance analysis is identified, that enables theoretical performance characteristics to be numerically computed. An analytical relationship between the average hand-off arrival rate and the new call origination rate is developed. The model is used to analyze an example system. Sample performance characteristics are shown and discussed.

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1. INTRODUCTION

The problem of resource allocation has been studied quite extensively, mostly in data multiplexing problems and circuit switching contexts. These studies concentrate on loss systems, in which calls arrive in a number of streams and contend for a usage of a single resource. A number of results have been obtained, one of the most general of which shows the product form solutions to hold for arbitrary holding time distributions as well as a wide class of resource sharing policies [9][10]. However, since the authors limited the considerations to policies giving rise to analytical solutions, a number of important classes of systems have not been discussed. Most notably, cut-off priorities and delay systems have been missing from wide consideration. Such systems are very attractive in cellular communications systems, where the goal is provide a good protection against hand-off failure without excessively compromising other performance measures. The solutions provided in [8] are developed for a system that queues only new call originations (under the restrictive implicit assumption that they are stationary) and provides cut-off priorities to hand-off attempts. The approach does not generalize to multiple call types, contending for multiple resources. Moreover, in modeling of cellular systems additional constraints are placed on the system arrival rates, due to coupling of traffic components between cells. Such a requirement not only complicates the numerical solutions but also ruins the simplicity of known analytical solutions.

In this paper we present a formulation of a multiple resource allocation problem, that includes cut-off priorities and hand-off queuing with quite general queuing discipline. We start by formulating the problem in its full generality and then we show how some special cases can be solved analytically. We then proceed to discussing the numerical solution and a method to accelerate the numerical algorithm by utilizing analytical results obtained for related problems. The numerical results give some insight into the interdependence of mobility factors and call resource requirements.

The approach taken in this report further develops the framework outlined in references [1],[2],[3],[4],[5] and [12].

2. PROBLEM DESCRIPTION

We consider a large geographical region, tessellated by cells that are defined by proximity to designated network gateways (base stations). We consider a cell to be an area in which a communication link of acceptable quality can be established between a mobile platform and the cell gateway. Generally, the cell coverage areas overlap. A cell can be conceptually partitioned into two zones. In one, *ONLY* the cell's own gateway is able to provide service. We call this an *inner zone*. In the other, called the *transition zone*, at least two gateways can provide a platform with a link of acceptable quality.

The service region is traversed by large numbers of mobile platforms. The platforms are of several types, differing in mobility. There are G platform types, labeled g=1,2,3 ... G. Each platform can support one call at any given time. Mobility of a g-type platform can be described by defining a *dwell time in an inner zone* and a *dwell time in a transition zone*. The *dwell time in an inner zone* is the time that a g-type platform remains in communication range of only a single gateway. This is a random variable denoted $T_D(g)$. The *dwell time in a transition zone* is the time that a g-type platform remains in the transition zone. It is also a random variable denoted $T_T(g)$.

A platform that has a call in progress is called a *communicating platform*. There are I types of calls, labeled i=1,...I, differing in the resources needed to support the call.

In each cell there are K types of resources used to support calls. There are R_k, k=0 ... K-1, units of resource k at every gateway. Resources that may be required include bandwidth (measured in appropriate units such as channels), buffer space and call supervising processors.

A communicating platform with a link in a *source* cell can move to a *target* cell. If an i type call is in progress on a platform that needs resources in the target cell, a demand for a set of resources is placed at the target gateway. The event is called a *hand-off attempt of type i*. In general the service facility can be of *loss type*, in which calls that cannot be assigned resources in the target cell immediately are cleared from the system. An alternative is a *delay type* service in which a platform with hand-off needs is placed in queue awaiting the release of a sufficient number of resources to satisfy its demand. The source gateway is able to provide service to such a platform for as long as the platform remains in the transition zone. If the platform leaves the transition zone before sufficient resources are available at the target gateway, the call in progress on the platform will be terminated. When a call in progress is terminated, a *hand-off failure* is said to occur. It should be emphasized that calls can be successfully completed while the supporting platform is in the transition zone. Such an event is not a hand-off failure. Many variations of the two basic strategies described above are possible e.g. hybrid delay/loss system, i.e. a system with a hand-off queue and limited waiting space of size Q. New call attempts are not queued.

A call (hand-off) of type i occurring on a platform of type g is called a call (hand-off) of type (g,i). There are G·I possible call (hand-off) types.

The priority rule employed determines the order in which queued hand-offs are given access to resources at the target gateway. The approach presented here allows for analysis of priority systems in which calls do not change priority level in the course of a call. We will use index j to denote priority level. The set of all priority levels is denoted $\mathcal{J}$.

Some of the priority ordering schemes combined with delay/loss type service facility, can lead to calls being displaced from the queue.

The *unencumbered* session duration is the amount of time that a call would remain in progress if it were not terminated by hand-off failure. It is a random variable $T(i)$.

At any given instant, a single cell in the system can be characterized by its state. The state is specified by G+2 n-tuples of integers. The first G n-tuples describe the conditions prevailing at the target gateway. The g-th n-tuple consists of I integers: $v_{g1}\ v_{g2}\ \ldots\ v_{gI}$. The integer v_{gi} denotes the number of communicating platforms of type g with call of type i in progress (at the gateway). The remaining two n-tuples specify the conditions in the hand-off queue i.e. the status of platforms awaiting resources at the target gateway. Both consist of Q integers. The n-tuple $z_1 z_2 \ldots z_Q$ specifies the call types on queued platforms. In particular, z_q is the type of the call in progress on the q-th platform in queue. The n-tuple, $x_1 x_2 \ldots x_Q$, specifies the platform types of the queued platforms, that is, x_q is the platform type of the q-th platform in queue. Succinctly, the state of a cell can be written as

$$\begin{matrix} v_{11} & v_{12} & \cdot & \cdot & \cdot & v_{1I} \\ v_{21} & v_{22} & \cdot & \cdot & \cdot & v_{2I} \\ \cdot & \cdot & \cdot & v_{gi} & \cdot & \cdot \\ v_{G1} & v_{G2} & \cdot & \cdot & \cdot & v_{GI} \\ z_1 & z_2 & \cdot & z_q & \cdot & z_Q \\ x_1 & x_2 & \cdot & x_q & \cdot & x_Q \end{matrix} \tag{1}$$

It is convenient to order the states using an index $s=0,1,\ldots,s_{max}$. Then the state variables can be shown explicitly dependent on the state. That is, $v_{gi}=v(s,g,i)$, $z_q=z(s,q)$ and $x_q=x(s,q)$.

The number of units of resource of type k that is being used by calls of type i on platforms of type g when the gateway is in state s is $r_k(s,g,i)$. If $r_k(s,g,i)$ is proportional to $v(s,g,i)$ then

$$r_k(s,g,i) = r_{ki} \cdot v(s,g,i) \tag{2}$$

where r_{ki} is the amount of resource k required to service a single i-type call.

The quantity of resources of type k being used by calls of type i is

$$r_k(s,\cdot,i) = \sum_{g=1}^{G} r_k(s,g,i) \tag{3}$$

and the quantity of resources of type k being used by g-type platforms is

$$r_k(s,g,\cdot) = \sum_{i=1}^{I} r_k(s,g,i) \tag{4}$$

The total quantity of resources of type k being used in state s is

$$r_k(s,\cdot,\cdot) = \sum_{i=1}^{I} \sum_{g=1}^{G} r_k(s,g,i) \tag{5}$$

Every cell has R_k units of resource k. Clearly the total resource usage in any state is subject to *resource limit* which requires

$$r_k(s,\cdot,\cdot) \le R_k\,, \quad k = 1,2\ldots K. \tag{6}$$

In addition *resource quotas* can be imposed on the system which limit the quantity of resource of type k that can be used simultaneously by g-type platforms and require

$$r_k(s,g,\cdot) \le R_k^p(\mathrm{g})\,, \quad g = 1,2\ldots G;\ k = 1,2\ldots K \tag{7}$$

or limit the quantity of resource of type k used by i-type calls and require

$$r_k(s,\cdot,i) \le R_k^c(\mathrm{i})\,, \quad i = 1,2\ldots I;\ k = 1,2\ldots K \tag{8}$$

Additional constraints can be placed on the operation of the system by imposing cut-off priorities. These reserve a number of resources for exclusive hand-off use. It should be noted that specific resources are not reserved, just a number. The new call attempt of type (g,i) will fail if the number of resources of type k in use is $R_k - R_k^h(g,i)$ or greater. In contrast, the hand-off attempt of type (g,i) will fail only if its admission would violate a resource limit or quota.

Permissible states correspond to all possible collections of n-tuples in the form of (1), for which all the resource limits and quotas are satisfied. The set of permissible states Φ and transitions between states are determined by the resource sharing policy as well as call admission policy and queuing discipline. The resource sharing policy is in turn described by a set of constraints, governing the allocation of resources. Some of the constraints change the set Φ (e.g. (6),(7) and (8)) while others modify the state transitions without changing Φ (e.g. cut-off priorities).

The state of a cell changes as time progresses. The state transitions are driven by underlying random processes, which include the following:

(1) Generation of new calls - new calls are assumed to arrive in $G \cdot I$ Poisson streams of intensity $\Lambda(g,i)$. Defining Λ_0=$\Lambda(1,1)$ and $\alpha(g,i) = \Lambda(g,i)/\Lambda_0$, $\Lambda(g,i)$ can be written as

$$\Lambda(g,i) \;=\; \alpha(g,i) \;\cdot\; \Lambda_0 \tag{9}$$

(2) Call completions;

(3) Arrival of communicating platforms at the cell (*hand-off arrivals)* - similarly to new call originations, hand-off arrivals are assumed to arrive in $G \cdot I$ Poisson streams of intensity $\Lambda_h(g,i)=F(g,i) \cdot \Lambda_h$, where F(g,i) is the average fraction of hand-off arrivals that are i-type calls on g-type platforms.

(4) Departure of communicating platforms from the cell (*hand-off departures)*;

(5) Departure of communicating platforms from the transition zone (*defections from the hand-off queue).*

The entire cellular system, of course consists of many cells. Since most hand-off events (specifically all except for hand-offs which find the hand-off attempt queue full) will result in a change of state of two cells simultaneously, a complete model should characterize *system state*. System state is a concatenation of the corresponding cell states. However, this approach leads to a system description burdened by overwhelming dimensionality. An alternative approach is to consider only one cell and balance the average rate of hand-off attempt arrivals and departures [1], [2].

3. PROBABILITY FLOW EQUATIONS

In this section we develop probability flow equation.

In the following we assume that the cell is in state s and we find the rates of flow out of state s due to events generated by the underlying processes. We also show the form of the successor state, ξ as a function of the current state s.

3.1.Flow due to new call originations.

A new call of type i arising on a platform of type g can be served only if the resource usage is below the resource quotas and cut-off limits prescribed for the (g,i) type call and no hand-off attempts are queued awaiting resources. Otherwise the call is cleared from the system. As a result of admission of a new call of type i arising on a g-type platform we find the system in state ξ with $v(\xi,g,i)=v(s,g,i)+1$. The component of flow out of state s due to new call arrivals of type i on g-type platforms is

$$\gamma_n(s,g,i)=\begin{cases}\alpha(g,i)\cdot\Lambda_0 & r_k(s,g,\cdot)\le R_k^p(\mathrm{g}) \text{ and } r_k(s,\cdot,i)\le R_k^c(\mathrm{i}),\\ & \text{and } r_k(s,\cdot,\cdot)\le R_k-R_k^h(g,i) \text{ for all k,i,g}\\ 0 & \text{otherwise}\end{cases} \tag{10}$$

and the corresponding successor state, ξ is

$$\begin{matrix}\cdot & \cdot & \cdot\\ \cdot & v_{gi}+1 & \cdot\\ \cdot & \cdot & \cdot\end{matrix} \tag{11}$$

The dots denote all state variables that do not change in the state transition.

3.2.Flow due to call completions.

Call completions (at the gateway) of type i on g-type platforms when the cell is in state s occur at the rate

$$\gamma_c(s,g,i)=\mu(i)\cdot v(s,g,i) \tag{12}$$

The expression for the successor state can be determined by a recursive procedure. A recursive procedure is needed due to the fact that resources released by the completed call may be assigned to a number of queued calls. With no hand-off attempts in the queue, after a call completion of an i-type call on a g-type platform (located in the inner zone) we find the system in a state ξ with $v(\xi,g,i)=v(s,g,i)-1$. If, however, the queue exists, the pool of available resources will be assigned to a possibly large number of queued calls. The procedure assigns the resources according to the priority rule employed until the queue is empty or the resources exhausted.

For example consider a system in which calls contend for a single resource. A type (g,i) call is completed while the system is in state s (in which all resources are in use), releasing 2 resource units. There are two hand-offs of type (h,k) in the queue, each needing single unit of the resource. The cell state ξ, after the call completion is found to be

$$
\begin{array}{ccccc}
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & v_{gi}-1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & v_{hk}+2 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & & & \\
0 & 0 & & &
\end{array}
$$

Calls can also be successfully completed while the supporting platform waits in the hand-off queue. Call completions on the q-th platform in queue occur at the rate

$$\gamma_c(s,q)=\mu(z(s,q)) \tag{13}$$

Clearly, this event can only occur while the system is in state s such that $z(s,q)\neq 0$. The completed call leaves the queue and all calls waiting in positions $q+1$, $q+2$, ... Q advance one position. Completion of the call with the highest priority, might allow the calls queued with the 2nd and lower priority levels to enter service utilizing resources previously reserved for the first call in the queue. A procedure similar to the one described in the previous paragraph can be used to determine the successor state.

3.3.Flow due to hand-off arrivals.

Flow due to hand-off arrivals of type i on g-type platforms when the system is in state s can be determined as

$$\gamma_h(s,g,i)=\begin{cases}\Lambda_h(g,i) & \text{when } z(s,Q)=0\\ 0 & \text{otherwise}\end{cases} \tag{14}$$

A hand-off arrival that finds no queue or is assigned a higher priority class than any hand-off waiting in the queue and if sufficient resources are available to serve it, will be given access to them immediately and continue service at the target gateway. The successor state can be determined as for new call arrivals. A hand-off that cannot be serviced immediately is placed in queue. Hand-offs that find the queue fully occupied are not allowed to contend for resources at the target gateway. They will continue to be supported by the source gateway for as long as they remain in the transition zone.

For example, when the priority rule employed is a simple head-of-the-line and the hand-off arrival occurs while system in state s such that there are q queued hand-offs, the call is placed at the end of the queue (provided the queue is not full) and the successor state, ξ is

$$\begin{matrix} v_{11} & \cdot & \cdot & \cdot & v_{1I} \\ \cdot & \cdot & v_{gi} & \cdot & \cdot \\ v_{G1} & \cdot & \cdot & \cdot & v_{GI} \\ z_1 & \cdot & z_q & z_{q+1} & \cdot \\ x_1 & \cdot & z_q & x_{q+1} & \cdot \end{matrix} \qquad (15)$$

3.4.Flow due to hand-off departures.

Hand-off departures of i-type calls on g-type platforms occur at the rate

$$\gamma_c(s,g,i) = \mu_D(g)\cdot v(s,g,i) \qquad (16)$$

The successor state can be determined using an identical (iterative) procedure as described above in the analysis of flow due to call completions at the gateway.

3.5.Flow due to defections from the hand-off queue.

Defections from the q-th position in the queue occur at the rate

$$\gamma_d(s,q) = \mu_T(x(s,q)) \qquad (17)$$

Event of this type can only occur in a state s such that $x(s,q)\neq 0$. The successor state can be determined identically as in the case of call completions of queued calls.

4. PERFORMANCE MEASURES.

4.1.Blocking probability.

New calls of type i originating on g-type platforms are not admitted into service and are cleared from the system, if the current resource demand (defined as the current resource use plus the resource requests registered at the gateway by queued hand-offs) is equal to or exceeds resource quota, limits or cut-off priority limits for any resource k. Define the following set of states B_{gi}

$$B_{gi} = \{ \text{s: } r_k(s,\cdot,i) \geq R_k^c(\text{i}) \text{ } or \text{ } r_k(s,g,\cdot) \geq R_k^p(\text{g}) \text{ } or \text{ } r_k(s,\cdot,\cdot) \geq R_k - R_k^h(g,i) \text{ for any } k \} \qquad (18)$$

If the cell is in state $s \in B_{gi}$ and a new call of type i arises on a g-type platform, blocking occurs. The blocking probability for i-type calls originating on g-type platforms is given by

$$P_b(g,i) = \sum_{s \in B_{gi}} p(s) \tag{19}$$

4.2.Hand-off failure probability.

A communicating platform arriving at a cell encounters one of the following conditions at the destination gateway.

a) The number of available resources of each type is equal or greater then the resource requirements of the arriving call and no hand-offs of higher priority are waiting. In this case the call is handed-off immediately.

b) For at least one resource type, the amount of available resource is less then the call requirements or the call is assigned to lower priority class then any of the hand-offs currently waiting in the queue. The call is placed in the queue and its service order is determined by its priority class. If the platform leaves the transition zone before the call can obtain the resources necessary, the call is terminated.

c) If the queue is full and all hand-off attempts queued are of equal or higher priority, the call will continue to be supported by the source gateway for as long as the platform remains in the transition zone. However, the call does not contend for the target gateway's resources.

It should be emphasized that there exists another source of hand-off failures. A hand-off arriving when the queue is fully occupied might displace the lowest priority hand-off from the queue if its priority is higher than the lowest priority hand-off in queue. As a result the lowest priority hand-off in queue is displaced from the queue after admission to the queue and before defection from the queue. The displaced hand-off will receive continued support from the source gateway for as long as it remains in the transition zone.

We have identified three events that lead to a hand-off failure. Since they are independent, their contributions to the hand-off failure probability can be determined as separate components. We start with hand-off failures caused by queue defections.

The average rate at which g-type platforms supporting i-type calls defect from the queue is determined to be

$$D(g,i,q) = \sum_{s \in H_{giq}} \mu_T(x(s,q)) \cdot p(s) \tag{20}$$

where H_{giq} is a set of states for which the q-th position in the queue is occupied by a g-type platform having an i-type call in progress i.e. $H_{giq} = \{s\text{: } x(s,q)=g \text{ and } z(s,q)=i\}$.

The i-type calls supported by g-type platforms arrive at the gateway at an average rate $\Lambda_h(g,i)=F(g,i) \cdot \Lambda_h$ (platforms/sec). The call hand-off failure probability for i-type calls due to defections of g-type platforms can be calculated as a ratio of the number of i-type calls terminated due to hand-off failure on g-type platforms to the total number of such calls arriving at the gateway on g-type platforms. This first component of hand-off failure probability (denoted by superscript 1) can be expressed as

$$ {}^{1}P_H(g,i) = \sum_{q=1}^{Q} D(g,i,q) \Big/ \Lambda_h(g,i) \tag{21} $$

The second component of the hand-off failure probability accounts for hand-offs displaced from the queue. When the cellular system is in state s, the rate at which calls of priority class j* displace calls of type (g,i) can be determined as

$$ E(s,g,i) = \sum_{j^* \in J^*(s)} F(j^*) \cdot \Lambda_h \cdot p(s) \tag{22} $$

where $\mathcal{J}^*(s)$ is a set of priority classes such that a hand-off arrival assigned j* priority class, $j^* \in \mathcal{J}^*(s)$, when the system is in state s displaces the (g,i) type hand-off from the queue. F(j*) denotes the fraction of hand-off attempts that are assigned to j* priority level. The component of hand-off failure probability due to displacement from the queue can be found as a ratio of the number of calls of type i supported by g-type platforms and terminated due to displacement to the total number of (g,i) type hand-offs impinging on the gateway. This probability is found to be

$$ {}^{2}P_H(g,i) = \sum_{s} E(s,g,i) \Big/ \Lambda_h(g,i) \tag{23} $$

The third component of hand-off failure is due to full queue. With Poisson arrivals a (g,i) type hand-off will not be admitted to the queue with probability ${}^{3}P_H(g,i)$ given by

$$ {}^{3}P_H(g,i) = \sum_{s \in H_F} p(s) \cdot \Pr\big((g,i) \text{ assigned lowest priority} \,\big|\, s\big) \tag{24} $$

In (24) H_F is the set of permissible states for which the queue is fully occupied. The probability that a hand-off arrival is assigned lower priority than any of the hand-offs present in the queue is dependent on the priority rule. In the simple case of priority assignment based solely on the order of arrival, this probability is equal to 1 for all $s \in H_F$.

Calls displaced from the queue or denied access to the queue, receive continued support from the source gateway while in the transition zone. Some of them might successfully complete before they leave the transition zone. Specifically, since both the dwell time and the unencumbered session time are exponentially distributed random variables, a call will continue holding resources at the source gateway for a time which also is an exponentially distributed random variable with mean $1/(\mu(i)+\mu_T(g))$. Moreover, the fraction of calls that are forced into termination can be found as

$$\mathrm{Prob}(\boldsymbol{T}_T(g) < \boldsymbol{T}(i)) = k(g) = \frac{\mu_T(g)}{\mu(i)+\mu_T(g)} \tag{25}$$

The hand-off failure probability for i-type calls supported by g-type platforms is given by

$$P_H(g,i) = {}^1P_H(g,i) + k(g)\cdot\left({}^2P_H(g,i) + {}^3P_H(i)\right) \tag{26}$$

Let us also define $\tilde{P}_H(g,i)$ as the probability that a call of type (g,i) never enters service at the target gateway. The model developed here allows calls that are refused entry to the queue or are displaced from the queue to be successfully completed while in the transition zone. For that reason $\tilde{P}_H(g,i)$ is different from hand-off failure probability. $\tilde{P}_H(g,i)$ is determined to be

$$\tilde{P}_H(g,i) = {}^1P_H(g,i) + {}^2P_H(g,i) + {}^3P_H(i) \tag{27}$$

4.3.Forced termination probability.

Probability of hand-off failure gives the average fraction of hand-off attempts that fail. From the individual user's point of view a more interesting measure of performance is the probability that a call, which was not initially blocked, will be allowed to continue until satisfactory completion. *Forced termination probability* P_{FT} can be defined as a probability that a call, which was not blocked, will be interrupted in its lifetime due to hand-off failure. If we let a(g) denote the probability that an i-type call on g-type platform will make a hand-off attempt and will fail on that attempt. Similarly, b(g) denotes the probability that an i-type call on g-type platform will make a hand-off attempt and succeed. Using the Markovian properties of the model we get

$$a(g,i) = \mu_D(g)\cdot P_H(g,i)/(\mu(i)+\mu_D(g)) \tag{28}$$

and

$$b(g,i) = \mu_D(g)\cdot(1-P_H(g,i))/(\mu(i)+\mu_D(g)) \quad . \tag{29}$$

Assuming hand-offs independence we get

$$P_{FT}(g,i) = \sum_{i=0}^{\infty} a(g,i)\cdot b(g,i)^i \quad . \tag{30}$$

Summing (30) and using (24), (29) one finds

$$P_{FT}(g,i) = \mu_D(g) \cdot P_H(g,i) / \left(\mu(i) + \mu_D(g) \cdot P_H(g,i)\right) \quad . \tag{31}$$

4.4.Carried traffic.

The traffic carried by the i-type calls on g-type platforms, $A_C(g,i)$, is the average number of such calls in progress and is given by

$$A_C(g,i) = \sum_{s=0}^{s_{max}} v(s,g,i) \cdot p(s) \quad . \tag{32}$$

The total carried traffic can be found as

$$A_C(TOTAL) = \sum_{(g,i)} A_C(g,i) \quad . \tag{33}$$

5. CLOSED FORM SOLUTION TO THE LOSS SYSTEM WITH NO CUT-OFF PRIORITIES.

The previous sections describe a very large class of systems. Some of these systems have been analyzed in the literature, primarily loss systems (i.e. Q=0 in current notation) with complete and partial sharing (which is a special case of (7) and (8) corresponding to representing , say $R_k^p(i)$ as a sum of two components $R_k^{p,1}(i)$ and $R_k^{p,0}$) and no priority reservations [9][10]. Calls of type (g,i) arrive into the service facility in two streams: a stream of hand-off arrivals and a stream of new call originations. They also depart in two streams: hand-off departures and call completions. In this context, the stationary distribution of **v** (since in this case z(s,q)=0 and x(s,q)=0 for all s and q, the state s can be described by the first G n-tuples, denoted **v**) is given by

$$p(\mathbf{v}) = G(R_k, R_{gk}, R_{ik}) \prod_{(g,i)\in\mathcal{J}} \frac{1}{v_{gi}!} \cdot \left(\frac{\Lambda(g,i) + \Lambda_h(g,i)}{\mu(i) + \mu_D(g)} \right)^{v_{gi}} \tag{34}$$

where $\mathbf{v} \in \Phi(R_k, R_{gk}, R_{ik})$ and $\Phi(R_k, R_{gk}, R_{ik})$ is the state space determined by the constraints (6),(7) and (8).

However, in cellular systems modeling an additional constraint is placed on the system's operation, balancing the traffic due to hand-off departures with the traffic due to hand-off arrivals of the same hand-off type [1]. In this work we model the hand-off arrival process as J independent Poisson streams, with average arrival rate equal to

$$\Lambda_h(g,i) = \Theta(g,i) \cdot \Delta_h(g,i) \tag{35}$$

This allows for modeling of both homogenous (i.e. all Θ(g,i)=1) and non-homogenous systems[1]. The average hand-off departure rate of *(g,i)* type calls is found to be [see Appendix A].

$$\Delta_h(g,i) = \frac{\Lambda(g,i)+\Lambda_h(g,i)}{\mu(i)+\mu_D(g)} \cdot \mu_D(g) \cdot (1-P_B(g,i))$$

$P_B(g,i)$ denotes blocking probability for (g,i) type. For simplicity we will concentrate on a homogenous system. The analysis is presented in its full generality in the Appendix A. Recalling that for a homogenous system $\Lambda_h(g,i)=\Delta_h(g,i)$, we have

$$\Lambda_h(g,i) = \Lambda(g,i) \cdot \frac{1-P_B(g,i)}{P_B(g,i)+\dfrac{\mu(i)}{\mu_D(g)}} \tag{36}$$

and

$$\Lambda_h(g,i)+\Lambda(g,i) = \Lambda(g,i)\frac{1+\mu(i)/\mu_D(g)}{P_B(g,i)+\mu(i)/\mu_D(g)} \tag{37}$$

Substituting (37) into (34) we obtain the p($\mathbf{v}$)

$$p(\mathbf{v}) = G(R_k, R_{gk}, R_{ik}) \prod_{(g,i)\in\mathcal{J}} \frac{1}{v_{gi}!} \cdot \left(\frac{\Lambda(g,i)}{\mu_D(g)} \cdot \frac{1}{P_B(g,i)+\mu(i)/\mu_D(g)} \right)^{v_{gi}} \tag{38}$$

Since in most situations of interest $\mu(i)/\mu_D(g) \geq 1$ and $P_B(g,i) \leq 0.01$ we can neglect $P_B(g,i)$ and obtain the following approximate expression for $p(\mathrm{v})$

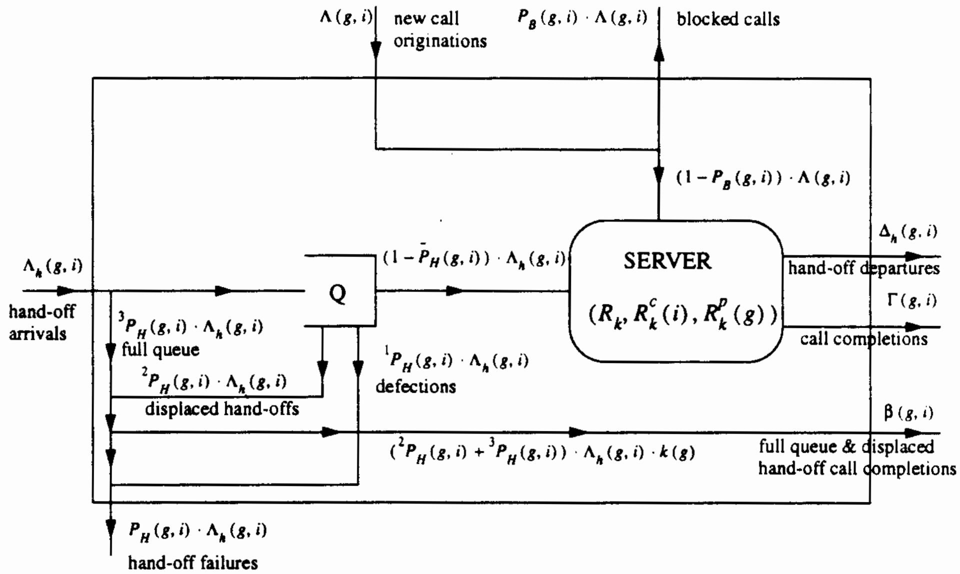
$$p(\mathbf{v}) \cong G'(R_k, R_{gk}, R_{ik}) \prod_{(g,i)\in\mathcal{J}} \frac{1}{v_{gi}!} \cdot \left(\frac{\Lambda(g,i)}{\mu(g)} \right)^{v_{gi}} \tag{39}$$

The system thus behaves as a stationary (i.e. non mobile) system and the state probability distribution is largely insensitive to mobility factors.

The reversibility property [11] allowed for a relatively simple solution to the type of systems considered in this section. However, imposition of priority reservations and/or hand-off queuing destroys reversibility. For these general cases we will be forced into numerical solutions. However, the analytical results obtained in this section will prove valuable in improving the efficiency of the numerical algorithm.

6. GENERALIZATION OF THE RELATIONSHIP BETWEEN HAND-OFF ARRIVAL RATES AND NEW CALL ORIGINATION RATES.

The relationship (36) between average hand-off arrival rate of type (g,i) and new call origination rate of the same type can be extended to hold for an arbitrary system. Applying a conservation law to the queuing model of a cell, a generalization of (36) can be obtained. The queuing model of a single gateway is shown below. The outputs from the bottom of the queue symbol depict hand-off defections and hand-offs displaced from the queue. The average traffic intensities are also shown.



Applying Little's law to the <u>server</u> for (g,i) type calls we obtain (for details see [Appendix B])

$$\left(\Lambda(g,i)\cdot\left(1-P_B(g,i)\right)+\Lambda_h(g,i)\cdot\left(1-\tilde{P}_H(g,i)\right)\right)\cdot\left(\mu(i)+\mu_D(g)\right)^{-1}=\bar{N}(g,i) \quad (40)$$

It has been shown [12] and [Appendix B] that

$$\Delta_h(g,i)=\sum_s v(s,g,i)\cdot\mu_D(g)\cdot p(s)=\mu_D(g)\cdot\bar{N}(g,i) \quad (41)$$

For simplicity we concentrate on the homogenous system (see Appendix B for a general development), we have a relationship between major system parameters

$$\Lambda_h(g,i)=\Lambda(g,i)\cdot\frac{1-P_B(g,i)}{\tilde{P}_H(g,i)+{}^{\mu(i)}\!/_{\mu_D(g)}} \quad (42)$$

It should be noted that both $P_B(g,i)$ and $\tilde{P}_H(g,i)$ are implicit functions of $\Lambda(g,i)$.

7. NUMERICAL ALGORITHM.

Previous work on the subject [1] [12] used three nested loops of iteration to obtain the numerical solution. The innermost loop implemented an iterative solution to a system of linear equations, the middle loop calculated the fractions $F(g,i)$ and outermost loop balanced Λ_h. Numerical problems arising when the initial guess was to far form the actual solution prevented the two outer loops from being combined into one. Calculations for different value of Λ_0 where performed by using previously obtained values of F(g,i), Λ_h and p(s) as initial guesses.

Application of (30) allows for combining of the two outer loops, by providing a very tight upper bound on $\Lambda_h(g,i)$.

The numerical calculations are performed for increasing values of Λ_0. The procedure starts by estimating the initial value for $\Lambda_h(g,i)$ for all (g,i). To that end (30) is used with $P_B(g,i) = 0$ and $\tilde{P}_H(g,i) = 0$. The system of probability flow equations is then solved. Since the initial value of $\Lambda_h(g,i)$ is an upper bound, the values obtained for $P_B(g,i)$ and $\tilde{P}_H(g,i)$ are overestimated. They can be then inserted back into (30) to provide a lower bound on $\Lambda_h(g,i)$. This process could in theory be continued, providing tighter and tighter upper and lower bounds, ultimately leading to a solution. However, numerical accuracy of the linear system solver might not allow for such an approach, in particular if the initial upper bound is very tight. A safer approach is therefore to use a divide-and-conquer algorithm after establishing the upper and lower bounds.

The system of linear equations is solved using Richardson method (convergence problems in Gauss-Seidel method forced the authors to implement this less efficient method). The solution to (38) is used to determine the initial guesses for the state probabilities for all states with **z=0** and **x=0**. Initial probabilities of other states are set to zero. The initial values for p are regenerated every time Λ_0 is varied (i.e. the final values for p(s) from the previous iteration are not used). The upper bound for $\Lambda_h(g,i)$ is found by using $P_B(g,i)$ and $\tilde{P}_H(g,i)$ obtained from the previous top level iteration (since both $P_B(g,i)$ and $\tilde{P}_H(g,i)$ are monotone increasing functions of Λ_0).

The method outlined here leads to a 6-7 fold decrease in computation time. A comparison of several possible numerical approaches is presented below. This method (which we will denote as method 4) is compared to three other variations briefly described below:

1. Method used in previous work [12]. Initial p(s) are set to $1/s_{max}$. Initial F(g,i) are set to 1/G·I. The final results for p(s) and $\Lambda_h(g,i)$ are used as an initial guess for state probability calculations for the next value of Λ_0.

2. (30) is used to determine initial values for $\Lambda_h(g,i)$. Initial values for p(s) are set to $1/s_{max}$. The final values for p(s) are used as initial values for the next calculation with a different Λ_0. The $P_B(g,i)$ and $P_H(g,i)$ are used to upper-bound $\Lambda_h(g,i)$.

3. Same as 2. but for the first value of Λ_0 (38) is used to determine starting values for p(s).

The four approaches were used to generate performance curves consisting of 21 points along the Λ_0 axis. System 1 and System 2 consist of 6551 and 18087 states respectively. The time (in hours) required to generate the system performance curves on a SPARCstation LX are summarized in the table below.

Method	1	2	3	4
System 1	20:00	5:20	5:00	3:15
System 2	52:15	-	-	11:00

Table 1.

The application of (30) virtually eliminates the need for iterative solution to $\Lambda_h(g,i)$ for blocking probabilities below 10^{-3}. Above that level usually the second iteration provides an estimate of $\Lambda_h(g,i)$ which is within 1%. Comparison of columns 3 and 4 shows that using (38) provides the iterative linear equation solver with a much better initial guess that when using values of p(s) from preceding iteration.

8. NUMERICAL EXAMPLE AND DISCUSSION OF RESULTS.

The approach described was used to determine performance of a cellular system whose parameters are described below. There are two platform types in the system (G=2). Type 1 platform is a low mobility platform (e.g. pedestrians), while type 2 is a high mobility platform (e.g. passenger cars). The platforms mobility parameters are summarized in Table 2.

Platform type	$\bar{T}_D$	$\bar{T}_T$
1	1000s	100s
2	200s	20s

Table 2. Platform parameters.

Any of the two call types can occur in the system. The calls differ in the number of resource units of each of the two resources needed to support the call. The number of units of each resource needed is shown in Table 3.

Call type	$\bar{T}$	r_{i1}	r_{i2}
1	100s	1	2
2	100s	3	1

Table 3. Call parameters.

There are $R_{1,2}=30$ units of each resource available at the gateway. The system performance was determined by varying Λ_0 from 0.01 to 0.1 calls per second. The size of the queue waiting space was varied from Q=0 to Q=2. Two sets of cut-off priorities were used: $R_k^h(g,i)=0$ for all k,g,i and $R_1^h(g,i)=3$ and $R_2^h(g,i)=2$ for all g,i.

The results are presented on Figs. 1-8. Their contents is described below:

Fig1. Blocking and forced termination for call of type 1 (lower resource requirements) on platforms with different mobility and no cut-off priorities.

Fig2. Blocking and forced termination for call of type 2 (higher resource requirements) on platforms with different mobility and no cut-off priorities.

Fig3. Blocking and forced termination for calls of both types supported by low mobility platform and no cut-off priorities.

Fig4. Blocking and forced termination for calls of both types supported by high mobility platform and no cut-off priorities.

Fig5. As Fig1. but with cut-off priorities.

Fig6. As Fig2. but with cut-off priorities.

Fig7. As Fig3. but with cut-off priorities.

Fig8. As Fig4. but with cut-off priorities.

Blocking probability increases slightly in the entire range of call origination rates and for call types when hand-off attempts are queued, as compared to no queuing (Figs.1-4). However, increasing the buffer size to two does not result in any further, significant deterioration of blocking probability. Moreover, the deterioration suffered by the calls with lower resource requirements is larger then for calls with higher resource demands. In contrast, in a system with cut-off priorities, hand-off queuing has no noticeable effect on blocking, as can be seen on figs.5-8.

The improvements in the forced termination probability for Q=1 are clearly most pronounced for calls with low resource requirements and supported by slowly moving platforms. Increasing the queue buffer size to two again benefits that type of calls the most, while type 2 (high resource requirement) calls on fast moving platforms experience virtually no performance gain. Introduction of cut-off priorities allows for significant additional improvement (close to an order of magnitude).

Improvements due to both hand-off queuing and cut-off priority come at some cost in carried traffic. An introduction of hand-off queuing to a system with no cut-off priorities causes a significant drop in carried traffic at moderate and heavy loads. This penalty is the most severe for calls supported by the slow platforms. Penalty for increasing the waiting space size to two is much less. Interestingly, a system with combined hand-off queuing and cut-off priorities can outperform (in carried traffic and hand-off failure probability) a system with no cut-off and waiting space of identical size. This occurs in moderate and heavy traffic. In the same traffic range, with increasing new call origination rate, we observe a drop in the carried traffic due to high resource calls.

In general, cut-off priorities cause an increase in blocking probability when the impinging traffic is low. As traffic intensity increases this penalty becomes smaller. At the same time at high loads, we observe a drop in the carried traffic in systems with cut-off priority as compared to systems with no queuing and no cut-off priorities. However, queuing of hand-off calls also penalizes the carried traffic and to a smaller extent blocking.

For systems where the designer's objective is to minimize forced termination probability, the most promising is the system with combined hand-off queuing and cut-off priorities. Clearly, the downside of this architecture is the increased system complexity.

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Appendix A. Solution for the state probabilities for a non-homogenous system with no queuing and no cut-off priorities.

The average hand-off departure rate of (ξ,ρ) type calls can be calculated as (see also Appendix B)

$$\Delta_h(\xi,\rho)=\sum_{\mathbf{v}} p(\mathbf{v})\cdot v_{\xi\rho}\cdot\mu_D(\xi)=\sum_{\mathbf{v}} G(R_k,R_{gk},R_{ik})\prod_{(g,i)\in J}\frac{1}{v_{gi}!}\cdot\left(\frac{\Lambda(g,i)+\Lambda_h(g,i)}{\mu(i)+\mu_D(g)}\right)^{v_{gi}}\cdot v_{\xi\rho}\cdot\mu_D(\xi)=$$

$$G(R_k,R_{gk},R_{ik})\sum_{\mathbf{v}}\left(\prod_{(g,i)\in J}\frac{1}{v_{gi}!}\cdot\left(\frac{\Lambda(g,i)+\Lambda_h(g,i)}{\mu(i)+\mu_D(g)}\right)^{v_{gi}}\cdot\frac{1}{v_{\xi\rho}!}\cdot\left(\frac{\Lambda(\xi,\rho)+\Lambda_h(\xi,\rho)}{\mu(\rho)+\mu_D(\xi)}\right)^{v_{\xi\rho}}\cdot v_{\xi\rho}\cdot\mu_D(\xi)\right)=$$

$$\frac{\Lambda(\xi,\rho)+\Lambda_h(\xi,\rho)}{\mu(\rho)+\mu_D(\xi)}\cdot G(R_k,R_{gk},R_{ik})\sum_{\mathbf{v}}\left(\prod_{(g,i)\in J}\frac{1}{v_{gi}!}\cdot\left(\frac{\Lambda(g,i)+\Lambda_h(g,i)}{\mu(i)+\mu_D(g)}\right)^{v_{gi}}\cdot\frac{1}{(v_{\xi\rho}-1)!}\cdot\left(\frac{\Lambda(\xi,\rho)+\Lambda_h(\xi,\rho)}{\mu(\rho)+\mu_D(\xi)}\right)^{v_{\xi\rho}}\cdot\mu_D(\xi)\right)=$$

$$\frac{\Lambda(\xi,\rho)+\Lambda_h(\xi,\rho)}{\mu(\rho)+\mu_D(\xi)}\cdot\mu_D(\xi)\cdot\sum_{\mathbf{v}} p(\mathbf{v}-\mathbf{1}_{\xi\rho})=\frac{\Lambda(\xi,\rho)+\Lambda_h(\xi,\rho)}{\mu(\rho)+\mu_D(\xi)}\cdot\mu_D(\xi)\cdot(1-P_B(\xi,\rho))$$

where $\mathbf{1}_{\xi\rho}$ is a matrix of all zeros, except at the (ξ,ρ) position which is equal one (Note that if no queuing is allowed **v** can be treated as a matrix). $P_B(\xi,\rho)$ denotes blocking probability for (ξ,ρ) type. Using (35) we get

$$\Lambda_h(g,i)=\Theta(g,i)\cdot\frac{\Lambda(g,i)+\Lambda_h(g,i)}{\mu(i)+\mu_D(g)}\cdot\mu_D(g)\cdot(1-P_B(g,i))$$

$$\Lambda_h(g,i)=\frac{\Theta(g,i)\cdot\Lambda(g,i)\cdot\mu_D(g)\cdot(1-P_B(g,i))}{\mu(i)+\mu_D(g)-\Theta(g,i)\cdot\mu_D(g)\cdot(1-P_B(g,i))}=\frac{\Lambda(g,i)\cdot(1-P_B(g,i))}{P_B(g,i)+\dfrac{\mu(i)}{\Theta(g,i)\cdot\mu_D(g)}+\dfrac{1-\Theta(g,i)}{\Theta(g,i)}}=$$

$$\Lambda_h(g,i)=\Lambda(g,i)\cdot\frac{1-P_B(g,i)}{P_B(g,i)+\dfrac{\mu(i)+(1-\Theta(g,i))\cdot\mu_D(g)}{\Theta(g,i)\cdot\mu_D(g)}}$$

and

$$\Lambda_h(g,i)+\Lambda(g,i)=\Lambda(g,i)\cdot\left(\frac{1-P_B(g,i)}{P_B(g,i)+\dfrac{\mu(i)+(1-\Theta(g,i))\cdot\mu_D(g)}{\Theta(g,i)\cdot\mu_D(g)}}+1\right)=\Lambda(g,i)\cdot\left(\frac{\dfrac{\mu(i)+\mu_D(g)}{\Theta(g,i)\cdot\mu_D(g)}}{P_B(g,i)+\dfrac{\mu(i)+(1-\Theta(g,i))\cdot\mu_D(g)}{\Theta(g,i)\cdot\mu_D(g)}}\right)$$

Hence

$$p(\mathbf{v}) = G(R_k, R_{gk}, R_{ik}) \cdot \prod_{(g,i)\in \mathcal{J}} \frac{1}{v_{gi}!} \cdot \left(\frac{\Lambda(g,i)}{\Theta(g,i)\cdot \mu_D(g)} \right) \cdot \left(\frac{1}{P_B(g,i) + \frac{\mu(i) + (1-\Theta(g,i))\cdot \mu_D(g)}{\Theta(g,i)\cdot \mu_D(g)}} \right)$$

Appendix B. GENERALIZATION OF THE RELATIONSHIP BETWEEN $\Lambda_h(g,i)$ and $\Lambda(g,i)$ FOR A NON-HOMOGENOUS SYSTEM.

Applying Little's law to the server of the system depicted in section 6 we obtain for (g,i) type calls

$$\left(\underbrace{\Lambda(g,i)\cdot(1-P_B(g,i))}_{\text{new (g,i) call originations which are not blocked}} + \underbrace{\Lambda_h(g,i)\cdot\left(1-\tilde{P}_H(g,i)\right)}_{\text{(g,i) hand-offs which are assigned resources at the gateway}} \right) \cdot \underbrace{(\mu(i)+\mu_D(g))^{-1}}_{\text{average service time for (g,i) type calls}} = \underbrace{\overline{N}(g,i)}_{\text{average number of (g,i) type calls at the server}}$$

Similarly to [12] we find that the average hand-off departure rate for type (g,i) can be determined as follows. Let $d_{gi|s}$ denote the probability of a hand-off departure of type (g,i) when the cell is in state s. This is just the ratio of flow out of state s due to hand-off departures to the total flow out of state s. Denoting the total flow out of state s (taken as a positive quantity) by r(s), we get

$$d_{gi|s} = \mu_D(g)\cdot v(s,g,i)/r(s)$$

The overall probability of a hand-off departure of type (g,i) is

$$d_{gi} = \sum_{s=0}^{s_{max}} \left[\mu_D(g)\cdot v(s,g,i)/r(s)\right]\cdot \tilde{p}(s)$$

where $\tilde{p}(s)$ is the probability that the birth-death process visits state s. The relationship between $\tilde{p}(s)$ and $p(s)$ can be expressed as

$$\tilde{p}(s) = r(s)\cdot p(s) \Big/ \sum_s r(s)\cdot p(s)$$

Hence

$$d_{gi} = \sum_{s=0}^{s_{max}} \mu_D(g)\cdot v(s,g,i)\cdot p(s) \Big/ \sum_s r(s)\cdot p(s)$$

The denominator can be recognized as an average rate at which transition events occur. The numerator is the average rate at which hand-off departures of type (g,i) occur. That is, hand-off departures of type (g,i) occur at an average rate given by

$$\Delta_h(g,i) = \sum_s v(s,g,i)\cdot \mu_D(g)\cdot p(s) = \mu_D(g)\cdot \overline{N}(g,i)$$

Using (35) we obtain the general relationship between $\Lambda_h(g,i)$ and $\Lambda(g,i)$

$$\Lambda_h(g,i) = \Lambda(g,i) \cdot \left(\frac{1 - P_B(g,i)}{\tilde{P}_H(g,i) + \frac{\Theta(g,i) \cdot \mu(i) - (1 - \Theta(g,i)) \cdot \mu_D(g)}{\mu_D(g)}} \right)$$

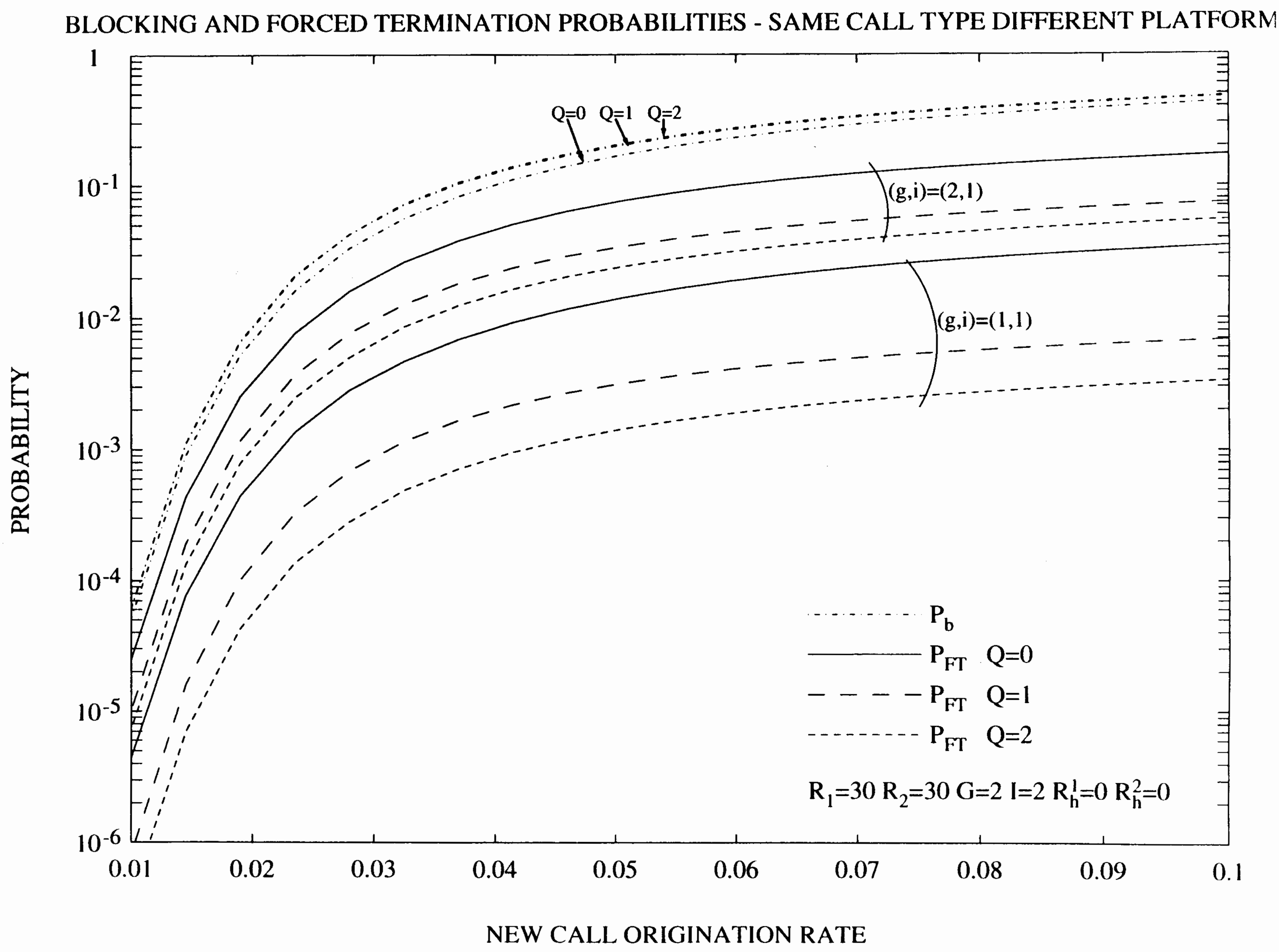

BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME CALL TYPE DIFFERENT PLATFORM
PROBABILITY
NEW CALL ORIGINATION RATE
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Q=0 Q=1 Q=2
(g,i)=(2,1)
(g,i)=(1,1)
Pb
PFT Q=0
PFT Q=1
PFT Q=2
R1=30 R2=30 G=2 I=2 Rh1=0 Rh2=0

FIG.1

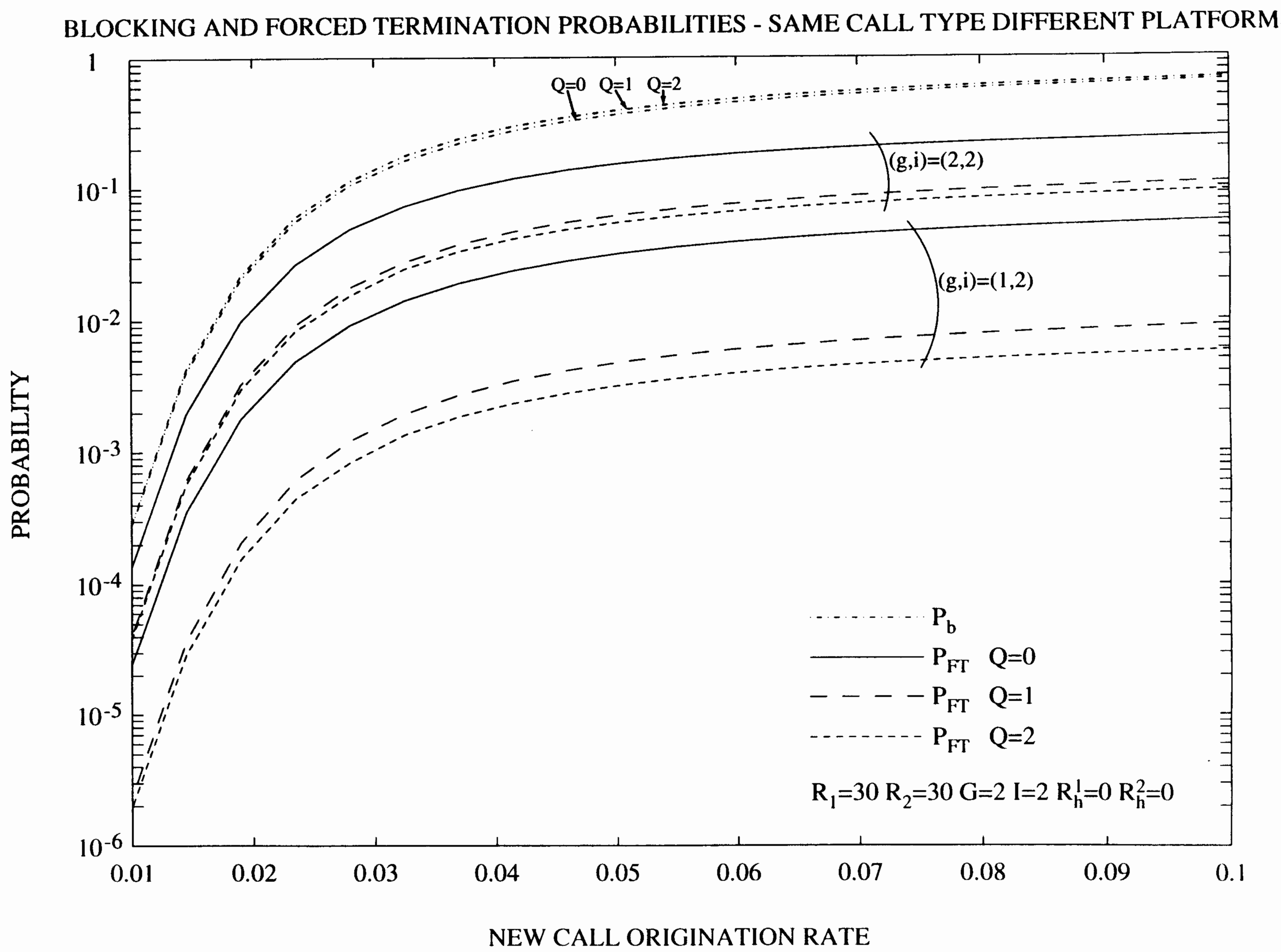
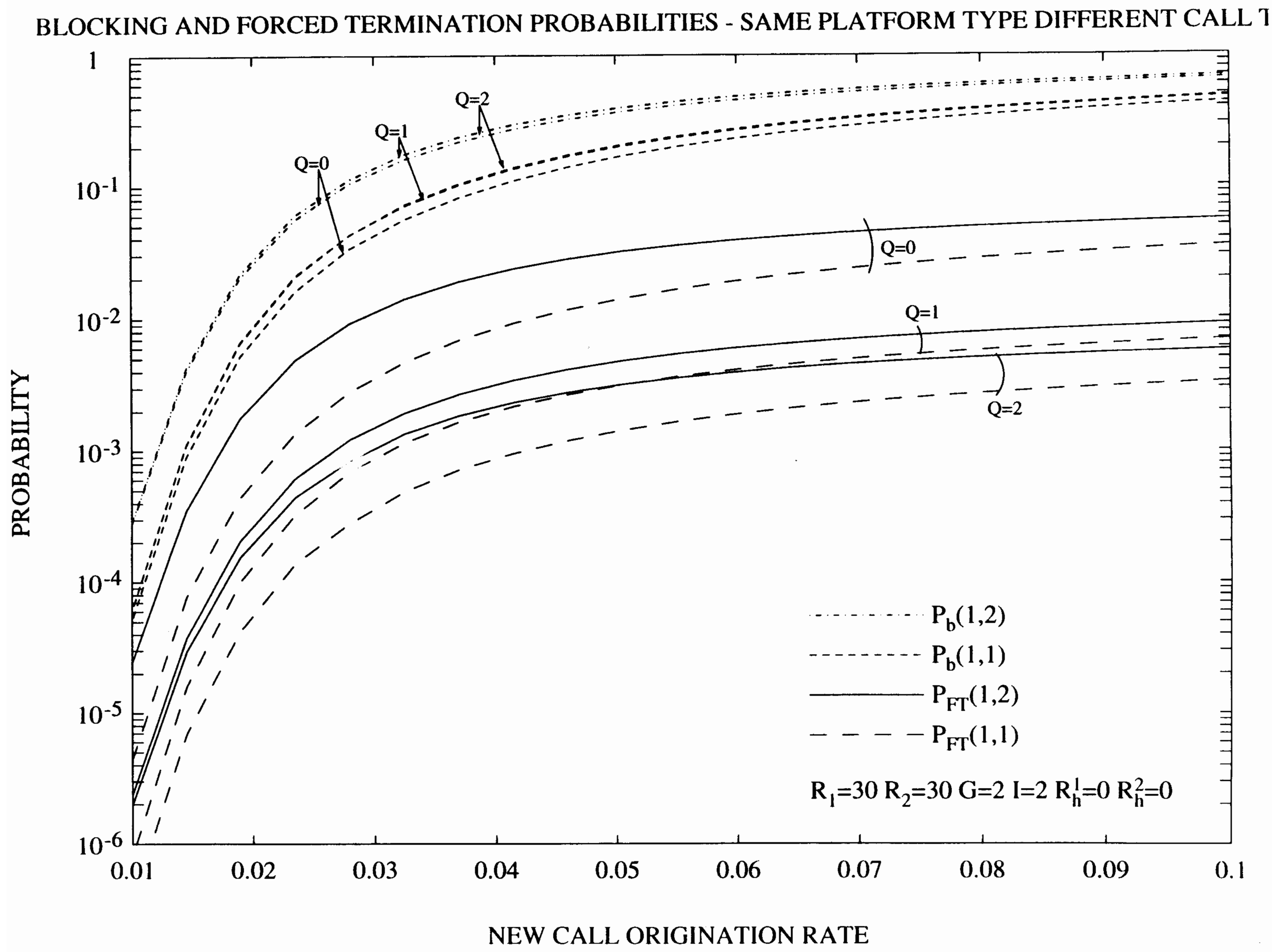
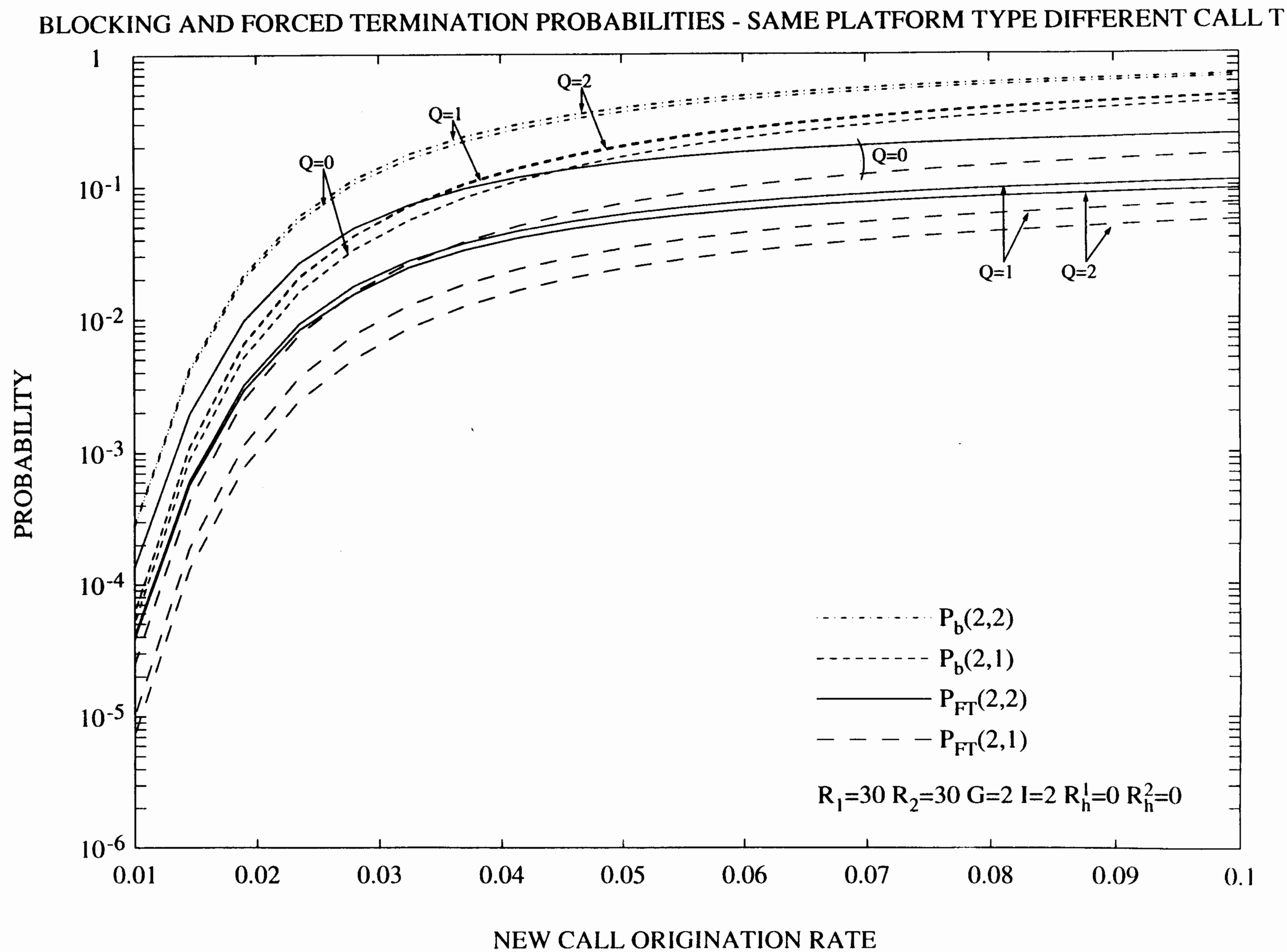

BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME CALL TYPE DIFFERENT PLATFORMS
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
NEW CALL ORIGINATION RATE
Q=0 Q=1 Q=2
(g,i)=(2,2)
(g,i)=(1,2)
Pb
PFT Q=0
PFT Q=1
PFT Q=2
R1=30 R2=30 G=2 I=2 Rh1=0 Rh2=0

FIG.2



BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME PLATFORM TYPE DIFFERENT CALL T
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Q=0
Q=1
Q=2
Q=0
Q=1
Q=2
Pb(1,2)
Pb(1,1)
PFT(1,2)
PFT(1,1)
R1=30 R2=30 G=2 I=2 Rh1=0 Rh2=0
NEW CALL ORIGINATION RATE

FIG.3



BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME PLATFORM TYPE DIFFERENT CALL T
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Q=0
Q=1
Q=2
Q=0
Q=1
Q=2
Pb(2,2)
Pb(2,1)
PFT(2,2)
PFT(2,1)
R1=30 R2=30 G=2 I=2 R1h=0 R2h=0
NEW CALL ORIGINATION RATE

FIG.4

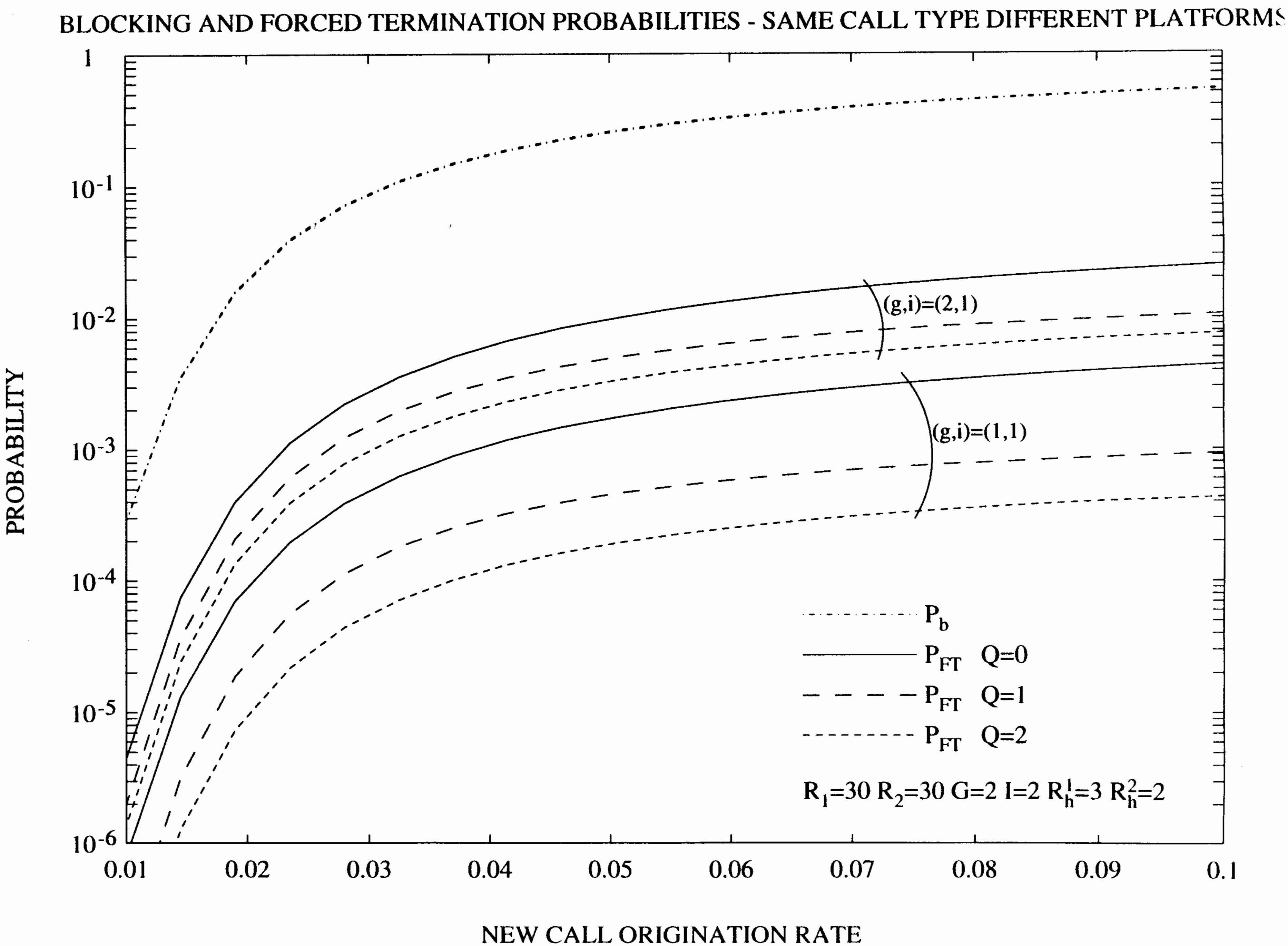

BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME CALL TYPE DIFFERENT PLATFORMS
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
(g,i)=(2,1)
(g,i)=(1,1)
P_b
P_FT Q=0
P_FT Q=1
P_FT Q=2
R_1=30 R_2=30 G=2 I=2 R_h^1=3 R_h^2=2
NEW CALL ORIGINATION RATE

FIG.5

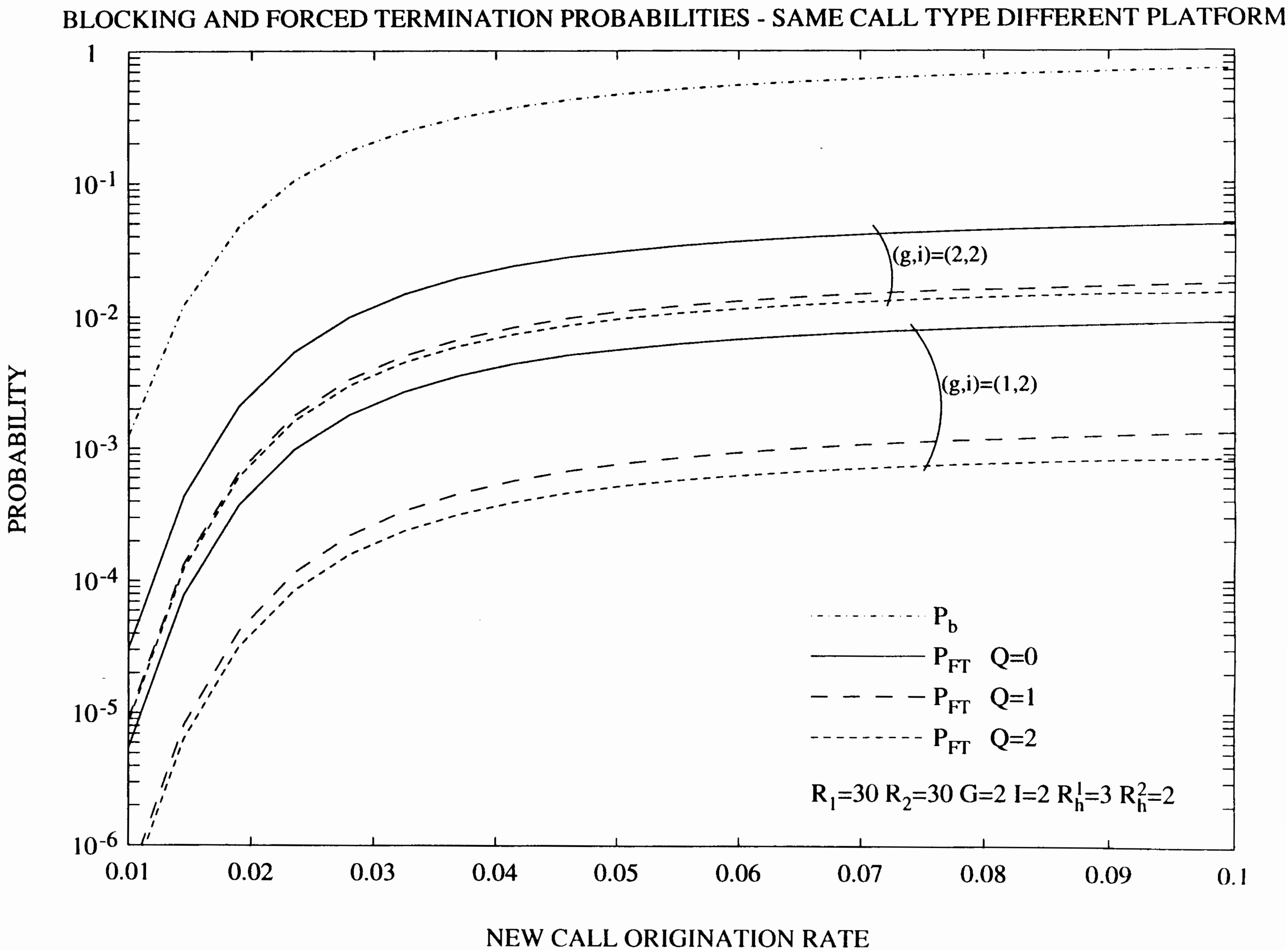
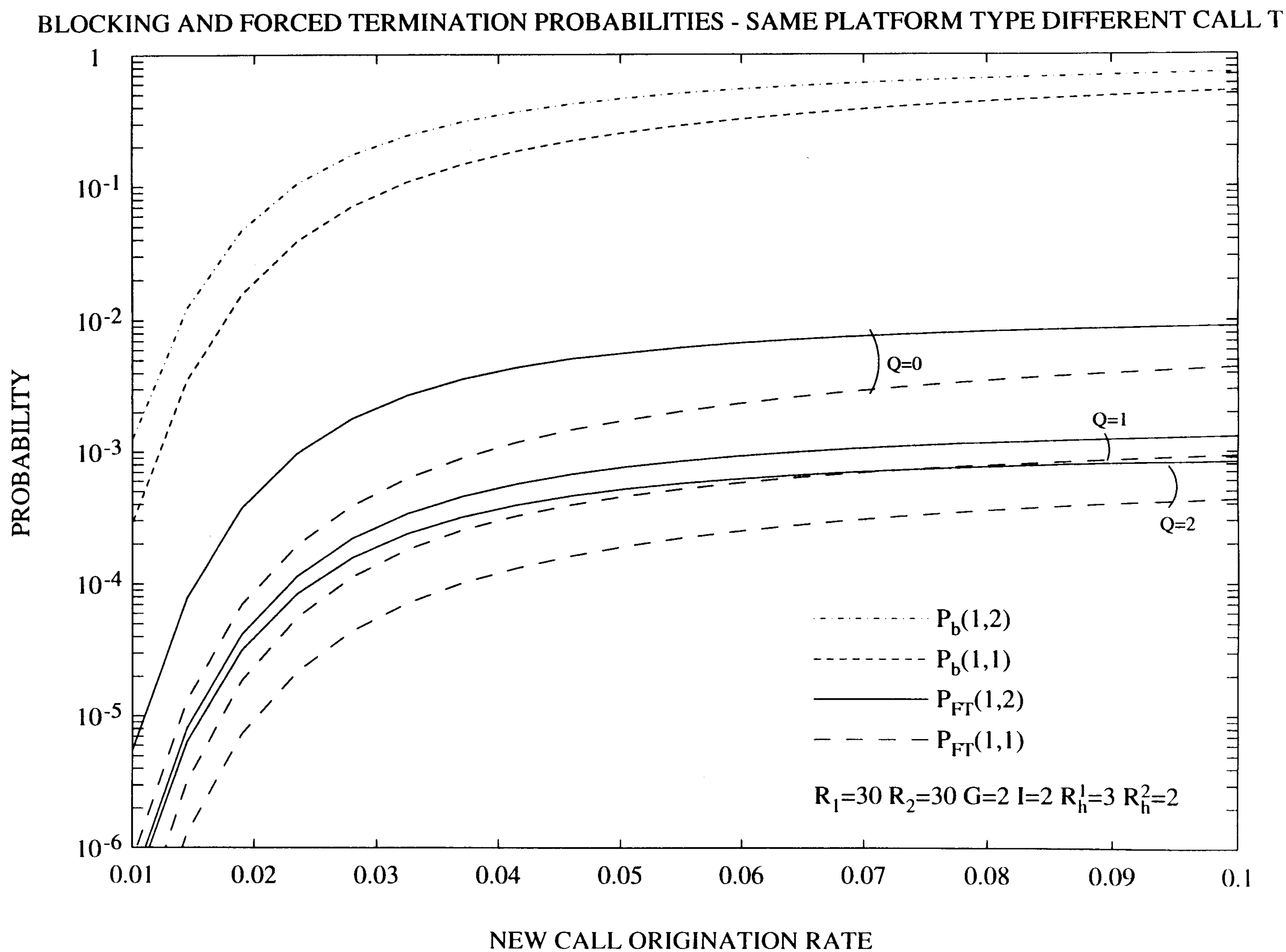

BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME CALL TYPE DIFFERENT PLATFORM
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
NEW CALL ORIGINATION RATE
(g,i)=(2,2)
(g,i)=(1,2)
P_b
P_{FT} Q=0
P_{FT} Q=1
P_{FT} Q=2
R_1=30 R_2=30 G=2 I=2 R_h^1=3 R_h^2=2

FIG.6



BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME PLATFORM TYPE DIFFERENT CALL T
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Q=0
Q=1
Q=2
Pb(1,2)
Pb(1,1)
PFT(1,2)
PFT(1,1)
R1=30 R2=30 G=2 I=2 Rh1=3 Rh2=2
NEW CALL ORIGINATION RATE

FIG.7

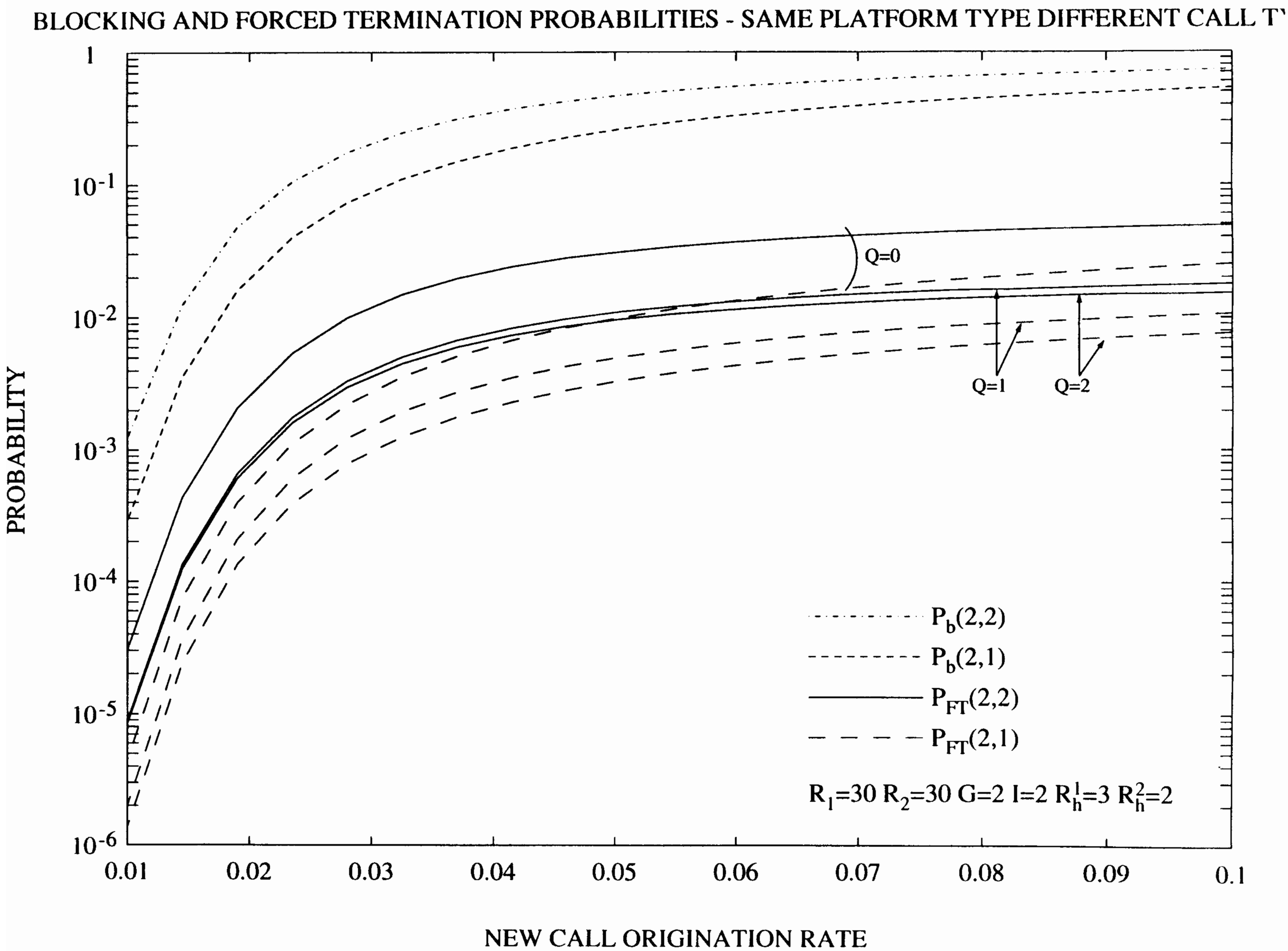
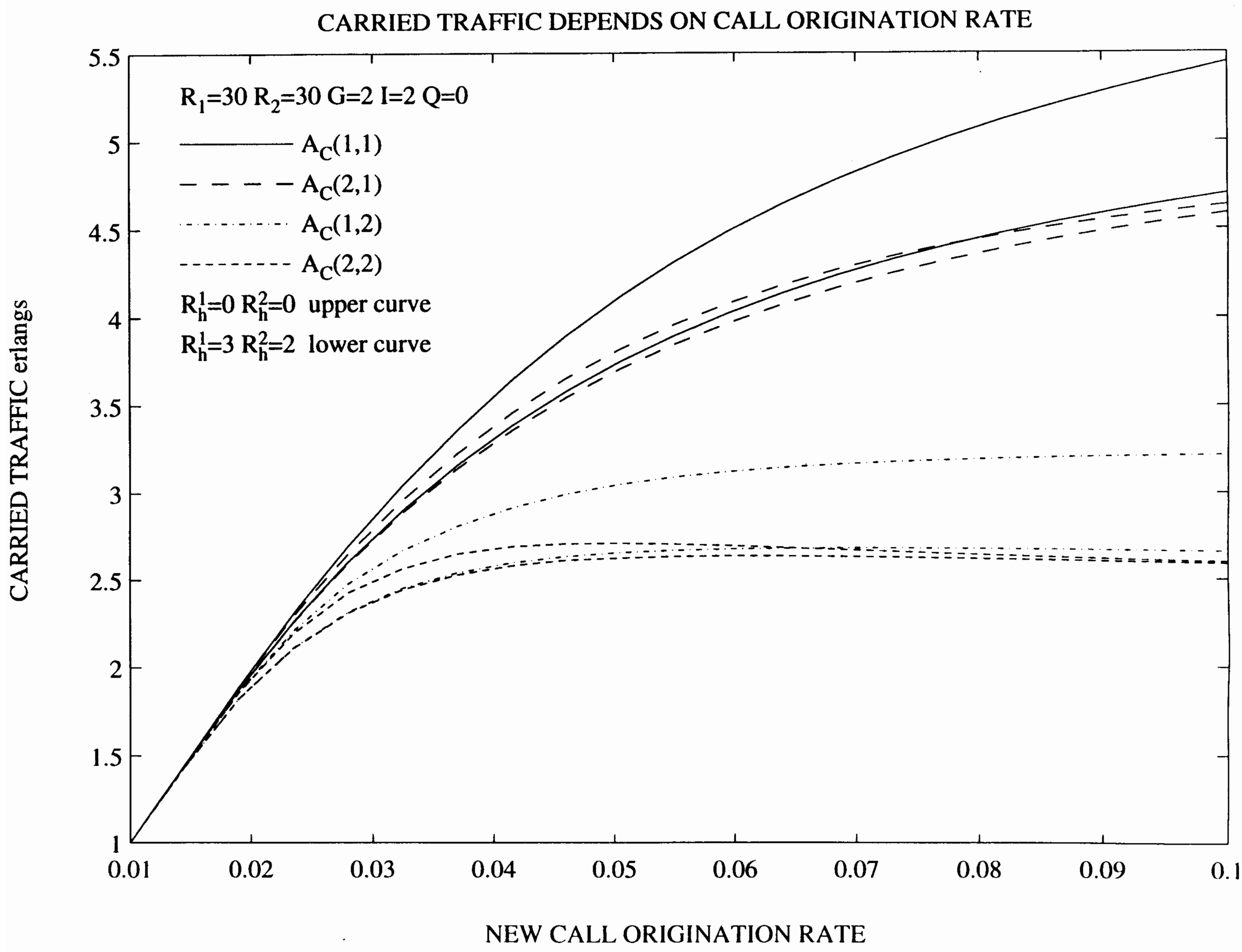

BLOCKING AND FORCED TERMINATION PROBABILITIES - SAME PLATFORM TYPE DIFFERENT CALL T
PROBABILITY
1
10-1
10-2
10-3
10-4
10-5
10-6
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Q=0
Q=1
Q=2
$P_b(2,2)$
$P_b(2,1)$
$P_{FT}(2,2)$
$P_{FT}(2,1)$
R_1=30 R_2=30 G=2 I=2 R_h^1=3 R_h^2=2
NEW CALL ORIGINATION RATE

FIG.8



CARRIED TRAFFIC DEPENDS ON CALL ORIGINATION RATE
R1=30 R2=30 G=2 I=2 Q=0
AC(1,1)
AC(2,1)
AC(1,2)
AC(2,2)
R1h=0 R2h=0 upper curve
R1h=3 R2h=2 lower curve
CARRIED TRAFFIC erlangs
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
NEW CALL ORIGINATION RATE

FIG.9

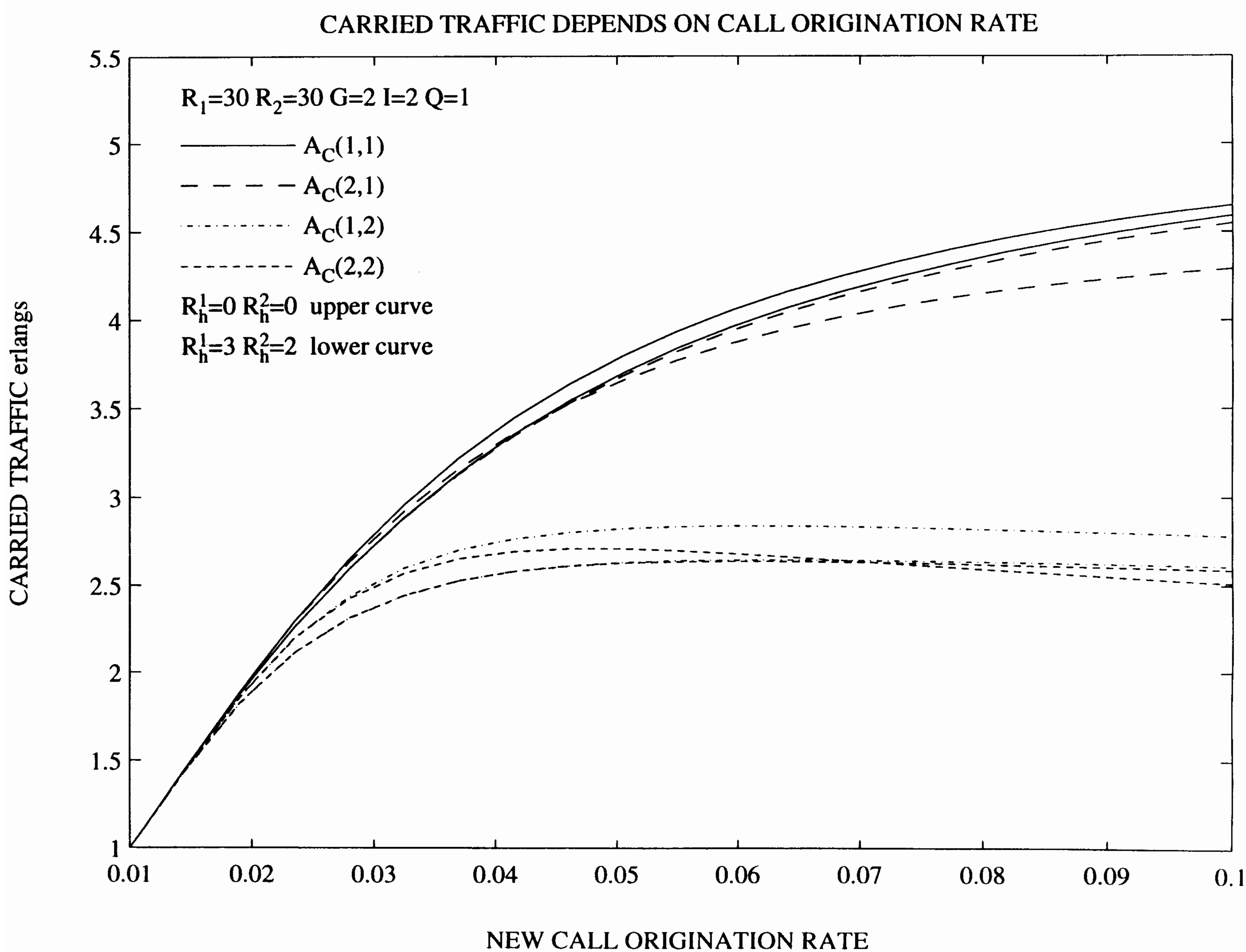

CARRIED TRAFFIC DEPENDS ON CALL ORIGINATION RATE
R1=30 R2=30 G=2 I=2 Q=1
AC(1,1)
AC(2,1)
AC(1,2)
AC(2,2)
Rh1=0 Rh2=0 upper curve
Rh1=3 Rh2=2 lower curve
CARRIED TRAFFIC erlangs
NEW CALL ORIGINATION RATE
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

FIG.10

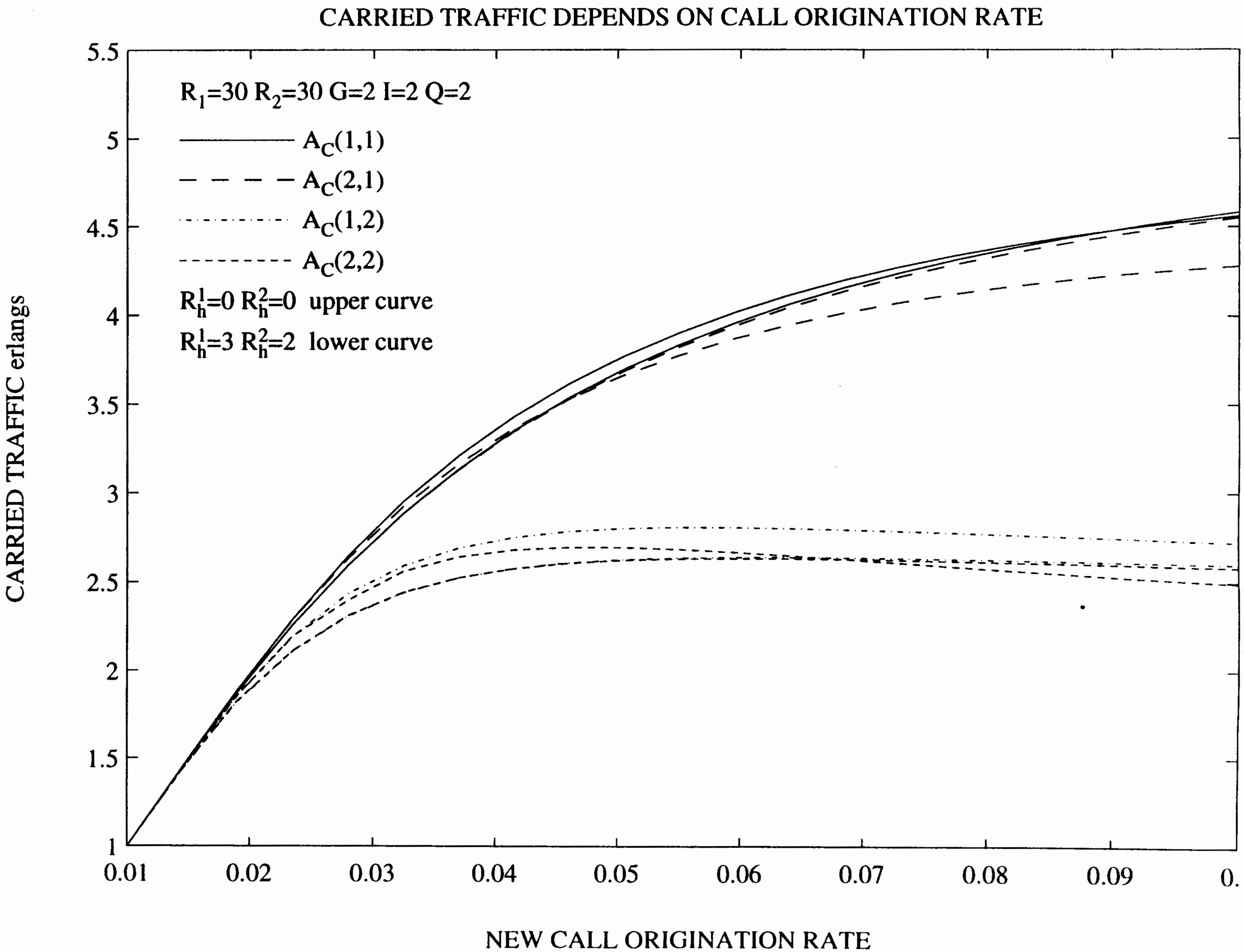

CARRIED TRAFFIC DEPENDS ON CALL ORIGINATION RATE
R_1=30 R_2=30 G=2 I=2 Q=2
$A_C(1,1)$
$A_C(2,1)$
$A_C(1,2)$
$A_C(2,2)$
R_h^1=0 R_h^2=0 upper curve
R_h^1=3 R_h^2=2 lower curve
CARRIED TRAFFIC erlangs
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
NEW CALL ORIGINATION RATE

FIG.11