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Mobile User Registration in Cellular Systems with Overlapping Location Areas

by

Daqing Gu and Stephen S. Rappaport

Department of Electrical and Computer Engineering
State University of New York
Stony Brook, New York 11794-2350

e-mail: dgu@sbee.sunysb.edu, rappaport@sunysb.edu

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Abstract

A family of registration schemes for location management in cellular systems with overlapping location areas (LAs) is considered. These schemes significantly reduce the required location updating signaling when compared with conventional schemes. This reduction is due to a decrease in the average number of mobile users who are in boundary cells of a *given* LA and are registered to this *given* LA. We present an analytical model to evaluate performance. An algorithm to compute the average location update rate per user is developed by using the concept of *dwell time*, i.e., the amount of time a mobile user remains in a cell. The analysis provides insight into the ability of overlapping LA structures to reduce the location updating signaling. Both one- and two-dimensional cellular systems are considered. Numerical results are obtained which show the dependence of average location update rate on the amount of overlap.

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1. Introduction

To complete a call to a mobile user in a cellular communication system, the current location of the mobile user (in the sense of network access) must be established. That is, a suitable network access node that allows signals to be routed to and from the mobile user must be identified. In some of the first generation mobile systems, the system pages a particular mobile user over the entire service area [3]. The specific mobile user responds to the page through a reachable node (access point), and a connection is set up through that node. This method is appropriate for small and low-user-density systems but it produces excessive paging signaling traffic for large and high-user-density systems.

A commonly used alternative strategy is to partition the entire service area into contiguous and distinct location areas, each consisting of a group of cells [1], [2], [3]. The current LA of each mobile user is maintained in the system. The LA of a mobile user is updated whenever the mobile user enters a new LA. When an incoming call to a mobile user arrives, the system pages the called mobile user in the user's current LA. This approach is a fixed scheme in the sense that the positions, sizes and shapes of LAs are fixed and predefined. The major drawback of this strategy is that location updating and paging signaling traffic is concentrated at certain fixed system nodes that are near LA boundaries. The signaling traffic processed at these nodes can be excessive.

Various location tracking strategies and location management schemes have been proposed for reducing the amount of location updating and paging signaling. Multilevel LAs for location updating in hierarchical cellular structures are discussed in [1]. In these, the registered level is dynamically changed according to the past and present mobility of a mobile user. A dynamic location tracking strategy in which the size and shape of an LA for a mobile user are dynamically determined on the basis of the user's mobility pattern and current incoming call arrival rate is discussed in [2]. Extension of the approach to hierarchical cellular structures is developed in [3]. Multiple LA layers, which are arranged in an overlapped and staggered fashion to reduce and distribute the location updating signaling, are considered in [4]. In [5], the optimal size of square-shaped LAs for a mobile user is dynamically determined according to the mobile user's current mobility and incoming call arrival rate. Three dynamic location update strategies, *time-based*, *movement-based*, and *distance-based*, are investigated in [6]. In these, a mobile user performs location updating, respectively, according to the *time elapsed*, the number of *cell changes encountered*, and the *distance traveled* since its last location update. In [7], a combined distance-based location updating and paging scheme, in which the optimal update threshold distance is determined using an iterative algorithm is discussed.

Using overlapping LA structure to reduce location updating signaling was proposed in [4], in which, square cells and LAs are considered. The LAs are arranged in an overlapped fashion to reduce and distribute the location updating signaling traffic. A simulation method was used to show the improvement of the scheme over conventional schemes. In this paper, we consider overlapping LA structures both in one- and two-dimensional cellular systems. In two-dimensional cellular systems, the popular hexagonal cells and LAs are used. An analytically tractable performance model is developed. An algorithm is given to compute the average location update rate per user by using the concept of *dwell time* [2], [13], and [14]—the amount of time a mobile user remains in a cell. Numerical results show that the reduction of the average location update rate per user as a function of the amount of overlap. Overlapping LAs affect the geographical distribution of mobile users registered to a *given* LA. The approach provides insight into the ability of overlapping LA structures to reduce the location updating signaling. Although we illustrate the algorithm using a homogenous cellular system with one class of mobile users, the analysis can be extended to treat non-homogenous cellular systems having mixed classes of mobile users with different mobility characteristics.

2. System Description and Mobility Model

2.1. Mobility Model

A one-dimensional model is suitable for those cases in which users can only move in two directions such as users along a highway. Two-dimensional models are appropriate for situations where mobile users can move in any azimuthal direction (e.g., pedestrians in a downtown area). A one-dimensional service area is depicted in Fig. 1(a). The area is divided into cells of equal size. Each is served by a base station and has two neighbors. Fig. 1(b) shows a two-dimensional cell layout. The area is divided into hexagonal cells of equal size. Each cell is served by a base station and has six neighboring cells.

Mobile users in a cell communicate through the base station of the cell. Mobile users are free to move and change cells in the service area. We assume that cell changes can occur at any time instant and that the movements of mobile users are probabilistic and independent of one another. The mobility model is summarized below:

- A cell change of a mobile user can occur at any time instant.
- The time interval between successive cell changes is called the *dwell time* of a mobile user in a cell.
- The *dwell time* in any cell for a mobile user is an exponentially distributed random variable with the average value, $\overline{T}_d$.
- When the *dwell time* in its current cell expires, a mobile user moves to one of the neighboring cells with probability 1/2 in the one-dimensional case and 1/6 in the two-dimensional case.
- *Dwell times* of a mobile user are statistically independent.

We consider a homogenous cellular system with one class of mobile users (having average *dwell time* $\overline{T}_d$). We assume that the average number of mobile users in any cell is $\overline{K}$ when the system is in statistical equilibrium (That is, probabilities and expectations are not time dependent). The mobility patterns of a mobile user in one- and two-dimensional service areas are shown in Fig. 2.

2.2. *Overlapping Location Area Structure and Registration Mechanism*

As shown in Fig. 3, LAs under discussion are considered to have a rectilinear shape for one-dimensional case and a regular hexagonal shape for two-dimensional case. In order to describe the size of LAs, we use the concept of rings discussed in [7]. An LA having *d* rings is said to have ***size d***. For convenience, the rings of this LA are labeled 0, 1, 2, . . . *d*-1 from the innermost (center cell) to the outermost. For the one-dimensional case, even though cells do not form geometric rings, we still use the same terminology as in the two-dimensional case. The numbers in Fig. 3 show the ring numbering.

In order to reduce location updating signaling, LAs are arranged in an overlapped fashion as shown in Fig. 4. In the one-dimensional case, each LA is partially overlapped by its two neighboring LAs. In the two-dimensional geometry, each LA is partially overlapped by its six neighboring LAs. For easy visualization, large hexagons are used to show location areas in two-dimensional case as depicted in Fig. 4(b). We define w as the number of rings of an LA that are overlapped by its neighbors. In this paper, we assume a homogeneous system in which all LAs have the same shape and same size, and the amount of overlapping of all LAs are identical. For simplicity, when we develop the algorithm to evaluate the average location update rate, we only consider the case in which the *amount of overlap*, w, is less than the size of LAs, d. This allows each LA to only partially overlap its immediately adjacent LAs. However, our approach can be extended to non-homogeneous systems and to the case in which w is greater than d.

The overlapping LA structure is organized so that any cell in the service area belongs to at least one LA and some may belong to more than one. Each mobile user is registered to exactly one LA at any time instant. Each mobile user keeps track of the LA to which it is currently registered. When a mobile user enters a new cell that does not belong to its LA, it requires a location update. If the new cell belongs to only one LA, the mobile user registers to this LA. Otherwise, the user will register to the LA whose center cell is closest to the new cell. Ties are broken by a random choice.

This scheme can be implemented by assigning a unique identifier (ID) to each LA. The base station in each cell periodically broadcasts, via its downlink control channel, its cell ID and a list of IDs of LAs to which the cell belongs. The list of LA IDs is ordered according to the distance of the center cell of the respective LA to the center of the cell served by the base station. Ties are broken by a random choice made by the base station at the beginning of each broadcast of the list. Each mobile user needs only to store the ID of its current LA. When active, a mobile user monitors the strongest broadcast channel and reads the list of the LA IDs that is broadcast. If the user's current LA ID is not on the list, a location update is required. When updating, a mobile user will choose as its new LA, the one which is closest to its current cell. It will replace the LA ID that is in its memory with the new LA ID. The user will also send its new LA ID to the base station through the uplink control channel. The system will update the user's location information in the appropriate location registers. Note that even in the conventional (non-overlapping) schemes, each base station must also broadcast its cell ID and the ID of the LA to which it belongs, and, each user must do detection and comparisons similar to that of user in this scheme. The proposed registration scheme does not incur additional signaling and complexity at either the base station or the user side.

When an incoming call arrives, the system pages the called mobile user in the user's currently registered LA. Several paging strategies that reduce average paging signaling are discussed in [8], [9]. Although the total *combined* signaling load depends on both location updating and paging signaling, here we put the paging algorithm aside and only consider location updating signaling.

3. Location Update Rate in One-dimensional Cellular Geometry

3.4. Non-overlapping LA Registration Scheme (Refer to Fig. 5(a))

In conventional one-dimensional non-overlapping LA schemes, the cells in the service area are grouped into many distinct rectilinear LAs of the same size, and any cell in the service area belongs to exactly one LA. Therefore, the number of mobile users (in a cell of a *given* LA) that are registered to the *given* LA is equal to the average number of mobile users in the cell. Consider a *given* LA of size d (say *LA* 1 in Fig. 5(a) in which $d = 4$). The average location update rate for *LA* 1 is the average number of mobile users that move out of *LA* 1 per unit time. Note that only the mobile users in the boundary cells (cells in the outermost ring) of an LA contribute to location updating signaling, so the average location update rate for an LA is the average number of mobile users that move out of the LA from the boundary cells per unit time. Recall that the average *dwell time* of a mobile user in a cell is $\overline{T}_d$. The average number of mobile users in a boundary cell is $\overline{K}$. The average rate of mobile user departures from a boundary cell of *LA* 1 (the average number of mobile users moving out of a boundary cell per unit time) , λ_d , is given by

$$\lambda_d = \overline{K} \cdot \frac{1}{\overline{T}_d} \tag{1}$$

There are two boundary cells for each LA in the one-dimensional case. They are shown shaded for *LA* 1 in Fig. 5(a). Consider one of the boundary cells. According to the one-dimensional mobility model, only half (on average) of the mobile users who exit a boundary cell require a location update. The other half exits the boundary cell but remains in the same LA. The average location update rate for *LA* 1, $\overline{R}_{LA}$, is the sum of the update rates from the boundary cells. Since there are two boundary cells (in the one-dimensional case), we have

$$\overline{R}_{LA} = \frac{1}{2} \cdot \lambda_d + \frac{1}{2} \cdot \lambda_d = \overline{K} \cdot \frac{1}{\overline{T}_d} \tag{2}$$

The total number of location updates per unit time in the system depends on the number of LAs for a given user population. For different LA configurations, the number of LAs in the system varies, so the location update rate per LA is not a good criterion to evaluate the system's performance. Since we want to compare systems having different LA structures but with the same number of users, we normalize the location update rate on per user basis. The number of cells in *LA* 1 is $(2 \cdot d - 1)$, therefore, the total average number of mobile users that are registered to *LA* 1, $\overline{N}$, is

$$\overline{N} = (2 \cdot d - 1) \cdot \overline{K} \tag{3}$$

Normalizing, we find that the average location update rate per user, $\overline{R}_{MS}$, is given by

$$\overline{R}_{MS} = \frac{\overline{R}_{LA}}{\overline{N}} = \frac{1}{(2 \cdot d - 1)} \cdot \frac{1}{\overline{T}_d} \tag{4}$$

3.2. Overlapping LA Registration Scheme (Refer to Fig. 5(b) and Fig. 5(c))

From equations (1) and (2), we see that the average location update rate for a *given* LA is proportional to the average number of mobile users that are in boundary cells of the *given* LA and that are registered to the *given* LA. Therefore, if we can reduce the average number of mobile users that are in boundary cells of a *given* LA and that are registered to the *given* LA, then the average location update rate for the *given* LA and the average location update rate per user are both reduced. Suppose that we allow the boundary cells to belong to more than one LA and that a mobile user is registered to exactly one LA at any time instant. Since the average number of mobile users in a cell remains $\overline{K}$, then the average number of mobile users in a boundary cell that are registered to an *given* LA is less than $\overline{K}$. Thus, the average location update rate per user can be reduced.

Consider the overlapping LA structure shown in Fig. 5(b) in which d=4 and $w = 1$. The neighboring LAs overlap the outermost ring of each LA. Thus, each of the boundary cells of an LA belongs to two LAs. Any *given* LA (say *LA* 1) having a size *d*, overlaps its two neighboring LAs: *LA* 2 and *LA* 3. The mobile users that are in the boundary cells (shaded)of *LA* 1 are registered to either *LA* 1 or *LA* 2 (or *LA* 3). For convenience, we use MS1 to denote the mobile users that are registered to *LA* 1, MS2 to denote the mobile users that are registered to *LA* 2, and MS3 to denote the mobile users that are registered to *LA* 3. Because of the homogeneity, the average number of MS1 in each of the boundary cells of *LA* 1 is $\overline{K}/2$. Only the departures of MS1 in the boundary cells of *LA* 1, contribute to location updates for *LA* 1. Mobile users that are in the boundary cells of *LA* 1 but are registered to other LAs do not affect the location update rate for *LA* 1. In the following discussion we focus on MS1 in *LA* 1. The average rate of MS1 departures from each of two boundary cells of *LA* 1, λ_d, is

$$\lambda_d = \frac{\overline{K}}{2} \cdot \frac{1}{\overline{T_d}} \tag{5}$$

As in Section 3. 1, the average location update rate for *LA* 1, $\overline{R}_{LA}$, is

$$\overline{R}_{LA} = \frac{1}{2} \cdot \lambda_d + \frac{1}{2} \cdot \lambda_d = \frac{\overline{K}}{2} \cdot \frac{1}{\overline{T_d}} \tag{6}$$

There are $(2 \cdot d - 1)$ cells in an LA in the one-dimensional case. Two of them are boundary cells, the other $(2 \cdot d - 3)$ cells are not boundary cells. The average number of MS1 in each of the boundary cells of *LA* 1 is $\overline{K}/2$, and the average number of MS1 in each of the other $(2 \cdot d - 3)$ cells of *LA* 1 $\overline{K}$. So the total average number of MS1 in *LA* 1, $\overline{N}$, is

$$\overline{N} = (2 \cdot d - 3) \cdot \overline{K} + 2 \cdot \frac{\overline{K}}{2} = 2 \cdot (d - 1) \cdot \overline{K} \qquad (7)$$

Thus, the average location update rate per user, $\overline{R}_{MS}$, is

$$\overline{R}_{MS} = \frac{\overline{R}_{LA}}{\overline{N}} = \frac{1}{4 \cdot (d - 1)} \cdot \frac{1}{\overline{T}_d} \qquad (8)$$

Comparing equations (4) and (8), it is seen that the average location update rate per user decreases by overlapping one ring of each LA.

In the following discussion, we continue this reasoning and show that overlapping more rings of each LA can reduce the average location update rate per user further. Consider a homogeneous one-dimensional system, in which LA size is *d,* and the outermost *w* rings of each LA are overlapped by the outermost *w* rings of the two neighboring LAs. An example LA structure having more than one ring of overlap is shown in Fig. 5(c), in which $d = 8$ and $w = 4$. From our earlier formulation, we know that, in addition to user mobility, the location update rate per user also depends on; 1) the number of mobile users that are in boundary cells of a *given* LA and that are registered to the *given* LA; and, 2) the number of mobile users that are in other cells of the *given* LA and that are registered to the *given* LA. Consider *LA* 1 in Fig. 5(c). It overlaps its two neighboring LAs: *LA* 2 and *LA* 3. In order to find the average location update rate per user, we need to find the average number of MS1 in each cell of *LA* 1. For this purpose, a one-axis coordinate system whose origin coincides with the center cell of *LA* 1 is used (See Fig. 5(c)), so each cell in the system is associated with a unique coordinate (*i*). From now on, we focus on MS1 in *LA* 1. We consider the system to be in statistical equilibrium. That is, probabilities and expectations are not time dependent. Thus, the average number of MS1 in a cell is time independent. We let x_i denote the average number of MS1 in the cell with coordinate (*i*) in *LA* 1. Based on the equilibrium property, we develop an algorithm to find the x_i s.

Since each LA is symmetrical about its center cell, it is sufficient to find the average number of MS1 in each of the cells on one side of the origin of the coordinate system. We consider the right-hand side (including center cell) of the origin of the one-axis coordinate system in *LA* 1. As shown in Fig.5(c), this side of *LA* 1 is divided into two regions labeled R(1) and R(1,3) which include cells that belong only to *LA* 1 and those that belong to both *LA* 1 and *LA* 3 respectively. Consider cell (*i*) ($i = 0, 1, 2, \ldots d - w - 1$) in region R(1). This cell is not overlapped and belongs only to *LA* 1. So the average number of MS1 in this cell equals the average number of mobile users in this cell, $\overline{K}$. Therefore

$$x_i = \overline{K} \qquad i = 0, 1, 2, \ldots d\text{-}w\text{-}1. \tag{9}$$

Consider cell (*i*) in region R(1,3), where $i = d - w, d - w + 1, \ldots d\text{-}1$. This cell belongs to both *LA* 1 and *LA* 3. Since the average number of MS1 in cell (*i*) is x_i, the average rate of MS1 departures from this cell, λ_d^i, is

$$\lambda_d^i = x_i \cdot \frac{1}{\overline{T}_d} \tag{10}$$

Since (on average) only half of MS1 departures from each of the two neighboring cells (*i*-1) and (*i*+1) arrive at cell (*i*), the average rate of MS1 arrivals to cell (*i*), λ_a^i, is the sum of average rates of MS1 departures from cells (*i*-1) and (*i*+1) who arrive at cell (*i*). Therefore, we have

$$\lambda_a^i = \frac{1}{2} \cdot \lambda_d^{i-1} + \frac{1}{2} \cdot \lambda_d^{i+1} = \frac{1}{2} \cdot x_{i-1} \cdot \frac{1}{\overline{T}_d} + \frac{1}{2} \cdot x_{i+1} \cdot \frac{1}{\overline{T}_d} \tag{11}$$

$$i = d - w,\ d - w + 1, \ldots\ d\text{-}1.$$

Since cell (*i*) in region R(1,3) belongs to both *LA* 1 and *LA* 3, MS1 and MS3 will remain registered to their previous LAs upon entering cell (*i*). No mobile users who are registered to other LAs (excluding *LA* 1 and *LA* 3) arrive at cell (*i*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i*) equals the average rate of MS1 arrivals to the cell. Thus, we have

$$\lambda_d^i = \lambda_a^i \tag{12}$$

Equations (10), (11) and (12) give

$$x_i = \frac{1}{2} \cdot x_{i-1} + \frac{1}{2} \cdot x_{i+1} \qquad i = d - w,\ d - w + 1, \ldots\ d - 1.$$

(13)

Since cell (d) belongs to only *LA* 3 and ***not to*** *LA* 1, no mobile users in cell (d) are registered to *LA* 1. Therefore

$$x_d = 0 \tag{14}$$

Rearranging equations (9), (13) and (14), we get the following simultaneous equations about x_i s

$$x_i = \overline{K} \qquad i = 0, 1, 2, \ldots d\text{-}w\text{-}1. \tag{15a}$$

$$x_i = \frac{1}{2} \cdot x_{i-1} + \frac{1}{2} \cdot x_{i+1} \qquad i = d - w,\ d - w + 1, \ldots\ d - 1 \tag{15b}$$

$$x_d = 0 \tag{15c}$$

Solution of the simultaneous equations (15) by substituting for x_i s, gives

$$x_i = \overline{K} \qquad i = 0, 1, 2, \ldots d\text{-}w\text{-}1 \tag{16a}$$

$$x_i = \frac{w - (i - (d - w))}{w + 1} \cdot \overline{K} \qquad i = d\text{-}w, d\text{-}w\text{+}1, \ldots d\text{-}1 \tag{16b}$$

Knowing the average number of MS1 in each cell of *LA* 1, we can find the average location update rate for *LA* 1 and the average location update rate per user respectively. Similarly, the average rate of MS1 departures from each of two boundary cells of *LA* 1, λ_d, is

$$\lambda_d = x_{d-1} \cdot \frac{1}{\overline{T}_d} = \frac{1}{(w+1)} \cdot \overline{K} \cdot \frac{1}{\overline{T}_d} \tag{17}$$

As in Section 3. 1, the average location update rate for *LA* 1, $\overline{R}_{LA}$, is the sum of the average rates of MS1 departures from the two boundary cells of *LA* 1 who require location update

$$\overline{R}_{LA} = \frac{1}{2}\cdot\lambda_d + \frac{1}{2}\cdot\lambda_d = \frac{\overline{K}}{w+1}\cdot\frac{1}{\overline{T}_d} \tag{18}$$

The total average number of MS1, $\overline{N}$, is the sum of MS1 in all cells of *LA* 1. Thus,

$$\overline{N} = x_0 + 2\cdot\sum_{i=1}^{d-1} x_i = (2d - w - 1)\cdot\overline{K} \tag{19}$$

The average location update rate per user, $\overline{R}_{MS}$, is

$$\overline{R}_{MS} = \frac{\overline{R}_{LA}}{\overline{N}} = \frac{1}{(w+1)\cdot(2d - w + 1)}\cdot\frac{1}{\overline{T}_d} \tag{20}$$

4. Location Update Rate in Two-dimensional Cellular Geometry

4.1 Non-overlapping LA Registration Scheme (Refer to Fig. 6)

In conventional two-dimensional non-overlapping LA schemes, the cells in the service area are grouped into many distinct hexagonal LAs of the same size, and any cell in the service area belongs to exactly one LA. Therefore, the average number of mobile users (in a cell of a *given* LA) that are registered to the *given* LA is equal to the average number of mobile users in the cell, $\overline{K}$. Consider a *given* LA of size *d* (say *LA* 1 in Fig. 6 in which $d = 4$). The average location update rate for *LA* 1 is the average number of mobile users who exit *LA* 1 per unit time. As in the one-dimensional case, only the mobile users in the boundary cells of an LA contribute to location updating signaling, so the average location update rate for an LA is the average number of mobile users that move out of the LA from the boundary cells per unit time. The average rate of mobile user departures from a boundary cell of *LA* 1, λ_d, is given by

$$\lambda_d = \overline{K}\cdot\frac{1}{\overline{T}_d} \tag{21}$$

There are $(6\cdot(d-1))$ boundary cells for *LA* 1. As shown in Fig. 6, six of them are corner boundary cells. Each of the six corner boundary cells has three of its six edges part of *LA* 1 boundary. For each of other boundary cells, two of its six edges are part of *LA* 1 boundary. According to the two-dimensional mobility model, only 3/6 of the mobile users who exit a corner boundary cell perform location update, and 2/6 of the mobile users who exit each of the other $(6\cdot(d-1)-6)$ boundary cells require location update. Mobile users, who exit a boundary cell along the other edges, remain in the same LA. The average location update rate for *LA* 1, $\overline{R}_{LA}$, is the sum of these departures from the boundary cells who require a location update in a unit time. Therefore

$$\overline{R}_{LA} = 6\cdot\frac{3}{6}\cdot\lambda_d + [6\cdot(d-1)-6]\cdot\frac{2}{6}\cdot\lambda_d = (2\cdot d+1)\cdot\overline{K}\cdot\frac{1}{\overline{T}_d} \tag{22}$$

As in subsection 3.1, we want to normalize the location update rate on per user basis. Since the number of cells in *LA* 1 is $(3\cdot d\cdot(d-1)+1)$, the total average number of mobile users that are registered to *LA* 1, $\overline{N}$, is

$$\overline{N} = [3\cdot d\cdot(d-1)+1]\cdot\overline{K} \tag{23}$$

The average location update rate per user, $\overline{R}_{MS}$, is

$$\overline{R}_{MS} = \frac{\overline{R}_{LA}}{\overline{N}} = \frac{2\cdot d+1}{3\cdot d\cdot(d-1)+1}\cdot\frac{1}{\overline{T}_d} \tag{24}$$

4.2. Overlapping LA Registration Scheme

As in the one-dimensional case, overlapping LAs in two-dimensional geometry can also reduce the average location update rate per user. Consider the overlapping LA structure shown in Fig. 7 in which $d = 4$ and $w = 1$. The outermost ring of each LA overlaps its six neighbors. Any given LA (say *LA* 1) having a size d, overlaps its six neighboring LAs: *LA* 2, *LA* 3, *LA* 4, *LA*5, *LA* 6 and *LA* 7. Each of the six corner boundary cells of *LA* 1 belongs to three LAs, while each of the other boundary cells of *LA* 1 belong to two LAs. The mobile users that are in the boundary cells of *LA* 1 may register to different LAs. For convenience, we use MS1, MS2, MS3, MS4, MS5, MS6 and MS7 to denote the mobile users that are registered to *LA* 1, *LA* 2, *LA* 3, *LA*

4, $LA5$, LA 6 and LA 7, respectively. Because of the homogeneity, the average number of MS1 in each of the six corner boundary cells of LA 1 is $\overline{K}/3$, while the average number of MS1 in each of the other boundary cells is $\overline{K}/2$. The average rate of MS1 departures from each of six corner boundary cells of LA 1, λ_{d1}, is

$$\lambda_{d1} = \frac{\overline{K}}{3} \cdot \frac{1}{\overline{T}_d} \tag{25}$$

For each of the other boundary cells, the average departure rate of MS1, λ_{d2}, is

$$\lambda_{d2} = \frac{\overline{K}}{2} \cdot \frac{1}{\overline{T}_d} \tag{26}$$

Mobile users that are in boundary cells of LA 1 but are registered to other LAs do not affect the location update rate for LA 1. Proceeding similarly to subsection 4.1, the average location update rate for LA 1, $\overline{R}_{LA}$, is

$$\overline{R}_{LA} = 6 \cdot \frac{3}{6} \cdot \lambda_{d1} + \left[6 \cdot (d-1) - 6\right] \cdot \frac{2}{6} \cdot \lambda_{d2} = (d-1) \cdot \overline{K} \cdot \frac{1}{\overline{T}_d} \tag{27}$$

The average numbers of MS1 in the cells of LA 1 are as follows: 1) $\overline{K}/3$ in each of the six corner boundary cells; 2) $\overline{K}/2$ in each of other ($6 \cdot (d-1) - 6$) boundary cells; and, 3) $\overline{K}$ in each of other $(3 \cdot (d-1) \cdot (d-2) + 1)$ inner cells. Thus, the total average number of MS1 in LA 1, $\overline{N}$, is

$$\begin{aligned} \overline{N} &= 6 \cdot \frac{\overline{K}}{3} + \left[6 \cdot (d-1) - 6\right] \cdot \frac{\overline{K}}{2} + \left[3 \cdot (d-1) \cdot (d-2) + 1\right] \cdot \overline{K} \\ &= 3 \cdot (d-1)^2 \cdot \overline{K} \end{aligned} \tag{28}$$

The average location update rate per user, $\overline{R}_{MS}$, is

$$\overline{R}_{MS} = \frac{\overline{R}_{LA}}{\overline{N}} = \frac{1}{3 \cdot (d-1)} \cdot \frac{1}{\overline{T}_d} \tag{29}$$

Now, we consider the more general case in which the LA size *is d* rings while the outermost *w* rings of each LA are overlapped by the outermost *w* rings of the six neighboring

LAs. An example LA structure having more than one ring of overlap is shown in Fig. 4(b), in which $d = 5$ and $w = 2$. From section 3, we know that, in addition to user mobility, the location update rate per user depends on; 1) the number of mobile users that are in boundary cells of a *given* LA and that are registered to the *given* LA; and, 2) the number of mobile users that are in other cells and that are registered to the *given* LA. Consider *LA* 1 in Fig. 4(b). It overlaps its six neighboring LAs: *LA* 2, *LA* 3, *LA* 4, *LA*5, *LA* 6 and *LA* 7. In order to find the average location update rate per user, we need to find the average number of MS1 in each cell of *LA* 1. For this purpose, a 60° degree coordinate system is employed with the origin coinciding with the center cell of *LA* 1 (see Fig. 4(b)). Therefore, each cell in the system is associated with a unique coordinates (i, j).

In the following, we focus on MS1 in *LA* 1 (see Fig. 4(b)). We consider the system to be in statistical equilibrium. That is, probabilities and expectations are not time dependent. Thus, the average number of MS1 in a cell is time independent. We let $x_{i,j}$ denote the average number of MS1 in the cell with coordinates (i, j). Since each hexagonal LA has six symmetrical sectors, it is sufficient to find the average number of MS1 in each cell of one sector of *LA* 1. For a *given* sector of *LA* 1, all or part of this sector may be overlapped by other LAs for different combinations of d and w. Upon entering a cell of the *given* sector, a mobile user who is registered to *LA* 1 will remain registered to *LA* 1, while, a mobile user who is registered to an LA other than *LA* 1 will, 1) remain registered to its previous LA; or, 2) register to *LA* 1; or, 3) register to an LA which is neither *LA* 1 nor its previous LA. This depends on both the user and the cell's position in the *given* sector. Algorithmically, we define indices C[$(i_1, i_2,\ldots i_l)$; $(j_1, j_2,\ldots j_m)$; $(k_1,k_2,\ldots k_n)$] to represent a cell's attributes in the *given* sector. The string in the first parentheses represents that the cell belongs to *LA* i_1, *LA* i_2,...and *LA* i_l simultaneously, and to no other LA. The string in the second parentheses indicates that the cell is contiguous to the boundaries of *LA* j_1, *LA* j_2,...and *LA* j_m, and to no other LA boundaries. If the cell is not contiguous to the boundary of any LA, "null" is placed in the second parentheses. The string in the third parentheses denotes that the cell is tied to be closest to the center cells of *LA* k_1, *LA* k_2,...and *LA* k_n. If there is only one LA in the third parentheses, that means the cell is closest to the center cell of this LA. According to the definition of indices of a cell, several cells may have the same indices. For our specific situation in which w is less than d, the strings in the three parentheses are subsets of the set of numbers {1,2,3,4,5,6,7,}that represents the seven LAs: *LA* 1, *LA* 2, *LA* 3, *LA* 4, *LA* 5, *LA* 6, and *LA* 7. Each string within a parentheses is organized in ascending order of the LA index.

In order to utilize the property of the equilibrium system, we divide a *given* sector into several disjoint regions so that the same kind of mobile users (such as MS1, MS2, . . . MS7) have the same registration behavior upon entering any cell of the *given* region. For convenience, we use notation $R[(i_1, i_2,\ldots i_l); (j_1, j_2,\ldots j_m); (k_1,k_2,\ldots k_n)]$ to denote a region that is the union of all cells with indices $C[(i_1, i_2,\ldots i_l); (j_1, j_2,\ldots j_m); (k_1,k_2,\ldots k_n)]$. However, in some situations in which a region can be uniquely determined by the strings in the first one or two parentheses in the region's notation, we do not care what the strings in the remaining parentheses are. This is indicated in the region's notation by entering "*" in the corresponding parentheses. This means that the strings in the parentheses with "*" can be any set of possible LAs. Based on the above discussion, we develop an algorithm to find the $x_{i,j}$ s. In the following, we discuss this algorithm for all combinations of d and w.

Case I: $1 \le w < d-2$, w is odd (Refer to Fig. 8a. Fig. 8b is used to help with visualizations)

The discussion refers to Fig. 8a in which d=9 and w=5, however because it is difficult to discern LA boundaries in this figure, we also provide Fig. 8b which shows approximate LA boundaries as smooth closed curves.

We consider a given sector (the sector that lies between $i > 0$ and $j \geq 0$ in the coordinate system in Fig. 8a.) of *LA* 1. This sector is partially overlapped by both *LA* 2 and *LA* 3. We divide this sector into the following nine disjoint regions. These regions are shown in different shaded patterns in Fig. 8a. In the following, we consider these regions respectively.

(a). *Region* R[(1); (*); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1); (*); (*)]. Any cell in this region belongs only to *LA* 1 and is not overlapped by other LAs. Mobile users in this region are all registered to *LA* 1. The average number of MS1 in each cell of this region equals the average number of mobile users in the cell, $\overline{K}$, thus

$$x_{i,j} = \overline{K} \qquad \text{for } i = 1, 2, \ldots d-1-w \text{ and } j = 0, 1, \ldots d-1-w-i \tag{30a}$$

(b). *Region* R[(1,2,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (null); (*)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell *(i,j)* in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell *(i, j)*, $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell *(i,j)* is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. We can write an equation expressing this as follows.

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \tag{30b}$$

for $\quad i = d-w, d-w+1, \ldots, d-1$ and $\quad j = 0, 1, 2, \ldots d-1-i$

(c). *Region* R[(1,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (null); (*)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell *(i,j)* in this region. MS1 and MS3 will remain registered to their previous LAs upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell *(i, j)*, $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell *(i, j)*. The average rate of MS1 arrivals to cell *(i,j)* is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Thus, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (30c)$$

for $\quad i = 2, 3, \ldots, d-w-2 \quad$ and $\quad j = d-w-i, d-w-i+1, \ldots, d-i-1$

(d). *Region* R[(1,3); (2); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2); (3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 2, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell *(i,j)* in this region. MS1 and MS3 will remain registered to their previous LAs when they move into this cell. MS2 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs *(excluding LA 1, LA 2 and LA 3)* will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell *(i,j)* is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (30d)$$

for $\quad i = d-w-1 \quad$ and $\quad j = \frac{w+1}{2} + 1, \frac{w+1}{2} + 2, \ldots, w$

(e). *Region* R[(1,3); (4); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (4); (3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3 ,and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell *(i,j)* in this region. MS1and MS3 will remain registered to their previous LAs when they move into this cell. MS4 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs *(excluding LA 1, LA 3 and LA 4)* will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i*, *j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (*i*, *j*). The average rate of MS1 arrivals to cell (*i*,*j*) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (30e)$$

$$\text{for} \quad i = 1 \quad \text{and} \quad j = d - \frac{w+1}{2}, d - \frac{w+1}{2} + 1, \ldots, d - 2$$

(f). *Region* R[(1,3); (2); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2); (1)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 2, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (*i*,*j*) in this region. The neighboring cells (*i*+1,*j*-*1*) and (*i*+1,*j*) are both the boundary cells of *LA* 2. MS2 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (*i*,*j*). MS1and MS3 will remain registered to their previous LAs upon entering cell (*i*,*j*). No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into cell (*i*,*j*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i*,*j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (*i*,*j*) and the average rate of MS2 arrivals to cell (*i*,*j*). Because of the system's homogeneity, the average numbers of MS2 in the two neighboring cells (*i*+1,*j*-1) and (*i*+1,*j*) are $x_{i+j,d-1-(i+j)}$ and $x_{i+j+1,d-1-(i+j+1)}$ respectively. Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell (*i*,*j*), we have

$$\begin{aligned} &x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\ &- \frac{1}{6} \cdot (x_{i+j,d-1-(i+j)} + x_{i+j+1,d-1-(i+j+1)}) = 0 \end{aligned} \qquad (30f)$$

for $\quad i = d - w - 1 \quad$ and $\quad j = 1, 2, \ldots, w$

(g). *Region* R[(1,3); (4); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (4); (1)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (*i, j*) in this region. Either or both of the neighboring cells (*i*-1, *j*) and (*i*-1, *j*+1) are the boundary cells of *LA* 4. MS4 in these two neighboring cells will perform location updates and register to *LA* 1 upon entering cell (*i,j*). MS1and MS3 will remain registered to their previous LAs upon entering cell (*i,j*). No mobile users that are registered to other LAs (*excluding LA 1, LA 3 and LA 4*) will move into cell (*i,j*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i,j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (*i,j*) and the average rate of MS4 arrivals to cell (*i,j*). Because of the system's homogeneity, the average numbers of MS4 in these two neighboring cells (*i*-1,*j*) and (*i*-1,*j*+1) are equal to the average numbers of MS1 that are in cells (*d*-1-*j*+(*d*-*w*),*j*-(*d*-*w*)) and (*d*-2-*j*+(*d*-*w*),*j*-(*d*-*w*)+1), respectively. They are $x_{d-1-j+(d-w),j-(d-w)}$ and $x_{d-2-j+(d-w),j-(d-w)+1}$ correspondingly. Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell (*i,j*), we have

$$
\begin{aligned}
& x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\
& - \frac{1}{6} \cdot (x_{d-1-j+(d-w),j-(d-w)} + x_{d-2-j+(d-w),j-(d-w)+1}) = 0
\end{aligned}
\tag{30g}
$$

for $\quad i = 1 \quad$ and $\quad j = d - w - 1, d - w, \ldots, d - 2 - \frac{w+1}{2}$

(h). *Region* R[(1,3); (2); (1,3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2); (1,3)]. Any cell in this region, 1) belongs to *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 2, and to no other LA boundaries; and, 3) has equal minimum distance to the center cells of *LA* 1 and *LA* 3. In fact, this region includes only one cell which is cell (*i,j*) with $i = d - w - 1$ and $j = (w+1)/2$. MS1and MS3 will remain registered to their previous LAs upon entering this cell. The two neighboring cells (*i*+1,*j*-1) and (*i*+1,*j*) are the boundary cells of *LA* 2. When MS2 in these two neighboring cells move to cell (*i,j*) , half of them will perform location updates and register to *LA* 1, the other half will perform location update and register to *LA* 3. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into cell (*i,j*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i,j*) , $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (*i,j*) and half of the average rate of MS2 arrivals to cell (*i,j*). Because of the system's homogeneity, the average numbers of MS2 in the two neighboring cells (*i*+1,*j*-1) and (*i*+1,*j*) are $x_{i+j,d-1-(i+j)}$ and $x_{i+j+1,d-1-(i+j+1)}$ respectively. Since only 1/6 of the departures from each of its six neighboring cells arrive at cell (*i,j*), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) - \frac{1}{2} \cdot \frac{1}{6} \cdot (x_{i+j,d-1-(i+j)} + x_{i+j+1,d-1-(i+j+1)}) = 0 \qquad (30h)$$

$$\text{for} \quad i = d - w - 1 \quad \text{and} \quad j = \frac{w+1}{2}$$

(i). *Region* R[(1,3); (4); (1,3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (4); (1,3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) has equal minimum distance to the center cells of *LA* 1 and *LA* 3. Actually, this region includes only one cell which is cell (*i,j*) with $i = 1$ and $j = d - 1 - (w+1)/2$. MS1and MS3 will remain registered to their previous LAs upon entering this cell. The two neighboring cells (*i*-1,*j*) and (*i*-1,*j*+1) are the boundary cells of *LA* 4. When MS4 in these two neighboring cells move to cell (*i,j*) , half of them will perform location update and then register to *LA* 1, the other half will perform location update and register to *LA* 3. No mobile users that are registered to other LAs (*excluding LA 1, LA 3 and*

LA 4) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (i,j) and half of the average rate of MS4 arrivals to cell (i,j). The average numbers of MS4 in the two neighboring cells $(i\text{-}1,j)$ and $(i\text{-}1,j+1)$ are $x_{d-1-j+(d-w),j-(d-w)}$ and $x_{d-2-j+(d-w),j-(d-w)+1}$ respectively. Since only 1/6 of the departures from each of its six neighboring cells move to cell (i,j), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1})$$
$$- \frac{1}{2} \cdot \frac{1}{6} \cdot (x_{d-1-j+(d-w),j-(d-w)} + x_{d-2-j+(d-w),j-(d-w)+1}) = 0 \qquad (30i)$$

for $\quad i = 1 \quad$ and $\quad j = d - 1 - \dfrac{w+1}{2}$

The above equations constitute a set of simultaneous equations about $x_{i,j}$s for the given sector of *LA* 1. The boundary conditions for this set of equations are determined by the average number of MS1 in each of cells that are contiguous to but do not belong to the given sector. These cells are shown shaded in Fig. 9. Since the cellular system is homogeneous, the average number of MS1 in each of these cells can be found as follows:

Since cell (0,0) belongs ONLY to *LA* 1, then

$$x_{0,0} = \overline{K} \qquad (30j)$$

The average number of MS1 in a cell along the j-axis in shaded area is equal to the average number of MS1 in the corresponding cell along the i-axis.

$$x_{0,j} = x_{j,0} \qquad j = d\text{-}w,\ d\text{-}w,\ d\text{-}w+1, \ldots, d\text{-}1 \qquad (30k)$$

We consider a cell along the line j=-1 in shaded area in *LA* 1. The average number of MS1 in this cell equals the average number of MS1 in the corresponding cell along the line i=1 in *LA* 1.

$$x_{i,-1} = x_{1,i-1} \qquad i = d\text{-}w,\ d\text{-}w\text{+}1,\ \ldots,\ d\text{-}1 \tag{30l}$$

The other shaded cells do not belong to *LA* 1, so the average number of MS1 in each of these cells is zero.

$$x_{d,-1} = 0 \tag{30m}$$

$$x_{i,d-i} = 0 \qquad i = 1, 2, 3, \ldots, d \tag{30n}$$

Case II: 1 ≤ w< d-2, w is even (Refer to Fig. 10a. Fig. 10b is used to help with visualizations)

The discussion refers to Fig. 10a in which d=9 and w=4, however because it is difficult to discern LA boundaries in this figure, we also provide Fig. 10b which shows approximate LA boundaries as smooth closed curves.

We consider a given sector (the sector that lies between i >0 and j≥0 in the coordinate system in Fig. 10a.) of *LA* 1. This sector is partially overlapped by both *LA* 2 and *LA* 3. We divide this sector into the following seven disjoint regions. These regions are shown in different shaded patterns in Fig. 10a. Comparing this case with case I of this subsection, the only difference between these two cases is that there are no regions R[(1,3); (2); (1,3)] and R[(1,3); (4); (1,3)] in this case. Proceeding similarly, we can get a set of simultaneous equations for these seven regions.

(a). *Region* R[(1); (*); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1); (*); (*)]. Any cell in this region belongs only to *LA* 1 and is not overlapped by other LAs. Mobile users in this region are all registered to *LA* 1. The average number of MS1 in each cell of this region equals the average number of mobile users in the cell, $\overline{K}$, thus

$$x_{i,j} = \overline{K} \qquad \text{for } i = 1, 2, \ldots d-1-w \text{ and } j = 0, 1, \ldots d-1-w-i \tag{31a}$$

(b). *Region* R[(1,2,3);(null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (null); (*)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell (i,j) in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. We can write an equation expressing this as follows.

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (31b)$$

for $\quad i = d-w, d-w+1, \ldots, d-1 \quad$ and $\quad j = 0, 1, 2, \ldots d-1-i$

(c). *Region* R[(1,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (null); (*)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell (i,j) in this region. MS1and MS3 will remain registered to their previous LAs upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Thus, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (31c)$$

for $\quad i = 2, 3, \ldots, d-w-2 \quad$ and $\quad j = d-w-i, d-w-i+1, \ldots, d-i-1$

(d). *Region* R[(1,3); (2); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2); (3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; 2) is contiguous to the boundary of *LA* 2, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell (i,j) in this region. MS1and MS3 will remain registered to their previous LAs when they move into this cell. MS2 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (31d)$$

for $\quad i = d - w - 1 \quad$ and $\quad j = \frac{w}{2} + 1, \frac{w}{2} + 2, \ldots, w$

(e). *Region* R[(1,3); (4); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (4); (3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell (i,j) in this region. MS1and MS3 will remain registered to their previous LAs when they move into this cell. MS4 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \quad (31e)$$

for $\quad i = 1 \quad$ and $\quad j = d - \frac{w}{2} - 1, d - \frac{w}{2}, \ldots, d - 2$

(f). *Region* R[(1,3); (2); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2); (1)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 2, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (*i,j*) in this region. The neighboring cells (*i*+1,*j*-*1*) and (*i*+1,*j*) are both the boundary cells of *LA* 2. MS2 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (*i,j*). MS1and MS3 will remain registered to their previous LAs when they move into cell (*i,j*). No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into cell (*i,j*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i,j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (*i,j*) and the average rate of MS2 arrivals to cell (*i,j*). Because of the system's homogeneity, the average numbers of MS2 in the two neighboring cells (*i*+1,*j*-1) and (*i*+1,*j*) are $x_{i+j,d-1-(i+j)}$ and $x_{i+j+1,d-1-(i+j+1)}$ respectively. Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell (*i,j*), we have

$$\begin{aligned} &x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\ &- \frac{1}{6} \cdot (x_{i+j,d-1-(i+j)} + x_{i+j+1,d-1-(i+j+1)}) = 0 \end{aligned} \quad (31f)$$

for $\quad i = d - w - 1 \quad$ and $\quad j = 1, 2, \ldots, \frac{w}{2}$

(g). *Region* R[(1,3); (4); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (4); (1)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (i, j) in this region. Either or both of the neighboring cells $(i-1, j)$ and $(i-1, j+1)$ are the boundary cells of *LA* 4. MS4 in these two neighboring cells will perform location updates and register to *LA* 1 upon entering cell (i,j). MS1and MS3 will remain registered to their previous LAs when they move into cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 3 and LA 4*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (i,j) and the average rate of MS4 arrivals to cell (i,j). Because of the system's homogeneity, the average numbers of MS4 in these two neighboring cells $(i-1,j)$ and $(i-1, j+1)$ are equal to the average number of MS1 that are in cells $(d-1-j+(d-w), j-(d-w))$ and $(d-2-j+(d-w), j-(d-w)+1)$, respectively. They are $x_{d-1-j+(d-w),j-(d-w)}$ and $x_{d-2-j+(d-w),j-(d-w)+1}$ correspondingly. Since (on average) only 1/6 of the departures from each of the neighboring cells arrive at cell (i,j), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1})$$
$$- \frac{1}{6} \cdot (x_{d-1-j+(d-w),j-(d-w)} + x_{d-2-j+(d-w),j-(d-w)+1}) = 0 \qquad (31g)$$

for $\quad i = 1 \quad$ and $\quad j = d - w - 1, d - w, \ldots, d - \frac{w}{2} - 2$

As before, the boundary conditions for this set of simultaneous equations are:

$$x_{0,0} = \overline{K} \qquad (31h)$$

-- Cell (0,0) belongs ONLY to *LA* 1, the average number of MS1 in this cell is $\overline{K}$.

$$x_{0,j} = x_{j,0} \qquad j = d\text{-}w,\ d\text{-}w\text{+}1, \ldots, d\text{-}1 \qquad (31i)$$

-- The average number of MS1 in a cell along j-axis equals the average number of MS1 in a corresponding cell along i-axis.

$$x_{i,-1} = x_{1,i-1} \qquad i = d\text{-}w,\ d\text{-}w\text{+}1,\ \ldots,\ d\text{-}1 \tag{31j}$$

-- The average number of MS1 in a cell along the line j=-1 equals the average number of MS1 in a corresponding cell along the line i=1.

$$x_{d,-1} = 0 \tag{31k}$$

-- This cell does not belong to *LA* 1, the average number of MS1 in this cell is zero.

$$x_{i,d-i} = 0 \qquad i = 1, 2, 3, \ldots, d \tag{31l}$$

-- These cells do not belong to *LA*1, the average number of MS1 in each of these cells is zero.

Similar derivations of the simultaneous equations about $x_{i,j}$s for the other cases are given in the Appendix.

Equations (30) and (31) as well as equations in appendix are sets of simultaneous equations which can be easily solved with the boundary conditions. For a *given* LA of size d with w rings overlapped in two-dimensional cellular structure, once the $x_{i,j}$s are determined, the average location update rate per user can be found as follows.

As in subsection 4.1, only 3/6 of MS1 departures from each of the six corner boundary cells and 2/6 of MS1 departures from each of the other ($6 \cdot (d-1) - 6$) boundary cells cause location update for *LA* 1. The average location update rate for *LA* 1, $\overline{R}_{LA}$, is the sum of these departures in a unit time.

$$\overline{R}_{LA} = 6 \cdot \frac{3}{6} \cdot x_{d-1,0} \cdot \frac{1}{\overline{T}_d} + 6 \cdot \frac{2}{6} \cdot (\sum_{i=1}^{d-2} x_{d-1-i,i}) \cdot \frac{1}{\overline{T}_d} = (3 \cdot x_{d-1,0} + 2 \cdot \sum_{i=1}^{d-2} x_{d-1-i,i}) \cdot \frac{1}{\overline{T}_d} \qquad (32)$$

The total average number of MS1 in *LA* 1, $\overline{N}$, is the sum of MS1 in each cell of the *LA* 1. Thus,

$$\overline{N} = x_{0,0} + 6 \cdot \sum_{i=1}^{d-1} \sum_{j=0}^{d-1-i} x_{i,j} \qquad (33)$$

The average location update rate per user, $\overline{R}_{MS}$, is

$$\overline{R}_{MS} = \frac{\overline{R}_{LA}}{\overline{N}} = \frac{3 \cdot x_{d-1,0} + 2 \cdot \sum_{i=1}^{d-2} x_{d-1-i,i}}{x_{0,0} + 6 \cdot \sum_{i=1}^{d-1} \sum_{j=0}^{d-1-i} x_{i,j}} \cdot \frac{1}{\overline{T}_d} \qquad (34)$$

5. Numerical Results and Discussion

For both one-dimensional and two-dimensional cases, we assume that *dwell time*, $\overline{T}_d$, of mobile users in a cell are 1, 2, 4, 8 minutes. This corresponds to different classes of mobile users ranging from high-mobility users to low-mobility users. The average number of mobile users in a cell was taken as $\overline{K} = 100$.

Fig. 15 shows the spatial distribution of mobile users that are registered to **a given** LA for the one-dimensional case with various *w* (*d* = 10 is assumed). Fig. 17 is a similar display for the two dimensional hexagonal case (*d*=10, various *w*). For each figure, the ordinate shows the average number of mobile users (per cell) that are registered to **the given** LA. The abscissa shows the distance of a cell from the center cell of **the given** LA. We can see that the cells that are farther from the center have fewer mobile users registered to **the given** LA. Thus, for a *cell in the outermost ring*, the average number of mobile users who are in the cell and who are registered to **the given** LA is smaller than that in any of other cells.

Fig. 16 and Fig. 18 show the relationship between the average location update rate per user and amount of overlap for the one-dimensional and two-dimensional hexagonal cases respectively. For each case, it is seen that the average location update rate per user decreases as the amount of overlap increases. These results are consistent with the simulation results in [4]. The average location update rate is the highest when the *amount of overlap* is zero (non-overlapping case). The reduction in the average location update rate is very significant even for the cases in which two or three outermost rings are overlapped, but the gain is not much when the amount of overlap continues to increase. This suggests that LA structures with large amount of overlap, w, are not advantageous.

Appendix

Following subsection 4.2, we continue the derivations of the simultaneous equations about the $x_{i,j}$ s for other cases in this section.

Case III: $w = d-2$, w is odd (Refer to Fig. 11a. Fig. 11b is used to help with visualizations)

The discussion refers to Fig. 11a in which d=7 and w=5, however because it is difficult to discern LA boundaries in this figure, we also provide Fig. 11b which shows approximate LA boundaries as smooth closed curves.

We consider a given sector (the sector that lies between i >0 and j≥0 in the coordinate system in Fig. 11a.) of *LA* 1. This sector is partially overlapped by both *LA* 2 and *LA* 3. We divide this sector into the following five disjoint regions. These regions are shown in different shaded patterns in Fig. 11a. In the following, we consider these five regions respectively.

(a). *Region* R[(1); (*); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1); (*); (*)]. Any cell in this region belongs only to *LA* 1 and is not overlapped by other LAs. In fact, this region includes only one cell which is cell with coordinates (1,0). Mobile users in this cell are all registered to *LA* 1. The average number of MS1 in this cell equals the average number of mobile users in this cell, $\overline{K}$, thus

$$x_{i,j} = \overline{K} \qquad \text{for} \qquad i = 1 \quad \text{and} \qquad j = 0 \tag{35a}$$

(b). *Region* R[(1,2,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (null); (*)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell *(i,j)* in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell *(i,j)*, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \tag{35b}$$

for $\quad i = 2, 3, \ldots, d-1 \quad$ and $\quad j = 0, 1, 2, \ldots d-1-i$

(c). *Region* R[(1,3); (2,4); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2,4); (3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of *LA* 2 and *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell *(i,j)* in this region. MS1and MS3 will remain registered to their previous LAs when they move into this cell. MS2 and MS4 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 ,LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (35c)$$

for $\quad i = 1 \quad$ and $\quad j = \frac{w+1}{2} + 1, \frac{w+1}{2} + 2, \ldots, d-2$

(d). *Region* R[(1,3); (2,4); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2,4); (1)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of *LA* 2 and *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (i,j) in this region. The neighboring cells $(i+1,j-1)$ and $(i+1,j)$ are both the boundary cells of *LA* 2. MS2 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (i,j). Either or both of the two neighboring cells $(i-1,j)$ and $(i-1, j+1)$ of cell (i,j) are the boundary cells of *LA* 4. MS4 in these two neighboring cells will perform location update and then register to *LA* 1 when they move to cell (i,j). MS1and MS3 will remain registered to their previous LAs when they move into cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average arrival rates to cell (i,j) of MS1, MS2 and MS4. Because of the homogeneity, the average numbers of MS2 in the two neighboring cells $(i+1, j-1)$ and $(i+1, j)$ are $x_{i+j,d-1-(i+j)}$ and $x_{i+j+1,d-1-(i+j+1)}$ respectively. The average numbers MS4 in the two neighboring cells $(i-1,j)$ and $(i-1,j+1)$ are $x_{d-1-j+(d-w),j-(d-w)}$ and $x_{d-2-j+(d-w),j-(d-w)+1}$ respectively. Since (on average) only 1/6 of the departures from each of its six neighboring cells arrive at cell (i,j), then we have

$$
\begin{aligned}
& x_{i,j} - \frac{1}{6}\cdot(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\
& - \frac{1}{6}\cdot(x_{d-1-j+(d-w),j-(d-w)} + x_{d-2-j+(d-w),j-(d-w)+1}) \qquad (35d) \\
& - \frac{1}{6}\cdot(x_{i+j,d-1-(i+j)} + x_{i+j+1,d-1-(i+j+1)}) = 0
\end{aligned}
$$

for $\quad i = 1 \quad$ and $\quad j = 1, 2, \ldots, \frac{w-1}{2}$

(e). *Region* R[(1,3); (2,4); (1,3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2,4); (1,3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of both *LA* 2 and *LA* 4, and to no other LA boundaries; and, 3) has equal minimum distance to the center cells of *LA* 1 and *LA* 3. Actually, this region includes only one cell that is cell (i,j) with $i = 1$ and $j = (w+1)/2$. The two neighboring cells $(i+1, j-1)$ and $(i+1,j)$ of cell (i,j) are the boundary cells of *LA* 2. When MS2 in these two neighboring cells move to cell (i,j), half of them will perform location update and register to *LA* 1, the other half will perform location update and register to *LA* 3. The two neighboring cells $(i-1, j)$ and $(i-1,j+1)$ of cell (i,j) are the boundary cells of *LA* 4. When MS4 in these two neighboring cells move to cell (i,j), half of them will perform location update and register to *LA* 1, the other half will perform location update and register to *LA* 3. We use MS2(1) and MS4(1) to denote those MS2 and those MS4 that will register to *LA* 1 respectively. MS1and MS3 will remain registered to their previous LAs upon entering cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j}\cdot(1/\overline{T}_d)$, equals the sum of the average arrival rates to cell (i,j) of MS1, MS2(1) and MS4(1). Because of the system's homogeneity, the average numbers of MS2 in the two neighboring cells $(i+1,j-1)$ and $(i+1,j)$ are $x_{i+j,d-1-(i+j)}$ and $x_{i+j+1,d-1-(i+j+1)}$ respectively. The average numbers of MS4 in the two neighboring cells $(i-1,j)$ and $(i-1,j+1)$ are $x_{d-1-j+(d-w),j-(d-w)}$ and $x_{d-2-j+(d-w),j-(d-w)+1}$ respectively. Since only 1/6 of the departures from each of its six neighboring cells arrive at cell (i,j), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}$$
$$- \frac{1}{2} \cdot \frac{1}{6} \cdot (x_{d-1-j+(d-w),j-(d-w)} + x_{d-2-j+(d-w),j-(d-w)+1}) \tag{35e}$$
$$- \frac{1}{2} \cdot \frac{1}{6} \cdot (x_{i+j,d-1-(i+j)} + x_{i+j+1,d-1-(i+j+1)}) = 0$$

for $i = 1$ and $j = \frac{w+1}{2}$

As before, the boundary conditions for the set of simultaneous equation are:

$$x_{0,0} = \overline{K} \tag{35f}$$

-- Cell (0,0) belongs ONLY to *LA* 1, the average number of MS1 in this cell is $\overline{K}$.

$$x_{0,j} = x_{j,0} \qquad j = d\text{-}w, d\text{-}w\text{+}1, \ldots, d\text{-}1 \tag{35g}$$

-- The average number of MS1 in a cell along *j*-axis equals the average number of MS1 in a corresponding cell along *i*-axis.

$$x_{i,-1} = x_{1,i-1} \qquad i = d\text{-}w, d\text{-}w\text{+}1, \ldots, d\text{-}1 \tag{35h}$$

-- The average number of MS1 in a cell along the line *j*=-1 equals the average number of MS1 in a corresponding cell along the line *i*=1.

$$x_{d,-1} = 0 \tag{35i}$$

-- This cell does not belong to *LA* 1, the average number of MS1 in this cell is zero.

$$x_{i,d-i} = 0 \qquad i = 1, 2, 3, \ldots, d \tag{35j}$$

-- These cells do not belong to LA1, the average number of MS1 in each of these cells is zero.

Case IV: $w = d\text{-}2$, w is even (Refer to Fig. 12a. Fig. 12b is used to help with visualizations)

The discussion refers to Fig. 12a in which d=8 and w=6, however because it is difficult to discern LA boundaries in this figure, we also provide Fig. 12b which shows approximate LA boundaries as smooth closed curves.

We consider a given sector (the sector that lies between i >0 and j≥0 in the coordinate system in Fig. 12a.) of *LA* 1. This sector is partially overlapped by both *LA* 2 and *LA* 3. We divide this sector into the following four disjoint regions. These regions are shown in different shaded patterns in Fig. 12a. Comparing this case with case III in appendix, the only difference between these two cases is that there is no region R[(1,3);(2,4);(1,3)] in this case. Proceeding similarly, we can get a set of simultaneous equations about $x_{i,j}$ s for the four regions.

(a). *Region* R[(1); (*); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1); (*); (*)]. Any cell in this region belongs only to *LA* 1 and is not overlapped by other LAs. Actually, this region includes only one cell which is cell with coordinates (1,0). Mobile users in this cell are all registered to *LA* 1. The average number of MS1 in this cell equals the average number of mobile users in this cell, $\overline{K}$, thus

$$x_{i,j} = \overline{K} \qquad \text{for} \qquad i = 1 \quad \text{and} \qquad j = 0 \tag{36a}$$

(b). *Region* R[(1,2,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (null); (*)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell (i,j) in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell (i,j), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (36b)$$

for $i = 2, 3, \ldots, d-1$ and $j = 0, 1, 2, \ldots d-1-i$

(c). *Region* R[(1,3); (2,4); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2,4); (3)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of both *LA* 2 and *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell (i,j) in this region. MS1and MS3 will remain registered to their previous LAs when they move into this cell. MS2 and MS4 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 ,LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (36c)$$

for $i = 1$ and $j = \frac{w}{2}+1, \frac{w}{2}+2, \ldots, d-2$

(d). *Region* R[(1,3); (2,4); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,3); (2,4); (1)]. Any cell in this region, 1) belongs to both *LA* 1 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of both *LA* 2 and *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (i,j) in this region. The neighboring cells $(i+1,j-1)$ and $(i+1,j)$ are both the boundary cells of *LA* 2. MS2 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (i,j). Either or both of the two neighboring cells $(i\text{-}1,j)$ and $(i\text{-}1, j{+}1)$ of cell (i,j) are the boundary cells of *LA* 4. MS4 in these two neighboring cells will perform location update and then register to *LA* 1 when they move to cell (i,j). MS1and MS3 will remain registered to their previous LAs when they move into cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average arrival rates to cell (i,j) of MS1, MS2 and MS4. Because of the homogeneity, the average numbers of MS2 in the two neighboring cells $(i{+}1, j\text{-}1)$ and $(i{+}1, j)$ are $x_{i+j,d-1-(i+j)}$ and $x_{i+j+1,d-1-(i+j+1)}$ respectively. The average numbers MS4 in the two neighboring cells $(i\text{-}1,j)$ and $(i\text{-}1,j+1)$ are $x_{d-1-j+(d-w),j-(d-w)}$ and $x_{d-2-j+(d-w),j-(d-w)+1}$ respectively. Since (on average) only 1/6 of the departures from each of its six neighboring cells arrive at cell (i,j), then we have

$$
\begin{aligned}
&x_{i,j} - \frac{1}{6}\cdot(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\
&- \frac{1}{6}\cdot(x_{d-1-j+(d-w),j-(d-w)} + x_{d-2-j+(d-w),j-(d-w)+1}) \\
&- \frac{1}{6}\cdot(x_{i+j,d-1-(i+j)} + x_{i+j+1,d-1-(i+j+1)}) = 0
\end{aligned}
\tag{36d}
$$

$$\text{for} \quad i = 1 \quad \text{and} \quad j = 1, 2, \ldots, \frac{w}{2}$$

As before, the boundary conditions for this set of equations are:

$$x_{0,0} = \overline{K} \tag{36e}$$

-- Cell (0,0) belongs ONLY to *LA* 1, the average number of MS1 in this cell is $\overline{K}$.

$$x_{0,j} = x_{j,0} \qquad j = d\text{-}w,\ d\text{-}w{+}1, \ldots, d\text{-}1 \tag{36f}$$

-- The average number of MS1 in a cell along j-axis equals the average number of

MS1 in a corresponding cell along i-axis.

$$x_{i,-1} = x_{1,i-1} \qquad i = d\text{-}w,\ d\text{-}w\text{+}1,\ \ldots,\ d\text{-}1 \tag{36g}$$

-- The average number of MS1 in a cell along the line j=-1 equals the average number of MS1 in a corresponding cell along the line i=1.

$$x_{d,-1} = 0 \tag{36h}$$

-- This cell does not belong to *LA* 1, the average number of MS1 in this cell is zero.

$$x_{i,d-i} = 0 \qquad i = 1, 2, 3, \ldots, d \tag{36i}$$

-- These cells do not belong to LA1, the average number of MS1 in each of these cells is zero.

Case V: w = d -1, w is odd (Refer to Fig. 13a. Fig. 13b is used to help with visualizations)

The discussion refers to Fig. 13a in which d=8 and w=7, however because it is difficult to discern LA boundaries in this figure, we also provide Fig. 13b which shows approximate LA boundaries as smooth closed curves.

We consider a given sector (the sector that lies between i >0 and j≥0 in the coordinate system in Fig. 13a.) of *LA* 1. All of this sector is overlapped by both *LA* 2 and *LA* 3. We divide this sector into the following eight disjoint regions. These regions are shown in different shaded patterns in Fig. 13a. In the following, we consider these regions respectively.

(a). *Region* R[(1,2,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (null); (*)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell (i,j) in this region. MS1, MS2 and MS3 will remain registered to their previous LAs upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i, j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (*i, j*). The average rate of MS1 arrivals to cell (*i,j*) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. We can write an equation expressing this as follows.

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (37a)$$

for $\quad i = 2, 3, \ldots, d-2 \quad$ and $\quad j = 1, 2, \ldots, d-i-1$

(b). *Region* R[(1,2,3); (4); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4); (3)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell (*i,j*) in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. MS4 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i, j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (*i, j*). The average rate of MS1 arrivals to cell (*i,j*) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (37b)$$

for $\quad i = 1 \quad$ and $\quad j = \frac{w+1}{2} + 1, \frac{w+1}{2} + 2, \ldots, d-2$

(c). *Region* R[(1,2,3); (7); (2)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (7); (2)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 7, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 2 than to the center of any other LA. Consider cell (i,j) in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. MS7 will perform location update and register to *LA* 2 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 7*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \tag{37c}$$

$$\text{for} \quad i = \frac{w+1}{2} + 1, \frac{w+1}{2} + 2, \ldots, d-1 \quad \text{and} \quad j = 0$$

(d). *Region* R[(1,2,3); (4,7); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4,7); (1)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of both *LA* 4 and *LA* 7, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the centers of any other LAs. Actually, this region includes only one cell which is cell (i,j) with i=1 and j=0. The neighboring cell $(i-1,j+1)$ is the boundary cell of *LA* 4. MS4 in this neighboring cell will perform location update and register to *LA* 1 upon entering cell (i,j). The two neighboring cells $(i,j-1)$ and $(i+1,j-1)$ of cell (i, j) are the boundary cells of *LA* 7. MS7 in these two neighboring cells will perform location update and register to *LA* 1 when they move to cell (i,j). MS1, MS2 and MS3 will remain registered to their previous LAs upon entering cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3, LA 4 and LA 7*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of average arrival rates to cell (i,j) of MS1, MS4 and MS7. Because of the homogeneity, the average number of MS4 in the neighboring cell (i-1,j + 1) is $x_{d-1-j,j}$. The average numbers of MS7 in the two neighboring cells (i, j-1) and (i+1, j-1) are $x_{i,d-1-i}$ and $x_{i+1,d-1-(i+1)}$ respectively. Since (on average) only 1/6 of the departures from each of its six neighboring cells arrive at cell (i,j), then we have

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}$$
$$- \frac{1}{6} \cdot x_{d-1-j,j} - \frac{1}{6} \cdot (x_{i,d-1-i} + x_{i+1,d-1-(i+1)}) = 0 \quad (37d)$$

for $i = 1$ and $j = 0$

(e). *Region* R[(1,2,3); (4); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4); (1)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (i,j) in this region. Both of the neighboring cells (i-1,j) and (i-1,j+1) are the boundary cells of *LA* 4. MS4 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (i,j). MS1, MS2 and MS3 will remain registered to their previous LAs when they move into cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (i,j) and the average rate of MS4 arrivals to cell (i,j). Because of the system's homogeneity, the average numbers of MS4 in these two neighboring cells are $x_{d-j,j-1}$ and $x_{d-1-j,j}$ respectively. Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell (i,j), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1})$$
$$- \frac{1}{6} \cdot (x_{d-j,j-1)} + x_{d-1-j,j}) = 0 \quad (37e)$$

for $\quad i=1 \quad$ and $\quad j=1,2,\ldots,\frac{w-1}{2}-1$

(f). *Region* R[(1,2,3); (7); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (7); (1)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 7, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (i, j) in this region. The neighboring cells $(i,j\text{-}1)$ and $(i+1,j\text{-}1)$ are both the boundary cells of *LA* 7. MS7 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (i,j). MS1, MS2 and MS3 will remain registered to their previous LAs upon entering cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 7*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j) , $x_{i,j}\cdot(1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (i,j) and the average rate of MS7 arrivals to cell (i,j). Because of the system's homogeneity, the average numbers of MS7 in the two neighboring cells $(i,j\text{-}1)$ and $(i+1,j\text{-}1)$ are $x_{i,d-1-i}$ and $x_{i+1,d-1-(i+1)}$ respectively. Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive at cell (i,j), we have

$$x_{i,j}-\frac{1}{6}\cdot(x_{i,j-1}+x_{i,j+1}+x_{i-1,j}+x_{i+1,j}+x_{i-1,j+1}+x_{i+1,j-1}$$
$$-\frac{1}{6}\cdot(x_{i,d-1-i}+x_{i+1,d-1-(i+1)})=0 \qquad (37f)$$

for $\quad i=2,3\ldots,\frac{w-1}{2} \quad$ and $\quad j=0$

(g). *Region* R[(1,2,3); (4); (1,3)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4); (1,3)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) has equal minimum distance to the center cells of *LA* 1 and *LA* 3. In fact, this region contains only one cell which is cell (i,j) with $i = 1$ and $j = (w-1)/2$. MS1, MS2 and MS3 will remain registered to their previous LAs upon entering this cell. The two neighboring cells $(i\text{-}1,j)$ and $(i\text{-}1,j+1)$ are the boundary cells of *LA* 4. When MS4 in these two neighboring cells move to cell (i,j), half of them will perform location update and then register to *LA* 1, the other half will perform location update and register to *LA* 3. No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (i,j) and half of the average rate of MS4 arrivals to cell (i,j). The average numbers of MS4 in the two neighboring cells $(i\text{-}1,j)$ and $(i\text{-}1,j+1)$ are $x_{d-j,j-1}$ and $x_{d-1-j,j}$ respectively. Since (on average) only 1/6 of the departures from each of its six neighboring cells arrive at cell (i,j), we have

$$\begin{aligned} & x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\ & - \frac{1}{2} \cdot \frac{1}{6} \cdot (x_{d-j,j-1} + x_{d-1-j,j}) = 0 \\ & \text{for} \quad i = 1 \quad \text{and} \quad j = \frac{w-1}{2} \end{aligned} \tag{37g}$$

(h). *Region* R[(1,2,3); (7); (1,2)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (7); (1,2)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 7, and to no other LA boundaries; and, 3) has equal minimum distance to the center cells of *LA* 1 and *LA* 2. Actually, this region contains only one cell which is cell (i,j) with $i = (w+1)/2$ and $j = 0$. MS1, MS2 and MS3 will remain registered to their previous LAs upon entering this cell. The two neighboring cells $(i,j\text{-}1)$ and $(i+1,j\text{-}1)$ are the boundary cells of *LA* 7. When MS7 in these two neighboring cells move to cell (i,j), half of them will perform location updates and register to *LA* 1, the other half will perform location update and register to *LA* 2. No mobile users that are registered to other LAs (*excluding LA 1, LA 2,LA 3 and LA 7*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j) , $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (i,j) and half of the average rate of MS7 arrivals to cell (i,j). The average numbers of MS7 in the two neighboring cells $(i,j\text{-}1)$ and $(i+1,j\text{-}1)$ are $x_{i,d-1-i}$ and $x_{i+1,d-1-(i+1)}$ respectively. Since (on average) only 1/6 of the departures from each of its neighboring cells arrive in cell (i,j), we have

$$
\begin{aligned}
& x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\
& - \frac{1}{2} \cdot \frac{1}{6} \cdot (x_{i,d-1-i} + x_{i+1,d-1-(i+1)}) = 0
\end{aligned}
\tag{37h}
$$

for $\quad i = \dfrac{w+1}{2} \qquad$ and $\qquad j = 0$

As before, the boundary conditions for the above equations are:

$$x_{0,0} = \overline{K} \tag{37i}$$

-- Cell (0,0) belongs ONLY to *LA* 1, the average number of MS1 in this cell is $\overline{K}$.

$$x_{0,j} = x_{j,0} \qquad j = d\text{-}w,\ d\text{-}w\text{+}1, \ldots, d\text{-}1 \tag{37i}$$

-- The average number of MS1 in a cell along *j*-axis equals the average number of MS1 in a corresponding cell along *i*-axis.

$$x_{i,-1} = x_{1,i-1} \qquad i = d\text{-}w,\ d\text{-}w\text{+}1, \ldots, d\text{-}1 \tag{37j}$$

-- The average number of MS1 in a cell along the line j=-1 equals the average number of MS1 in a corresponding cell along the line i=1.

$$x_{d,-1} = 0 \tag{37k}$$

-- This cell does not belong to *LA* 1, the average number of MS1 in this cell is zero.

$$x_{i,d-i} = 0 \qquad i = 1, 2, 3, \ldots, d \tag{37l}$$

-- These cells do not belong to LA1, the average number of MS1 in each of these cells is zero.

Case VI: $w = d - 1$, w is even (Refer to Fig. 14a. Fig. 14b is used to help with visualizations)

The discussion refers to Fig. 14a in which d=7 and w=6, however because it is difficult to discern LA boundaries in this figure, we also provide Fig. 14b which shows approximate LA boundaries as smooth closed curves.

We consider a given sector (that lies between $i > 0$ and $j \geq 0$ in the coordinate system in Fig.14a.) of *LA* 1. All of this sector is overlapped by *LA* 2 and *LA* 3. We divide this sector into the following six disjoint regions. They are shown in different shaded patterns in Fig. 14a. Comparing this case with case V in appendix, the only difference between these two cases is that there are no regions R[(1,2,3); (4); (1,3)] and R[(1,2,3); (7); (1,3)] in this case. Similarly, we can get a set of simultaneous equations for the six regions.

(a). *Region* R[(1,2,3); (null); (*)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (null); (*)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is not contiguous to the boundary of any LA. Consider cell (i,j) in this region. MS1, MS2 and MS3 will remain registered to their previous LAs upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2 and LA 3*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. We can write an equation expressing this as follows.

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \quad (38a)$$

for $\quad i = 2, 3, \ldots, d-2 \quad$ and $\quad j = 1, 2, \ldots, d-i-1$

(b). *Region* R[(1,2,3); (4); (3)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4); (3)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 3 than to the center of any other LA. Consider cell *(i,j)* in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. MS4 will perform location update and register to *LA* 3 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell *(i, j)*, $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell *(i, j)*. The average rate of MS1 arrivals to cell *(i,j)* is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \quad (38b)$$

for $\quad i = 1 \quad$ and $\quad j = \frac{w}{2}+1, \frac{w}{2}+2, \ldots, d-2$

(c). *Region* R[(1,2,3); (7); (2)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (7); (2)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 7, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 2 than to the center of any other LA. Consider cell *(i,j)* in this region. MS1, MS2 and MS3 will remain registered to their previous LAs when they move into this cell. MS7 will perform location update and register to *LA* 2 upon entering this cell. No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 7*) will move into this cell. Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i, j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the average rate of MS1 arrivals to cell (i, j). The average rate of MS1 arrivals to cell (i,j) is 1/6 of the sum of MS1 departures from its six neighboring cells per unit time. Therefore, we have

$$x_{i,j} - \frac{1}{6}(x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) = 0 \qquad (38c)$$

for $i = \frac{w}{2}+1, \frac{w}{2}+2, ..., d-1$ and $j = 0$

(d). *Region* R[(1,2,3); (4,7); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4,7); (1)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundaries of both *LA* 4 and *LA* 7, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Actually, this region includes only one cell which is cell (i,j) with i=1 and j=0. The neighboring cell $(i\text{-}1, j + 1)$ of cell (i,j) is the boundary cell of *LA* 4. MS4 in this neighboring cell will perform location update and register to *LA* 1 upon entering cell (i,j). The two neighboring cells $(i,j\text{-}1)$ and $(i\text{+}1,j\text{-}1)$ of cell (i, j) are the boundary cells of *LA* 7. MS7 in these two neighboring cells will perform location update and register to *LA* 1 when they move to cell (i,j). MS1, MS2 and MS3 will remain registered to their previous LAs upon entering cell (i,j). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3, LA 4 and LA 7*) will move into cell (i,j). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (i,j), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average arrival rates to cell (i,j) of MS1, MS4 and MS7. Because of the homogeneity, the average number of MS4 in the neighboring cell $(i\text{-}1, j + 1)$ is $x_{d-1-j,j}$. The average numbers of MS7 in the two neighboring cells $(i, j\text{-}1)$ and $(i\text{+}1, j\text{-}1)$ are $x_{i,d-1-i}$ and $x_{i+1,d-1-(i+1)}$ respectively. Since (on average) only 1/6 of the departures from each of its six neighboring cells arrive at cell (i,j), then we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) - \frac{1}{6} \cdot x_{d-1-j,j} - \frac{1}{6} \cdot (x_{i,d-1-i} + x_{i+1,d-1-(i+1)}) = 0 \qquad (38d)$$

for $i = 1$ and $j = 0$

(e). *Region* R[(1,2,3); (4); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (4); (1)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 4, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (*i,j*) in this region. Both of the neighboring cells (*i*-1,*j*) and (*i*-1,*j*+1) are the boundary cells of *LA* 4. MS4 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (*i,j*). MS1, MS2 and MS3 will remain registered to their previous LAs when they move into cell (*i,j*). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 4*) will move into cell (*i,j*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i,j*), $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (*i,j*) and the average rate of MS4 arrivals to cell (*i,j*). Because of the system's homogeneity, the average numbers of MS4 in these two neighboring cells are $x_{d-j,j-1}$ and $x_{d-1-j,j}$ respectively. Since (on average) only 1/6 of the departures from each of the sixneighboring cells arrive at cell (*i,j*), we have

$$
\begin{aligned}
&x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) \\
&- \frac{1}{6} \cdot (x_{d-j,j-1)} + x_{d-1-j,j}) = 0
\end{aligned}
\tag{38e}
$$

for $i = 1$ and $j = 1, 2, \ldots, \frac{w}{2} - 1$

(f). *Region* R[(1,2,3); (7); (1)]:

This region consists of those cells (in the sector) which have the indices C[(1,2,3); (7); (1)]. Any cell in this region, 1) belongs to *LA* 1, *LA* 2 and *LA* 3, and to no other LA; and, 2) is contiguous to the boundary of *LA* 7, and to no other LA boundaries; and, 3) is closer to the center cell of *LA* 1 than to the center of any other LA. Consider cell (*i, j*) in this region. The neighboring cells (*i,j*-1) and (*i*+1,*j*-1) are both the boundary cells of *LA* 7. MS7 in these two neighboring cells will perform location update and register to *LA* 1 upon entering cell (*i,j*). MS1, MS2 and MS3 will remain registered to their previous LAs upon entering cell (*i,j*). No mobile users that are registered to other LAs (*excluding LA 1, LA 2, LA 3 and LA 7*) will move into cell (*i,j*). Since the system is in statistical equilibrium and the average number of MS1 in a cell is time independent, the average rate of MS1 departures from cell (*i,j*) , $x_{i,j} \cdot (1/\overline{T}_d)$, equals the sum of the average rate of MS1 arrivals to cell (*i,j*) and the average rate of MS7 arrivals to cell (*i,j*). Because of the system's homogeneity, the average numbers of MS7 in the two neighboring cells (*i,j*-1) and (*i*+1,*j*-1) are $x_{i,d-1-i}$ and $x_{i+1,d-1-(i+1)}$ respectively. Since (on average) only 1/6 of the departures from each of the six neighboring cells arrive in cell (*i,j*), we have

$$x_{i,j} - \frac{1}{6} \cdot (x_{i,j-1} + x_{i,j+1} + x_{i-1,j} + x_{i+1,j} + x_{i-1,j+1} + x_{i+1,j-1}) - \frac{1}{6} \cdot (x_{i,d-1-i} + x_{i+1,d-1-(i+1)}) = 0 \tag{38f}$$

for $\quad i = 2, 3, \ldots, \frac{w}{2} \quad$ and $\quad j = 0$

Similarly, the boundary conditions for the above equations are:

$$x_{0,0} = \overline{K} \tag{38g}$$

-- Cell (0,0) belongs ONLY to *LA* 1, the average number of MS1 in this cell is $\overline{K}$.

$$x_{0,j} = x_{j,0} \qquad j = d\text{-}w,\ d\text{-}w\text{+}1, \ldots, d\text{-}1 \tag{38h}$$

-- The average number of MS1 in a cell along *j*-axis equals the average number of MS1 in a corresponding cell along *i*-axis.

$$x_{i,-1} = x_{1,i-1} \qquad i = d\text{-}w,\ d\text{-}w\text{+}1, \ldots, d\text{-}1 \tag{38i}$$

-- The average number of MS1 in a cell along the line *j*=-1 equals the average number of MS1 in a corresponding cell along the line *i*=1.

$x_{d,-1} = 0$ (38j)

-- This cell does not belong to *LA* 1, the average number of MS1 in this cell is zero.

$x_{i,d-i} = 0 \qquad i = 1, 2, 3, \ldots, d$ (38k)

-- These cells do not belong to LA1, the average number of MS1 in each of these cells is zero.

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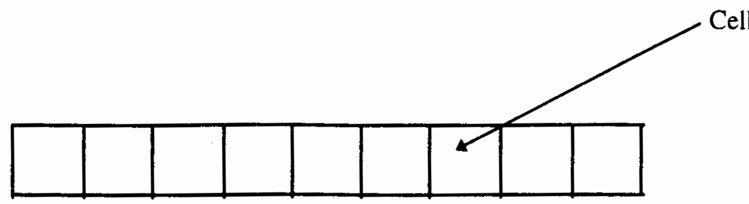
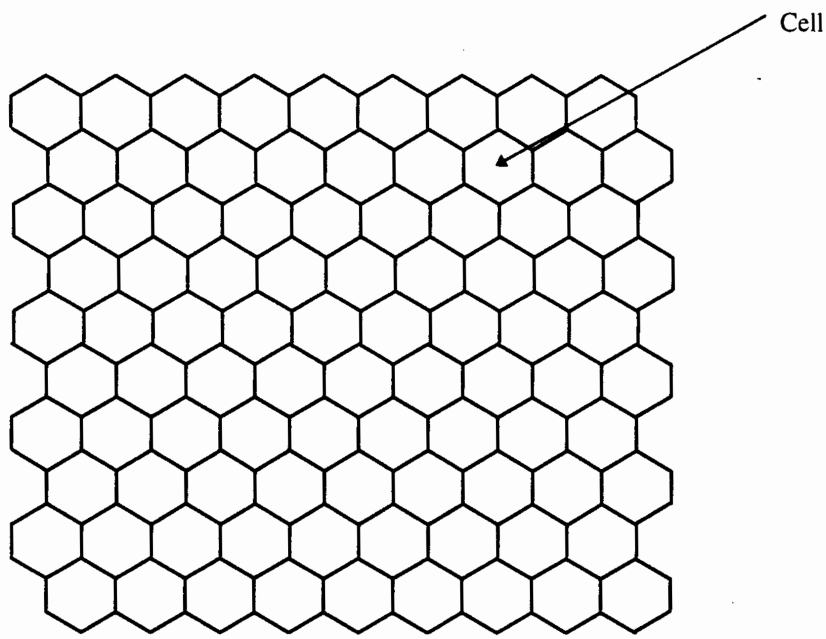
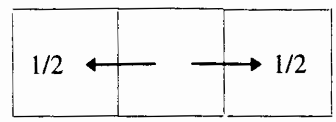


(a)



(b)

Fig. 1. Typical cell layouts: (a) one-dimensional cellular geometry (b) two-dimensional cellular geometry



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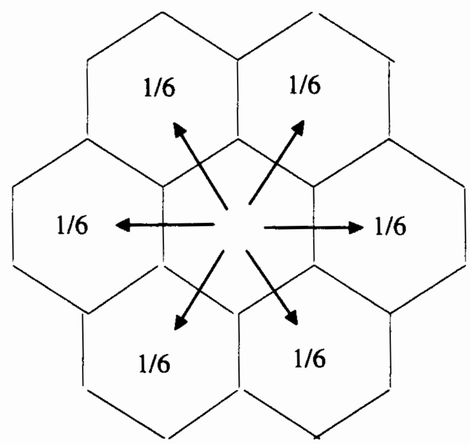
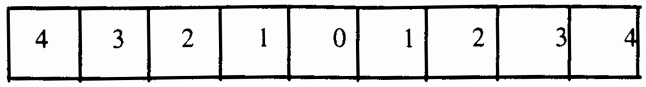
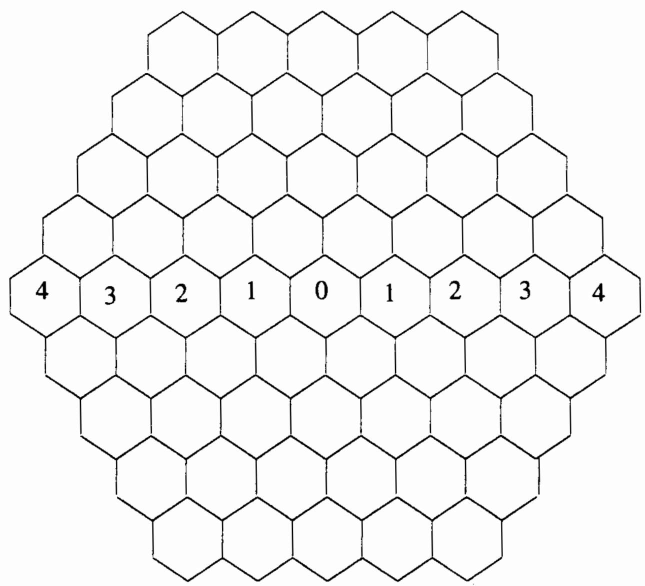


(b)

Fig. 2. Mobility patterns of a mobile user: (a) one-dimensional case (b) two-dimensional hexagonal case



(a)



(b).

Fig. 3. Typical location areas: (a) one-dimensional case (b) two-dimensional hexagonal case

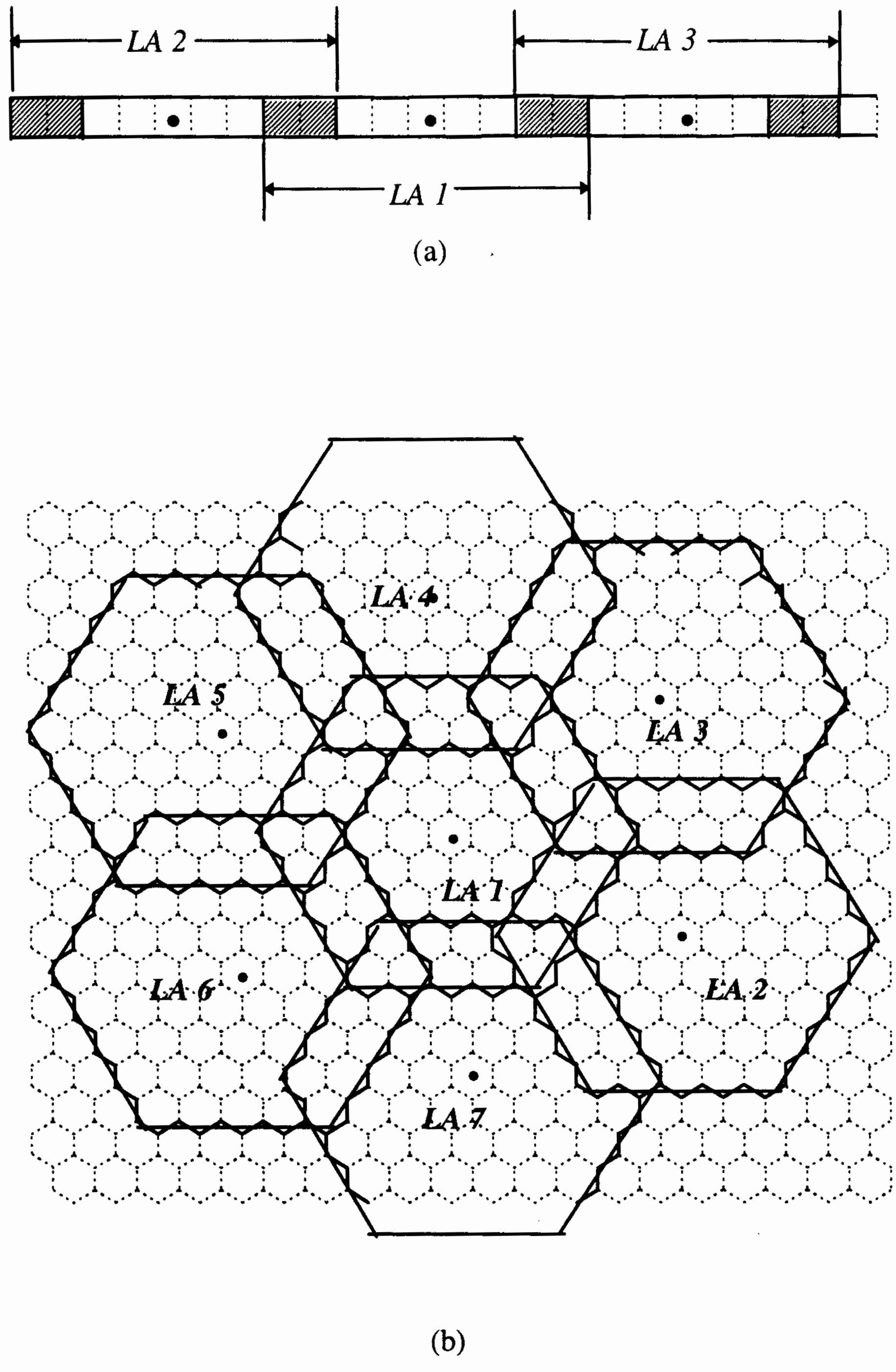


Fig. 4. Overlapping location areas: $d = 5$, $w = 2$. (a) one-dimensional case (b) two-dimensional hexagonal case

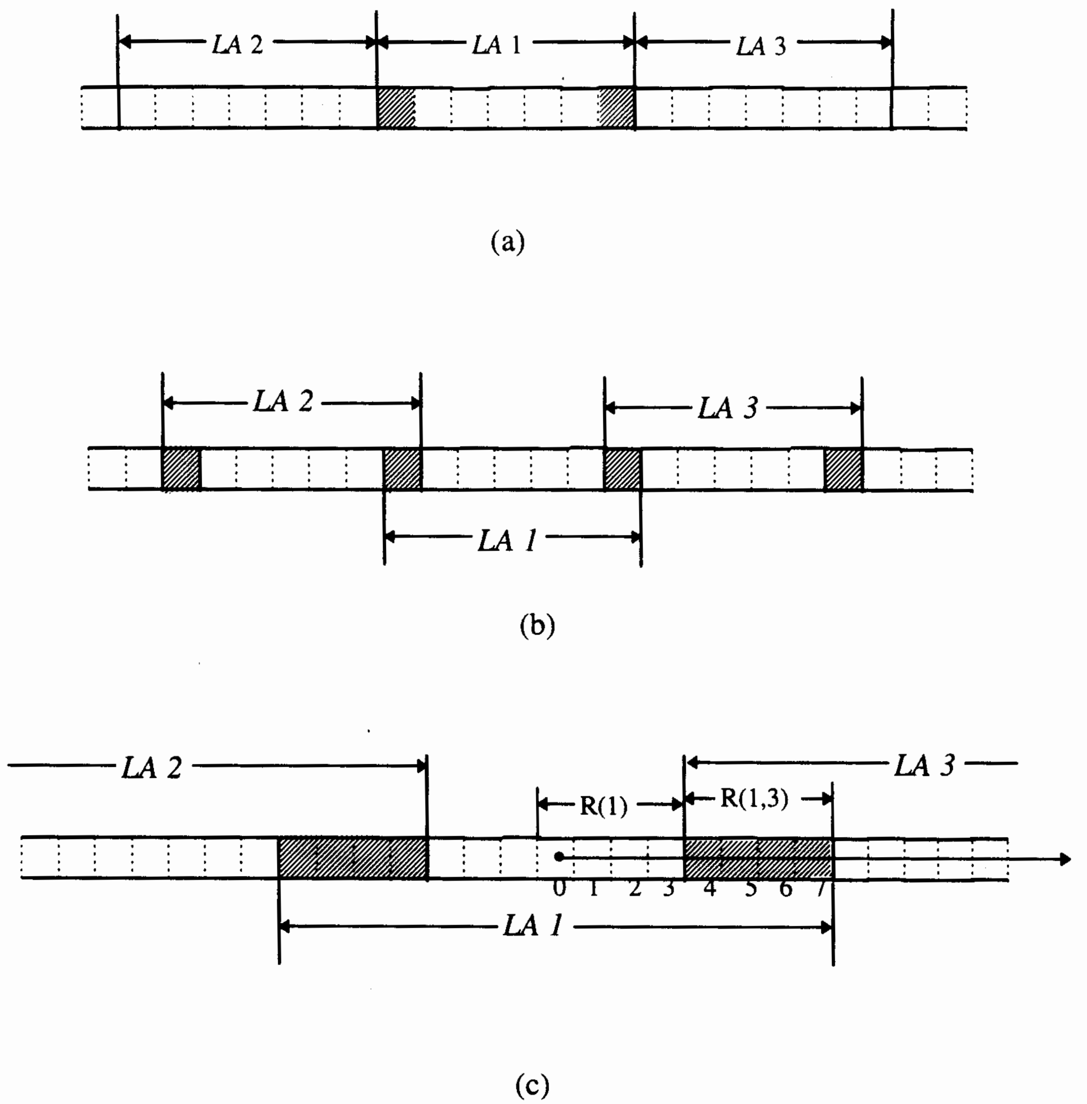


Fig. 5. One-dimensional LA structures: (a) non-overlapping LAs. (b) LAs with one ring overlapped. (c) overlapping LAs with $d = 8$ and $w = 4$.

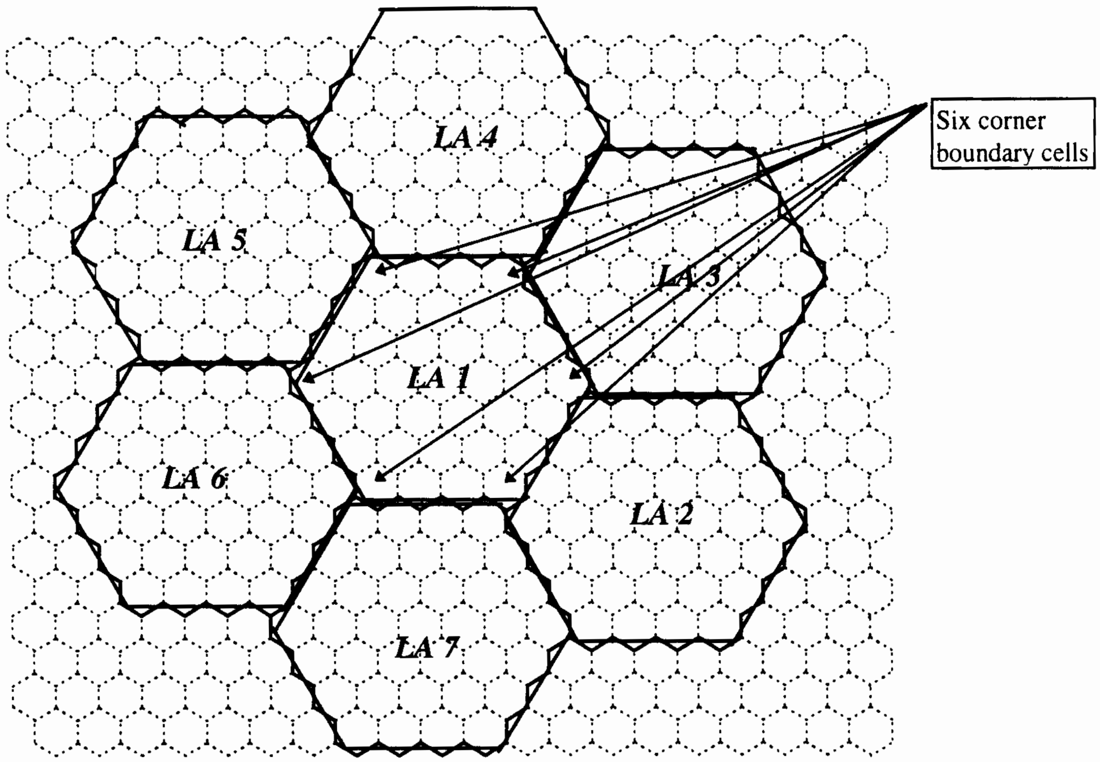


Fig. 6. Non-overlapping two-dimensional hexagonal LAs.
Parameters: d=4, w=0.

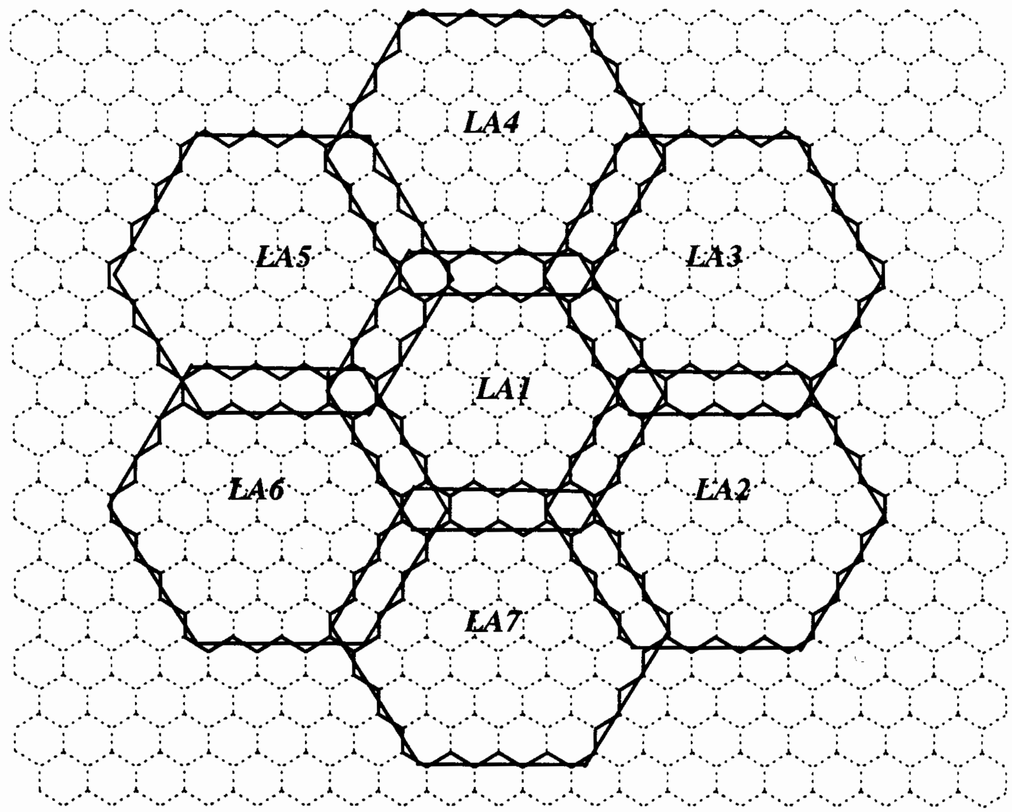


Fig. 7. Two-dimensional hexagonal LAs with one ring overlapped.
Parameters: d=4, w=1.

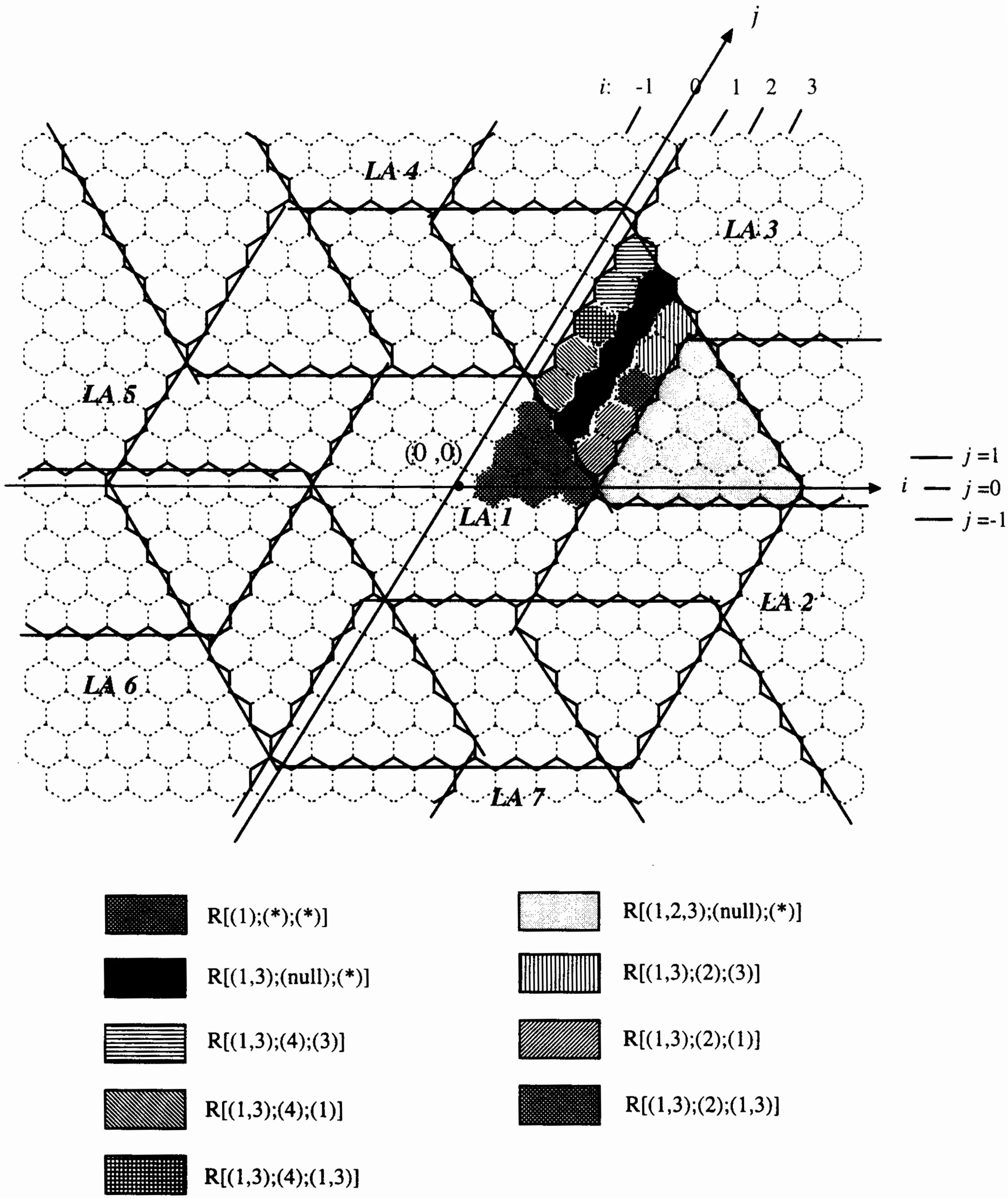


Fig. 8a. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 9$, $w = 5$.

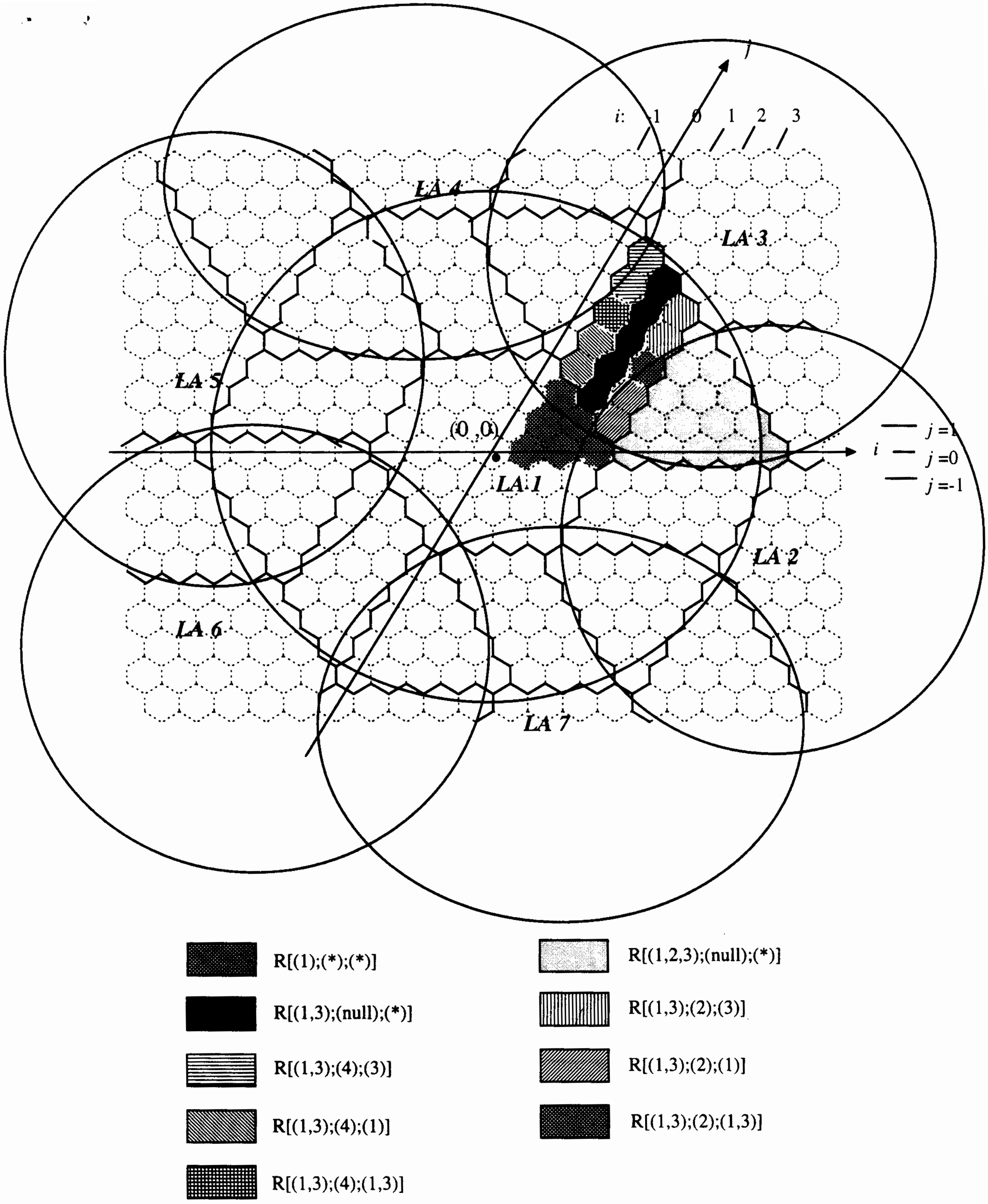


Fig. 8b. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 9$, $w = 5$.

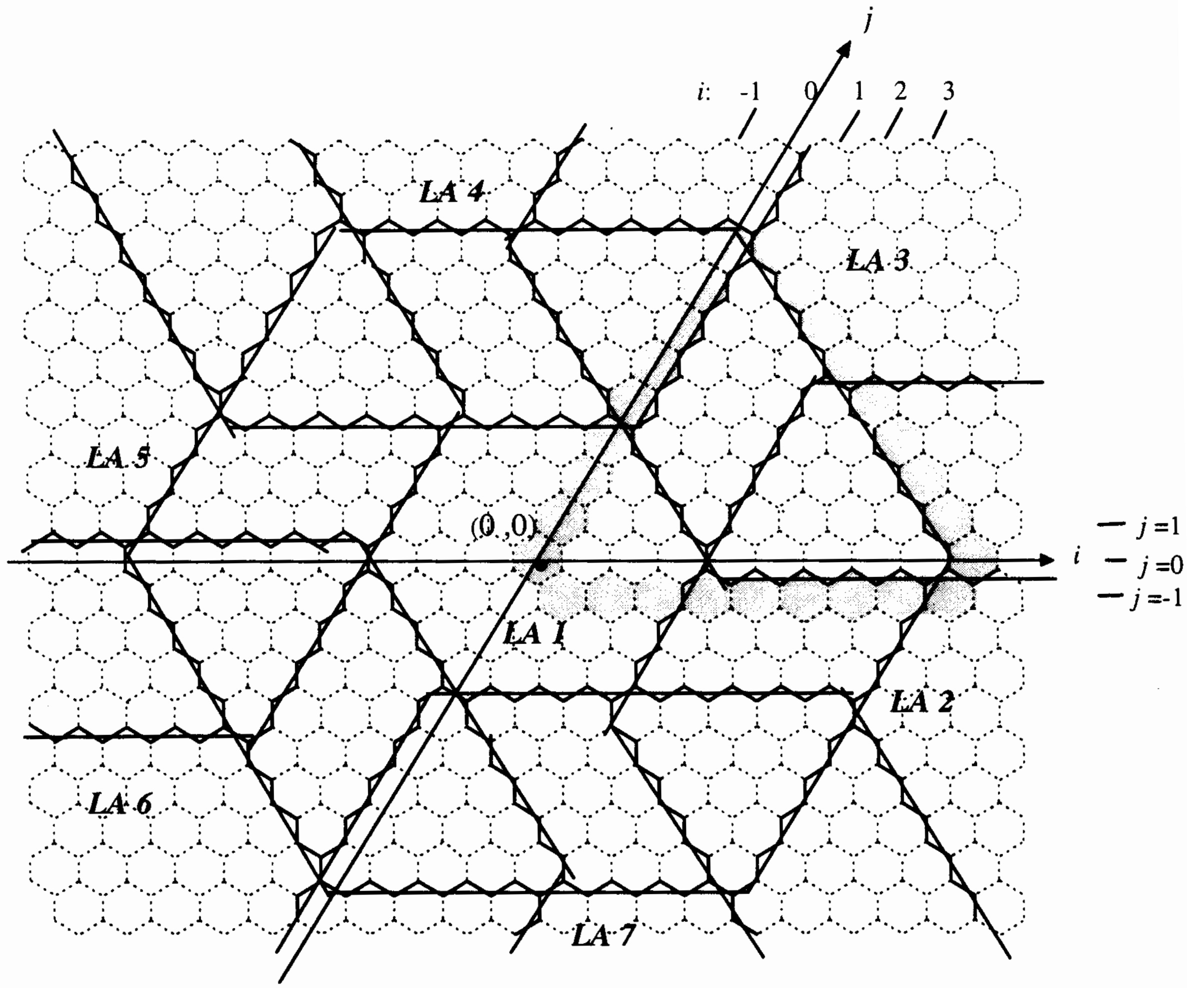


Fig. 9. Cells that are contiguous to the given sector of *LA* 1
Parameters: $d = 9$, $w = 5$.

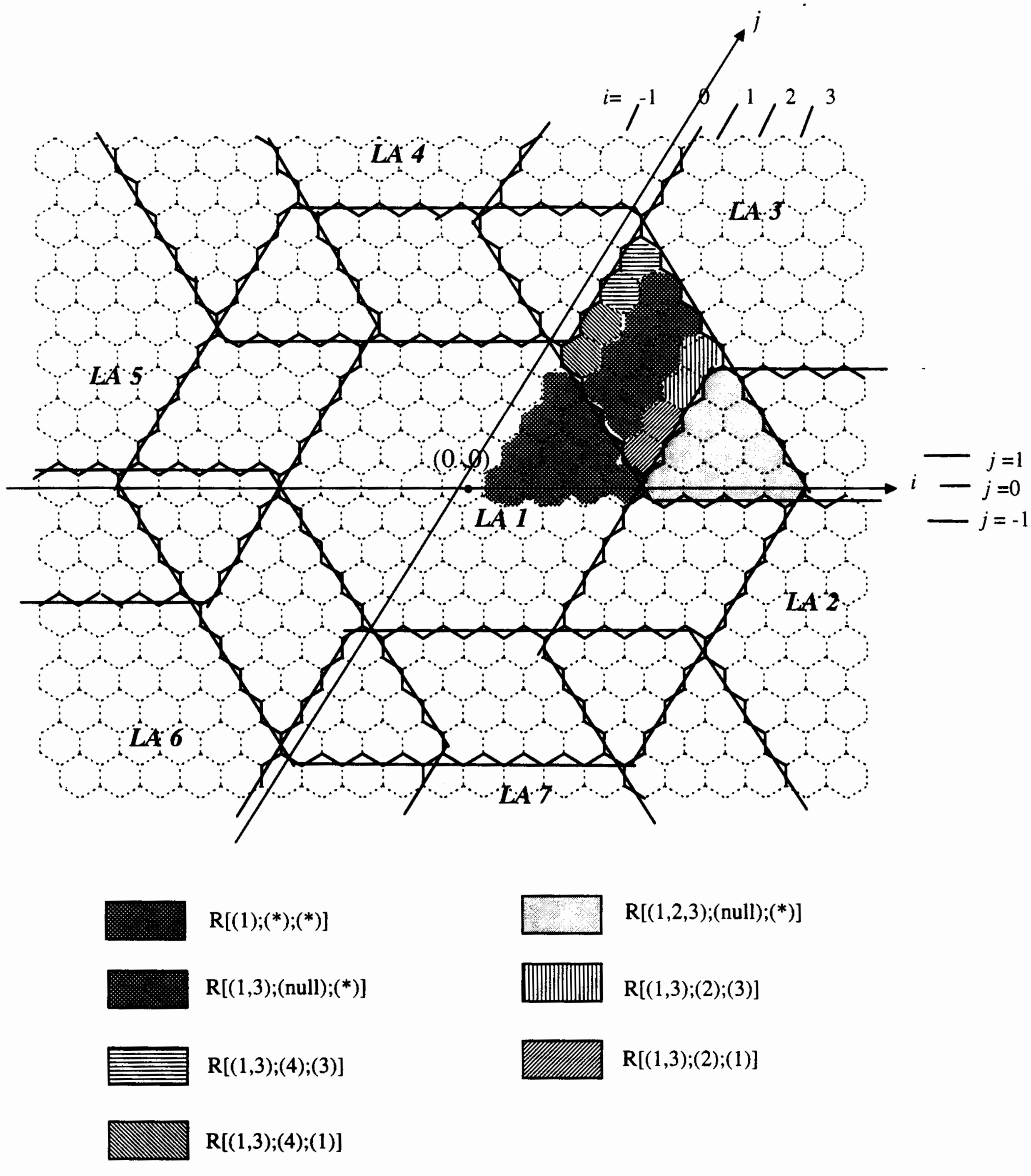


Fig. 10a. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 9$, $w = 4$.

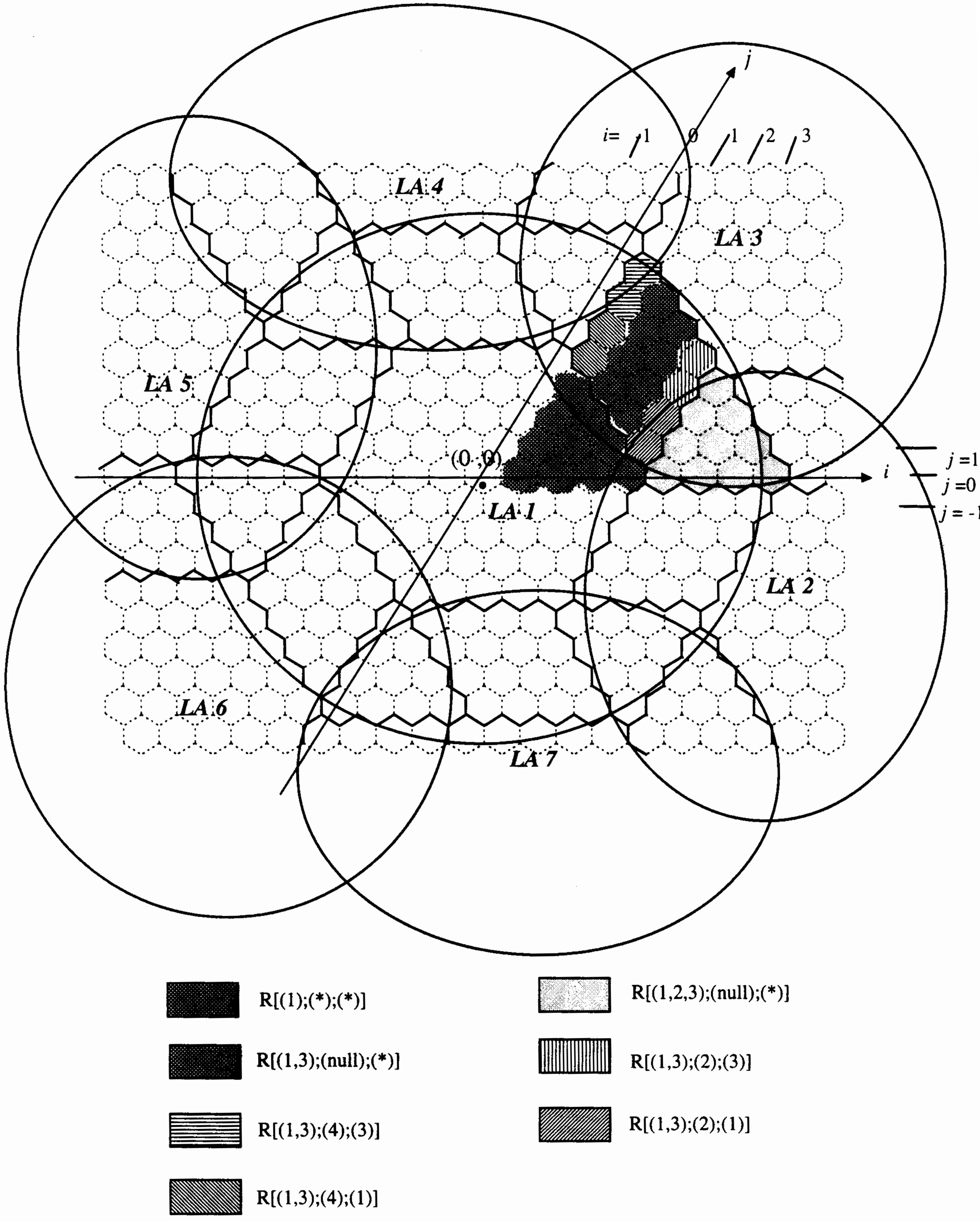


Fig. 10b. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 9$, $w = 4$.

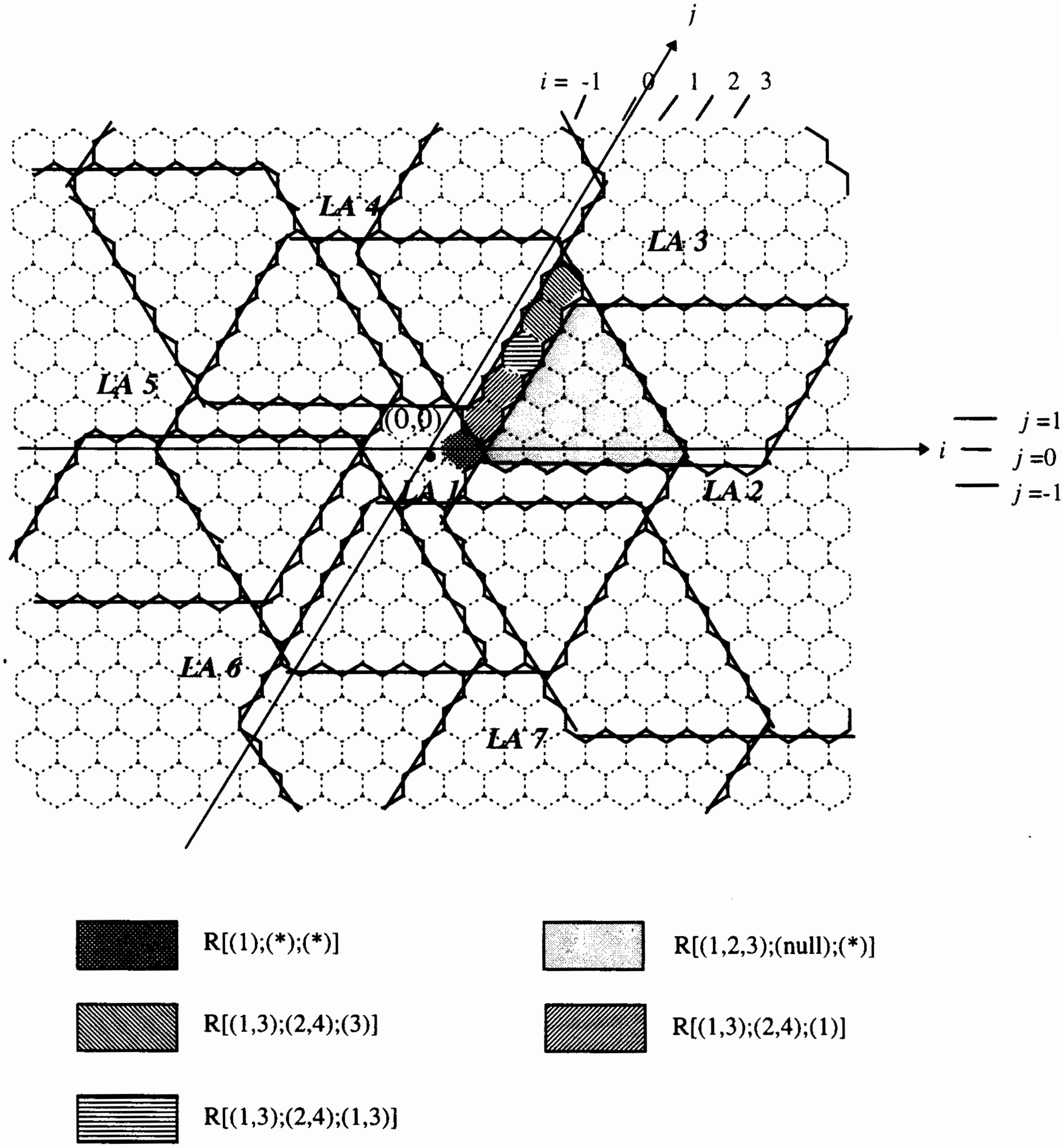


Fig.11a. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 7$, $w = 5$.

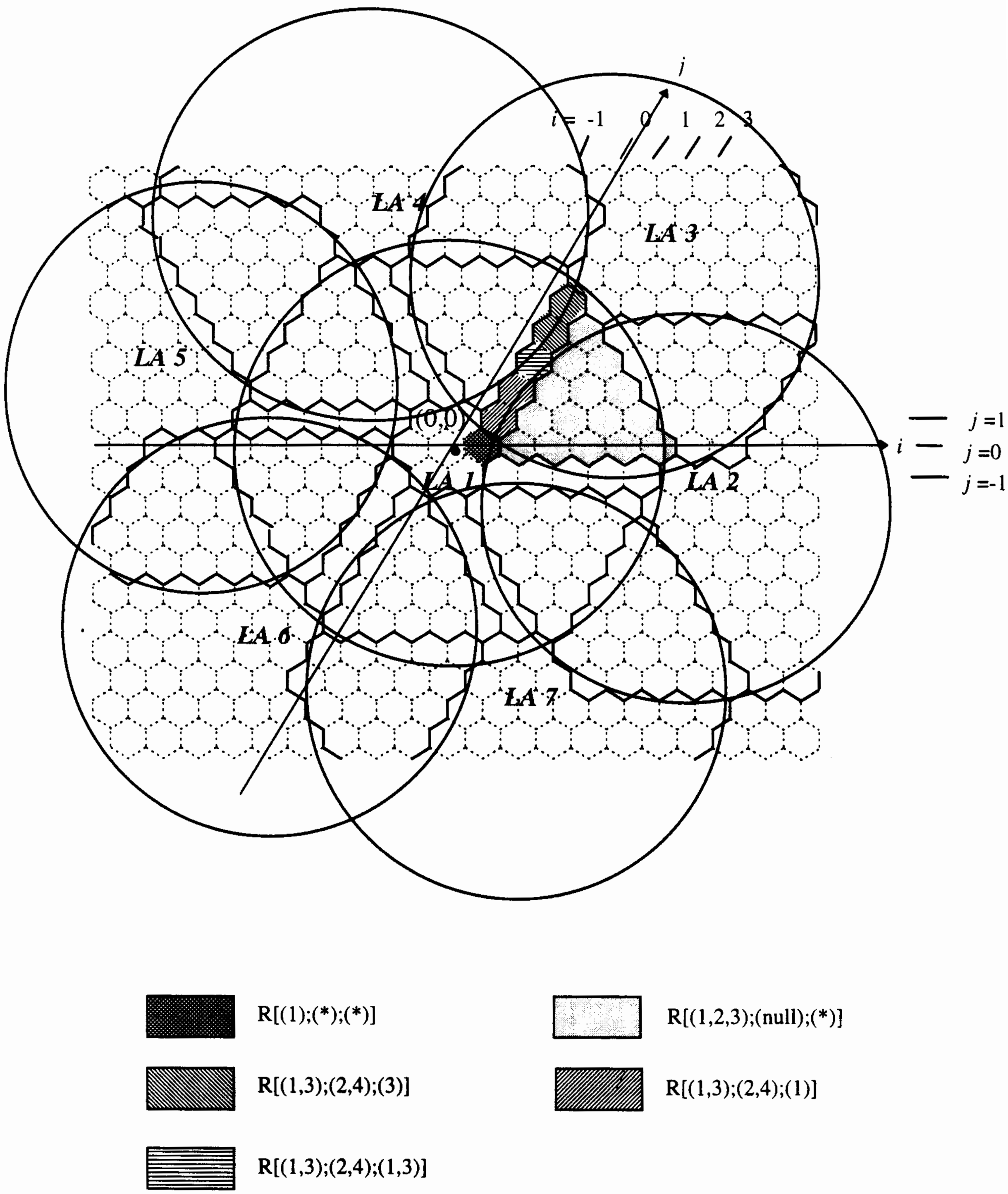


Fig.11b. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 7$, $w = 5$.

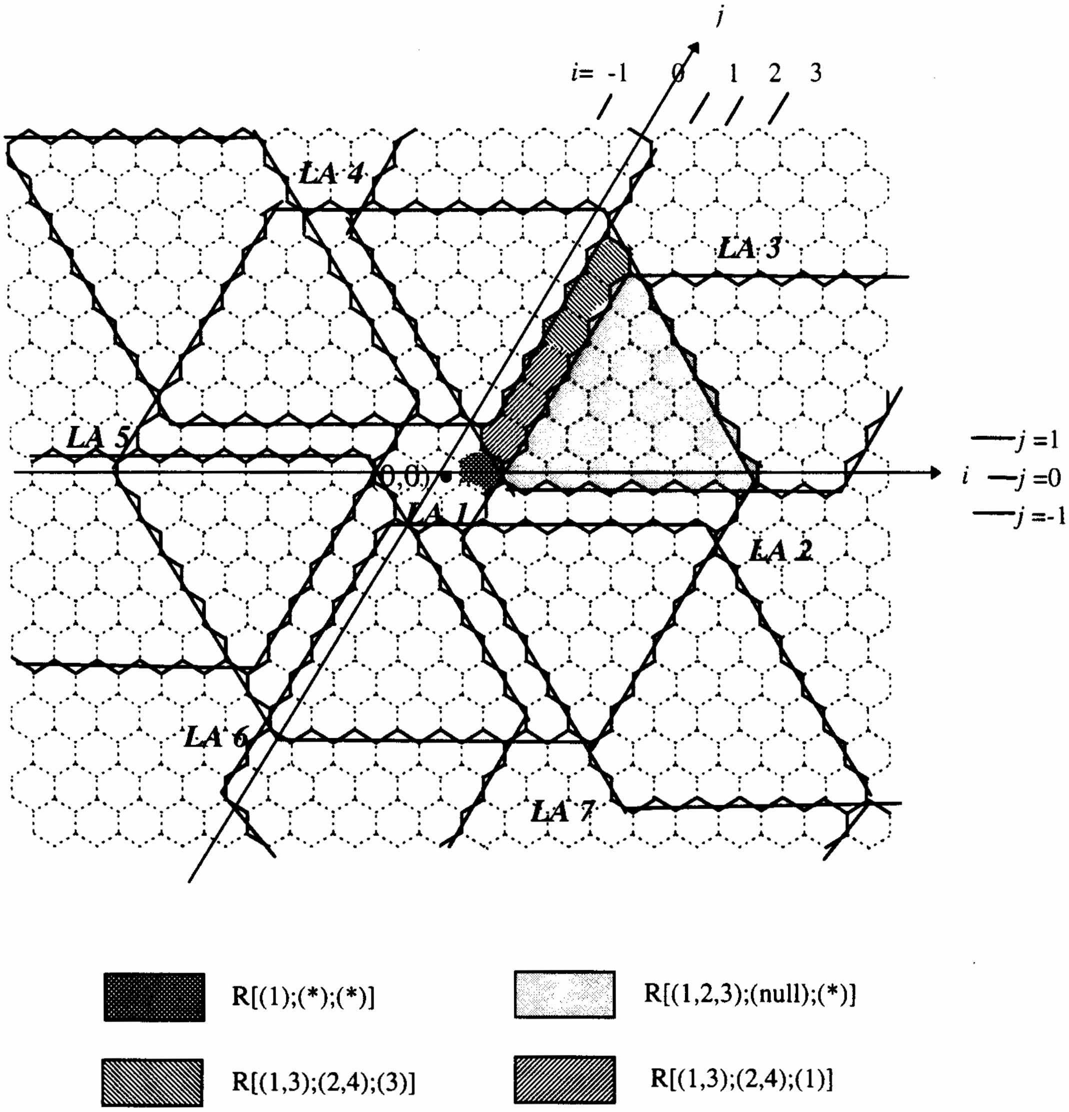


Fig. 12a. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 8$, $w = 6$.

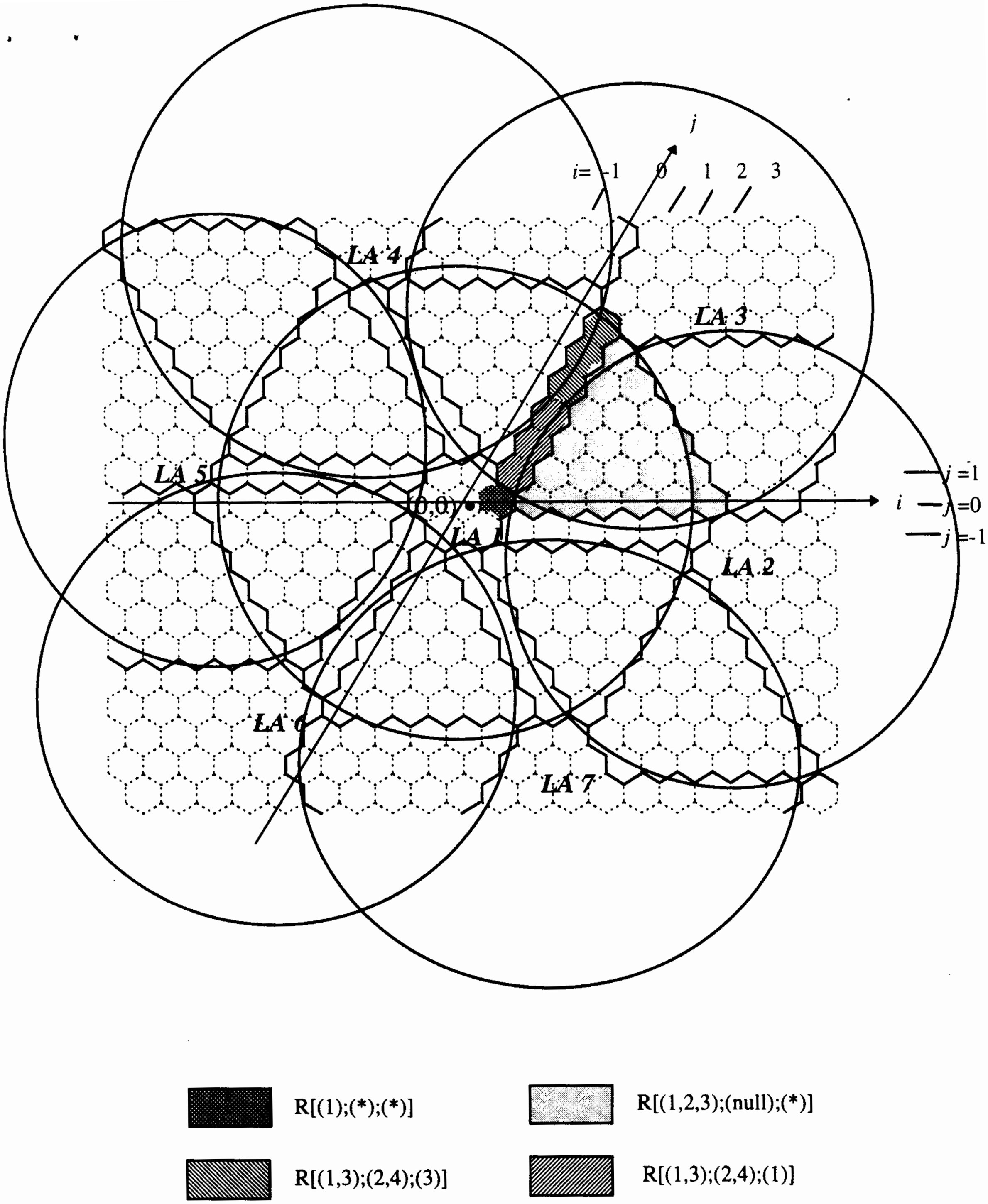


Fig. 12b. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 8$, $w = 6$.

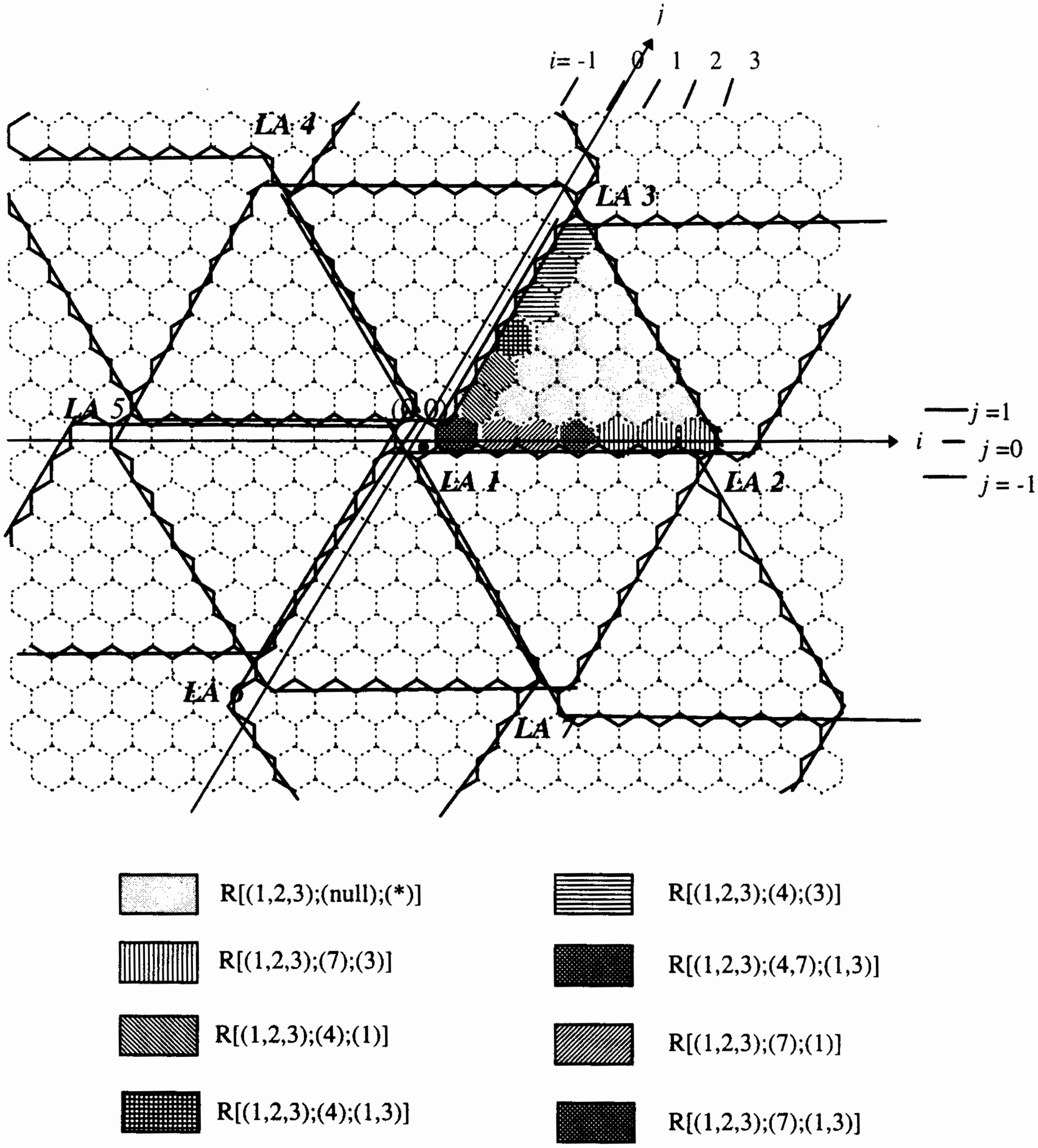


Fig. 13a. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 8$, $w = 7$.

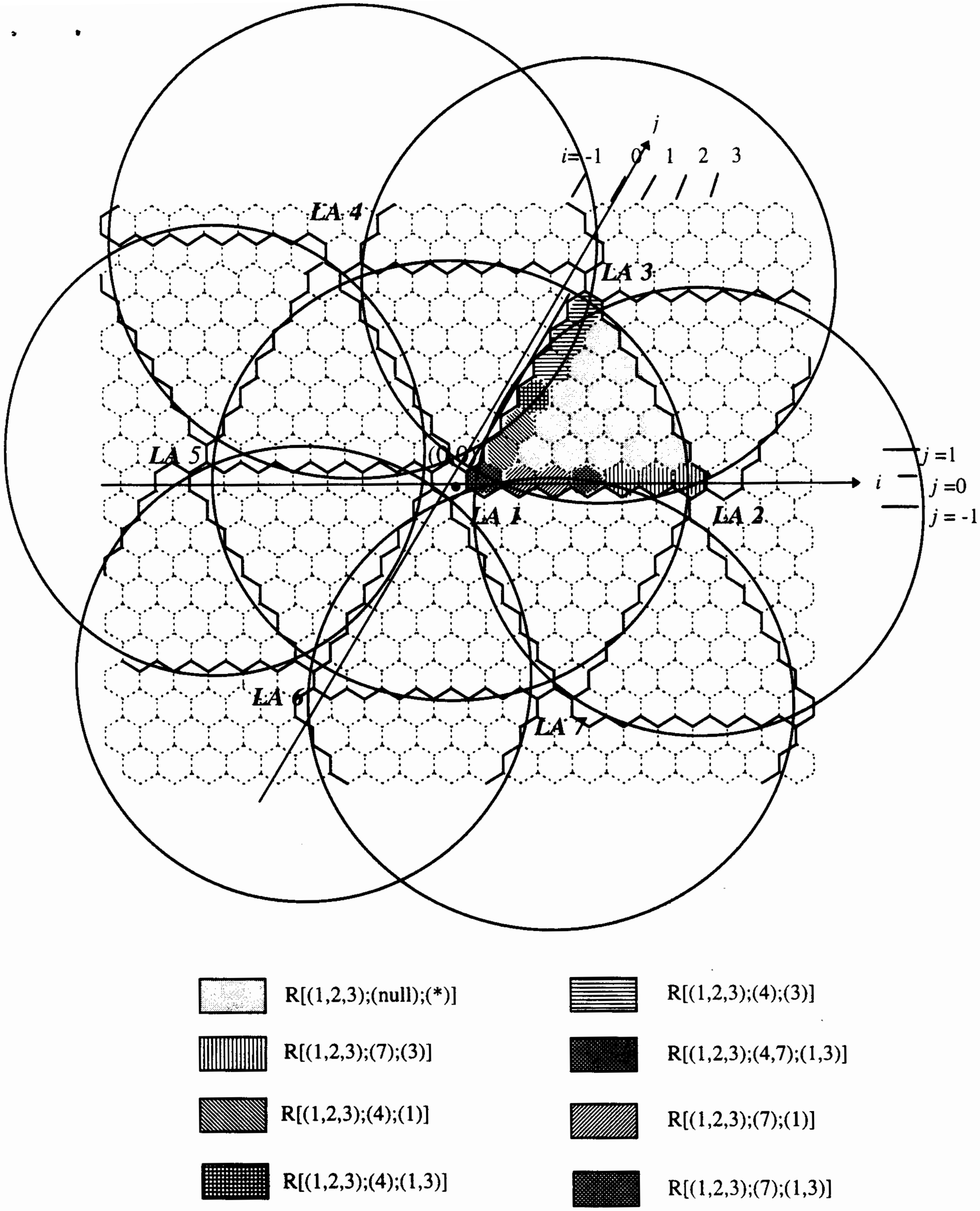


Fig. 13b. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 8$, $w = 7$.

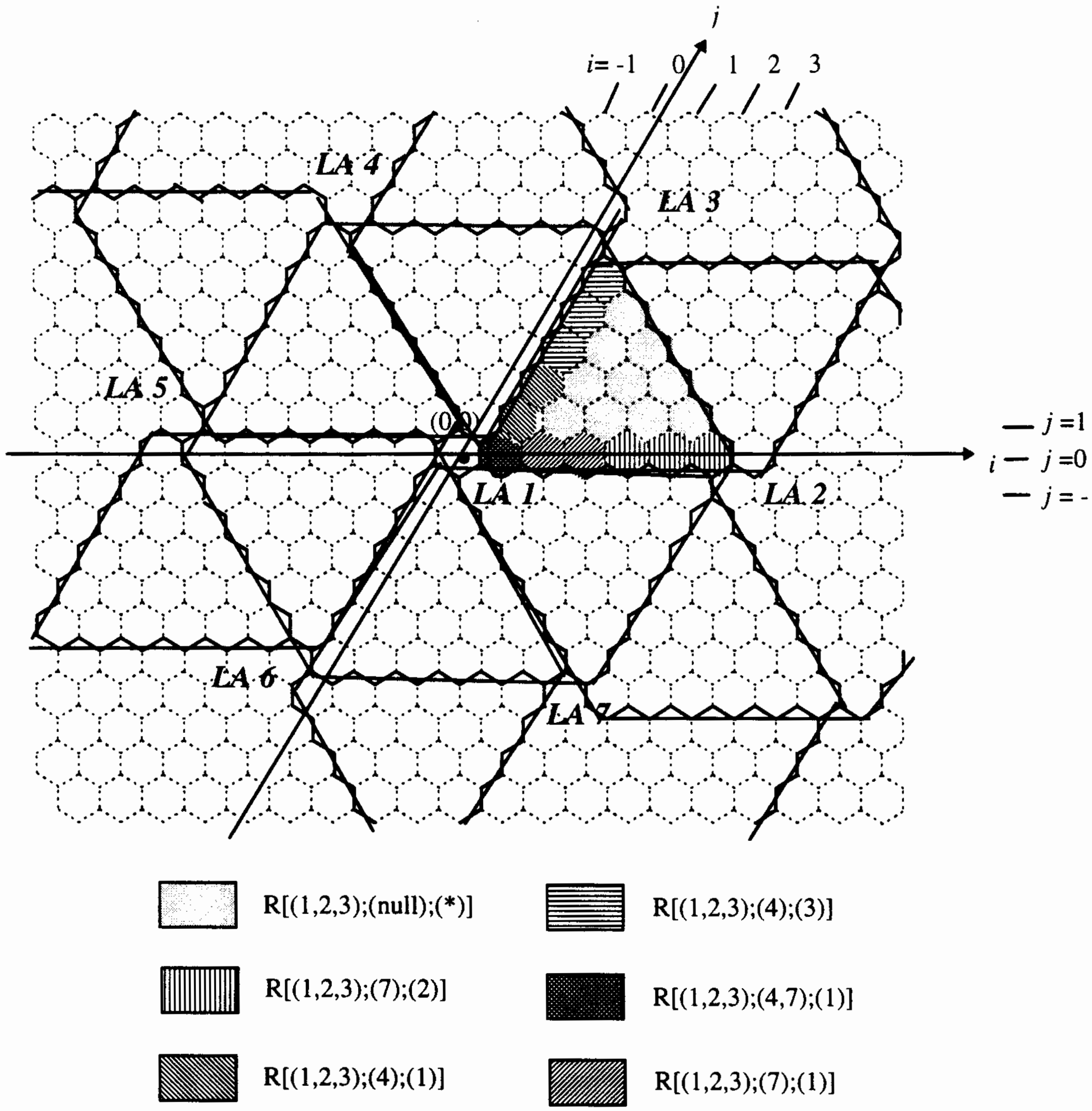


Fig. 14a. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 7$, $w = 6$.

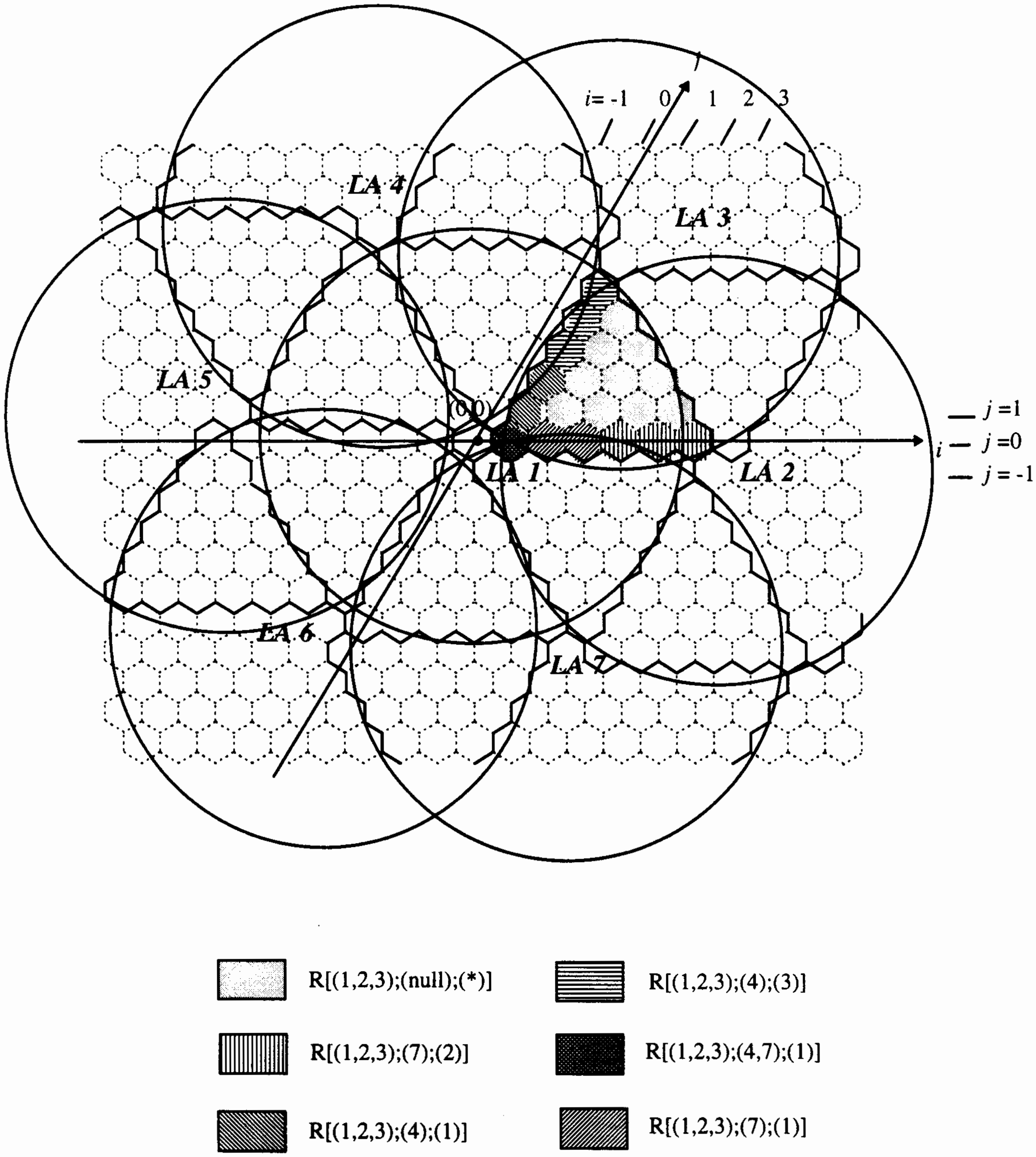


Fig. 14b. General overlapping two-dimensional hexagonal LAs.
Parameters: $d = 7$, $w = 6$.

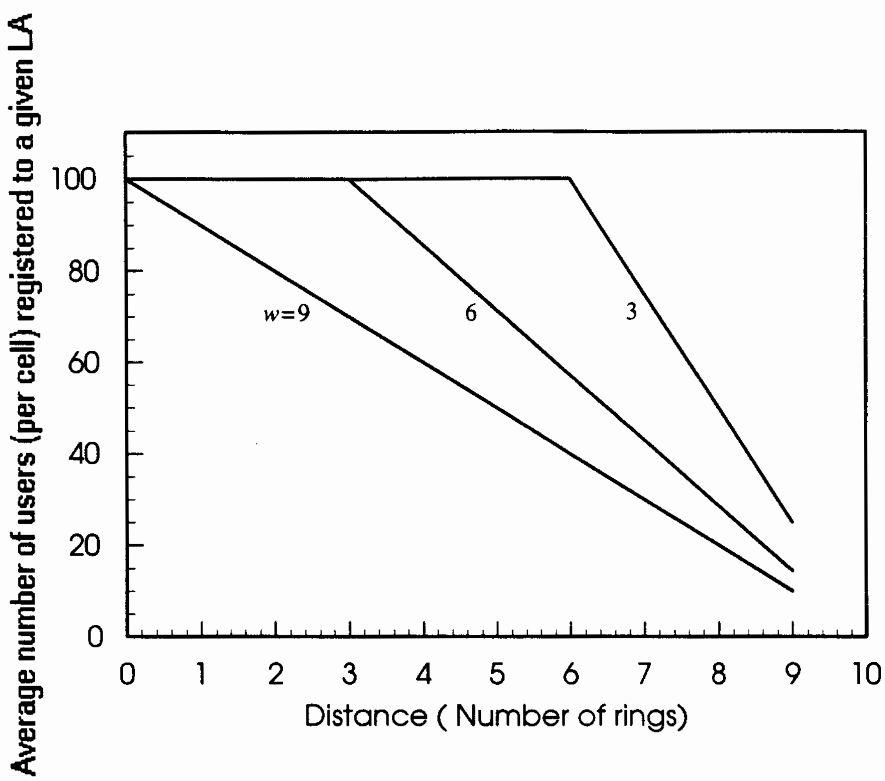


Fig. 15. Distribution of mobile users registered to a given LA for one-dimensional case.
Parameters: $\overline{K} = 100$, $d = 10$.

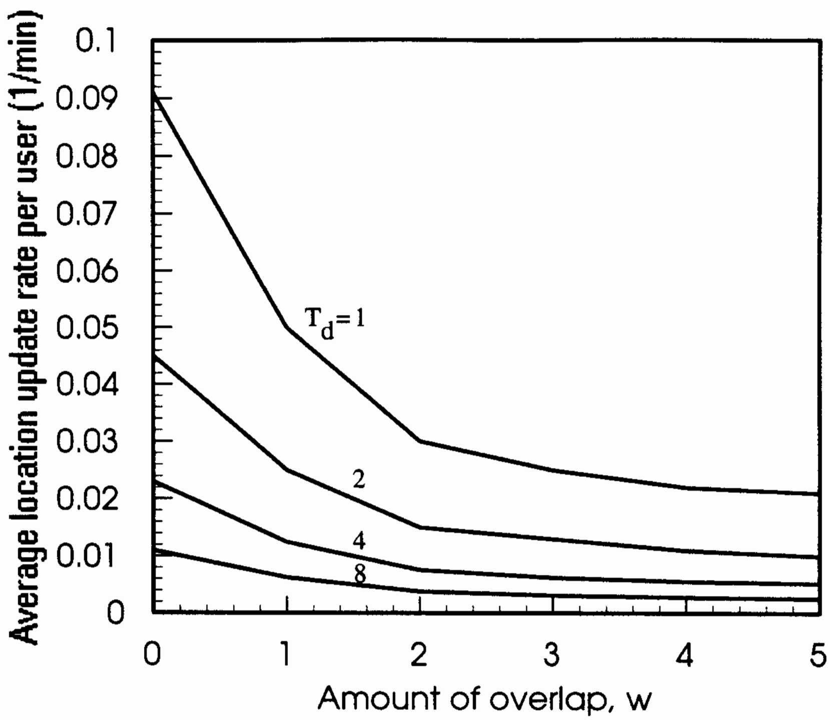


Fig. 16. Average location update rate for one-dimensional case.
Parameters: $\overline{K} = 100$, $d = 6$.

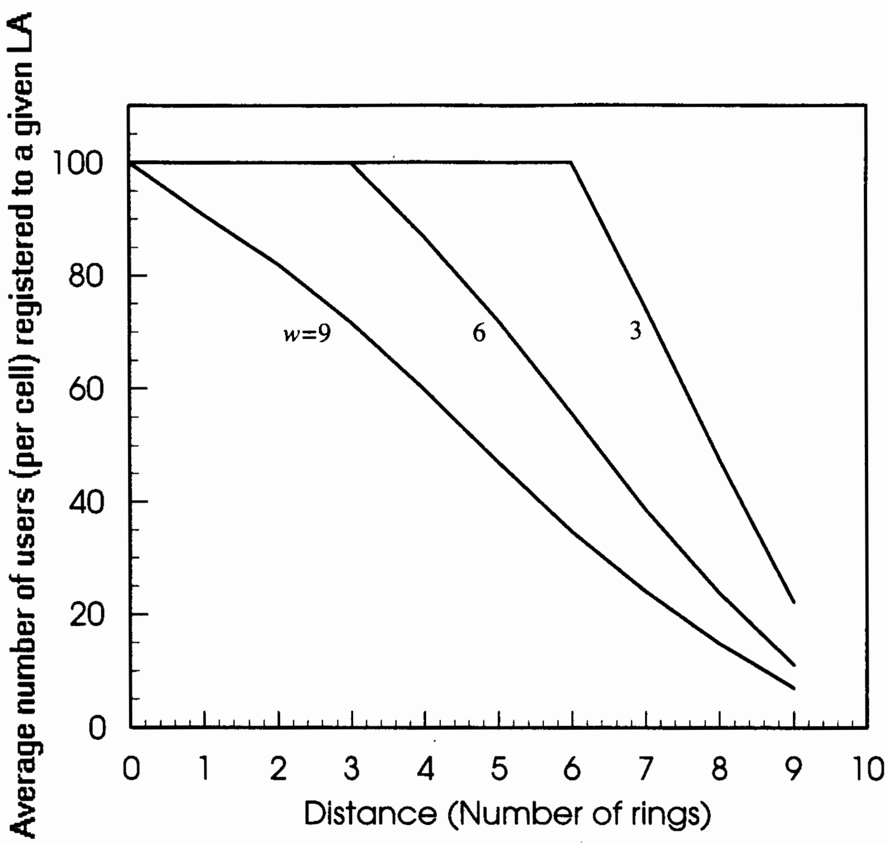


Fig.17. Distribution of mobile user registered to a given LA for two-dimensional hexagonal case.
Parameters: $\overline{K} = 100$, d=10.

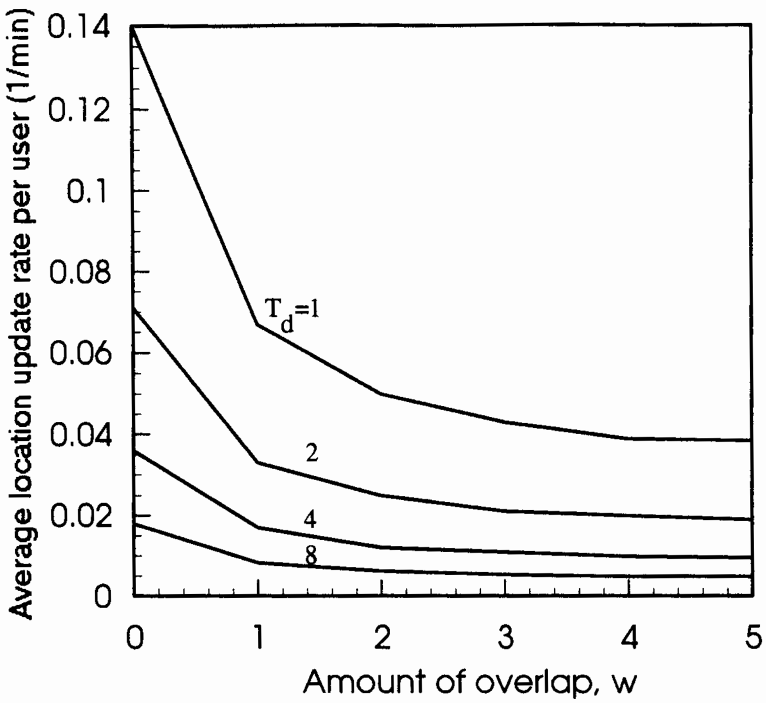


Fig. 18. Average location update rate per user for two-dimensional hexagonal case.
Parameters: $\overline{K} = 100,\ d = 6$.