State University of New York at Stony Brook
College of Engineering and Applied Sciences

Technical Report No. 773

Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Nonprioritized Handoff Procedures - Version 2a

by

Daehyoung Hong and Stephen S. Rappaport

Department of Electrical and Computer Engineering
State University of New York
Stony Brook, New York 11794-2350

e-mail: dhong@ccs.sogang.ac.kr, rappaport@sunysb.edu

Date: June 1, 1999

Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Nonprioritized Handoff Procedures - Version 2

Daehyoung Hong and Stephen S. Rappaport

PREFACE

This manuscript is an updated version of the frequently referenced VT-86 paper† by Hong and Rappaport, which is among the earliest published analytical treatments of teletraffic modeling for cellular systems. Much work has been done in this field since 1986, but it is not the intention of this manuscript to bring the reader up to date on the subject. Rather, the intention is to make available in one place a "clean" version of the paper which eliminates grammatical and typographical errors that appeared in the original publication and also to correct equation (34) of that paper which contains a mathematical error. The error in equation (34) potentially impacts the computation of several of the system performance curves that were presented. We have recalculated the performance curves using the corrected formula and have compared these to the original curves that were presented. In spite of the error, the original curves are only marginally different from the revised curves in the range of parameters that were presented. Nevertheless, to clarify the record, this manuscript contains a full set of *revised* figures, which has been calculated on the basis of the corrected formula. In preparing this revision, to the extent possible we have adhered to the wording and numbering of the original paper, except that we have also taken the opportunity to improve grammatical constructions in several places.

This revised manuscript will be listed and maintained as a technical report of the College of Engineering and Applied Sciences of the State University of New York and will be available upon request. In addition, an electronic version in Adobe's pdf format will be posted on the Web at the following URL's.

URL1: http://www.ece.sunysb.edu/~rappap/papers/start_papers.htm

URL2: http://eecom3.sogang.ac.kr/professor/home_eng.html

Researchers and others who wish to refer to this work should use the following citation:

> D. Hong and S.S. Rappaport, "Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Nonprioritized Handoff Procedures," IEEE Trans. on Vehicular Technology, August 1986, vol. VT-35, no. 3, pp. 77-92. See also: CEAS Technical Report No. 773, June 1, 1999, College of Engineering and Applied Sciences, State University of New York, Stony Brook, NY 11794, USA.

The authors wish to thank Joan Borras and Roy Yates for calling the error in equation (34) to their attention.

Daehyoung Hong and Stephen S. Rappaport
June 1999

---

† D. Hong and S.S. Rappaport, "Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Non-Prioritized Handoff Procedures," IEEE Trans. on Vehicular Technology, August 1986, vol. VT- 35, no. 3, pp. 77-92.

Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Nonprioritized Handoff Procedures – Version 2a

Daehyoung Hong and Stephen S. Rappaport

***Abstract*-- A traffic model and analysis for cellular mobile telephone systems with handoff are presented. Three mobile schemes for call traffic handling are considered. One is nonprioritized and two are priority oriented. Fixed channel assignment is considered. In the nonprioritized scheme the base stations make no distinction between new call attempts and handoff attempts. Attempts that find all channels occupied are cleared. In the first priority scheme considered, a fixed number of channels in each cell are reserved exclusively for handoff calls. The second priority scheme employs a similar channel assignment strategy, but, additionally, the queuing of handoff attempts is allowed. Appropriate analytical models and criteria are developed and used to derive performance characteristics. These show, for example, blocking probability, forced termination probability, and fraction of new calls not completed, as functions of pertinent system parameters. General formulas are given and specific numerical results for nominal system parameters are presented.**

I. Introduction

The performance of cellular mobile radio telephone systems in which cell size is relatively small and the handoff procedure has an important effect is investigated in this paper. Spectrally efficient mobile radio service for a large number of customers can be provided by cellular systems [1],[2]. The service area is divided into cells. Users communicate via radio links to base stations in the cells. Channel frequencies are reused in cells that are sufficiently separated in distance so that mutual interference is beneath tolerable levels.

Channel frequencies for mobile radio systems are allocated to base stations to be used in each cell by various channel assignment schemes. In fixed channel assignment (FCA) a group of channels is assigned to each base station. Different groups of channels are assigned to each cell according to definite rules. In dynamic channel assignment (DCA) no fixed relationship exists between the channel frequencies and the cells. Any channel can be used in any cell if no interference constraints are violated. In hybrid channel assignment (HCA) some channels are fixed assigned to cells, and others are assigned dynamically [1], [3]-[11]. Throughout this paper we assume that FCA is used.

When a new call is originated and attempted in a cell, one of the channels assigned to the base station of the cell is used for communication between the mobile user and the base station (if any channel is available for the call). If all the channels assigned to the base station are in use, the call attempt is assumed to be blocked and cleared from the system (blocked calls cleared (BCC)). When a new call gets a channel, it keeps the channel until the call is completed in the cell or the mobile moves out of the cell. When the call is completed in the cell, the channel is released and becomes available to serve another call. When the mobile crosses a cell boundary into an adjacent cell while the call is in progress, the call requires a new base station and channel frequency to continue. The procedure of changing channels is called “handoff”. If no channel is available in the new cell into which the mobile moves, the handoff call is forced to terminate before completion. Simulation studies of handoff schemes have appeared in literature [4]. For convenience in subsequent discussion we define the cell into which the mobile is moving and desires a handoff as the *target* cell for the handoff. Furthermore, we call the cell which the mobile is leaving, the *source* cell of the handoff attempt.

The required co-channel interference constraint is expressed as the ratio of the distance D between the centers of nearest neighboring cells that simultaneously can use the same channel to the cell radius R. This ratio, sometimes called the co-channel reuse ratio, is related to the number of cells per cluster N_c by $N_c = (D/R)^2/3$ [1]. When N_c is chosen from co-channel interference considerations, the capacity (e.g., erlangs carried per unit area at given performance level) of the mobile radio system depends on the cell radius. The cell radius R should be small for a high capacity system since this allows more frequency reuse in a given service area. On the other hand, in small cell systems, there are increased numbers of cell boundary crossings by mobiles. The average call duration (or channel holding time) in a cell $\overline{T}_H$ becomes less than the average unencumbered message duration $\overline{T}_M$. Also the distribution of channel holding time is different from that of message duration. In addition to the new call attempts, handoff call attempts are generated. The handoff attempt rate depends on cell radius as well as other system parameters. An important effect that should be considered is that some fraction of handoff attempts will be unsuccessful. Some calls will be forced to terminate before message completion.

In this paper we develop analytical models to investigate these effects and to examine the relationships between performance characteristics and system parameters. We wish to state at the outset that the analysis is approximate and contains simplifying assumptions made for the sake of analytical tractability. An exact analysis does not appear to be feasible. The informed reader will recognize the complexity of the problem. Even simulation studies of mobile communications traffic that have appeared in the technical literature, contain many simplifying assumptions, especially regarding distributions of certain random quantities which are needed to model physical reality. We feel that the underlying assumptions made in the development presented here are not unreasonable in view of the complexity of the real problem and the strength and tractability of the resulting analysis. A simulation study of the proposed handoff procedures is underway.

II. Traffic Model

A. *Calling Rates*

The basic system model assumes that the new call origination rate is uniformly distributed over the mobile service area. We denote the average number of new call originations per second per unit area as L_a. A very large population of mobiles is assumed, thus the average call origination rate is for practical purposes independent of the number of calls on progress. A hexagonal cell shape is also assumed for the system because it has some definite advantages over other possible shapes [1]. The cell radius R for a hexagonal cell is defined as the maximum distance from the center of a cell to the cell boundary. With the cell radius R, the average new call origination rate per cell L_R is

$$L_R = \frac{3\sqrt{3}}{2} R^2 L_a. \tag{1}$$

Additionally, handoff attempts are made, with an average handoff attempt rate per cell denoted Λ_{Rh}. This rate will be related to other system parameters. The ratio γ_o of handoff attempt rate to new call origination rate (per cell) is

$$g_o \underset{=}{D} \frac{L_{Rh}}{L_R}. \tag{2}$$

If a fraction P_B of new call origination is blocked and cleared from the system, the average rate at which new calls are carried is

$$L_{Rc} = L_R (1 - P_B). \tag{3}$$

Also, if a fraction P_{fh} of handoff attempts fails, the average rate at which handoff calls are carried is

$$L_{Rhc} = L_{Rh} (1 - P_{fh}). \tag{4}$$

The ratio γ_c of the average carried handoff attempt rate to the average carried new call origination rate is defined

$$g_c \underset{=}{D} \frac{L_{Rhc}}{L_{Rc}} = g_o \frac{(1 - P_{fh})}{(1 - P_B)}. \tag{5}$$

B. *Channel Holding Time in a Cell*

The channel holding time T_H in a cell is defined as the time duration between the instant that a channel is occupied by a call and the instant it is released by either completion of the call or a cell boundary crossing by the mobile. This is a function of system parameters such as cell size, speed and direction of mobiles, etc. The distribution of T_H is investigated in this section.

We let the random variable T_M, denote the unencumbered message duration, that is, the time an assigned channel would be held if no handoff is required. The random variable T_M is assumed to be exponentially distributed with the mean value $\overline{T}_M$ ($\triangleq 1/\mu_M$). Because of handoff, the distribution of this random variable will generally differ from that of the channel holding time. We assume that the velocity of a mobile is a random variable but remains constant during the mobile's travel in a cell. The speed in a cell is assumed to be uniformly distributed on the interval [0, $V_{\max}$]. Specifically, the probability density functions (pdf) of V and T_M are respectively,

$$f_{T_M}(t) = \begin{cases} m_M e^{-m_M t}, & for\ \ t \geq 0 \\ 0, & otherwise \end{cases} \tag{6}$$

$$f_V(u) = \begin{cases} \dfrac{1}{V_{max}}, & for\ \ 0 \leq u \leq V_{max} \\ 0, & otherwise \end{cases}. \tag{7}$$

When a mobile crosses a cell boundary, the model assumes that vehicular speed and direction change. The direction of travel is also assumed to be uniformly distributed and independent of speed.

We define the random variable T_n as the time (from the onset of a call) for which a mobile resides in the cell to which the call is originated. The time that a mobile resides in the cell in which the call is handed off is denoted T_h. In Appendix A we develop a mathematical model and expressions for the pdfs $f_{Tn}(t)$ and $f_{Th}(t)$.

When a call is originated in a cell and gets a channel, the call holds the channel until the call is completed in the cell or the mobile moves out of the cell. Therefore, the channel holding time T_{Hn} is either the unencumbered message duration T_M or the time T_n for which the mobile resides in the cell, whichever is less. For a call that has been handed off successfully, the channel is held until the call is completed in the cell or the mobile moves out of the cell again before call completion. Because of the memoryless property of the exponential distributions, the remaining message duration of a call after handoff has the same distribution as the unencumbered message duration. In this case the channel holding time T_{Hh} is either the remaining message duration T_M or mobile residing time T_h in the cell; whichever is less. The random variables T_{Hn} and T_{Hh} are therefore given by

$$\begin{aligned} T_{Hn} &= min(T_M, T_n) \\ T_{Hh} &= min(T_M, T_h). \end{aligned} \tag{8}$$

The cumulative distribution functions (cdf) of T_{Hn} and T_{Hh} can be expressed as

$$\begin{aligned} F_{T_{Hn}}(t) &= F_{T_M}(t) + F_{T_n}(t)(1 - F_{T_M}(t)) \\ F_{T_{Hh}}(t) &= F_{T_M}(t) + F_{T_h}(t)(1 - F_{T_M}(t)). \end{aligned} \tag{9}$$

The distribution of channel holding time can be written as

$$\begin{aligned} F_{T_H}(t) &= \frac{L_{Rc}}{L_{Rc} + L_{Rhc}} F_{T_{Hn}}(t) + \frac{L_{Rhc}}{L_{Rc} + L_{Rhc}} F_{T_{Hh}}(t) \\ &= \frac{1}{1+g_c} F_{T_{Hn}}(t) + \frac{g_c}{1+g_c} F_{T_{Hh}}(t) \\ &= F_{T_M}(t) + \frac{1}{1+g_c}(1 - F_{T_M}(t))(F_{T_n}(t) + g_c F_{T_h}(t)). \end{aligned} \tag{10}$$

From (6),

$$F_{T_H}(t)=\begin{cases} 1-e^{-m_M t}+\dfrac{e^{-m_M t}}{1+g_c}\left(F_{T_n}(t)+g_c F_{T_h}(t)\right), & for \quad t\geq 0 \\ 0, & elsewhere \end{cases}. \tag{11}$$

The complementary distribution function (or survivor function) $F^C{}_{T_H}(t)$ is

$$F^C{}_{T_H}(t)=1-F_{T_H}(t)=\begin{cases} e^{-m_M t}-\dfrac{e^{-m_M t}}{1+g_c}\left(F_{T_n}(t)+g_c F_{T_h}(t)\right), & for \quad t\geq 0 \\ 0, & elsewhere \end{cases}. \tag{12}$$

The probability function (pdf) of T_H is found by differentiating (11). Thus

$$f_{T_H}(t)=m_M e^{-m_M t}+\frac{e^{-m_M t}}{1+g_c}\left[f_{T_n}(t)+g_c f_{T_h}(t)\right]-\frac{m_M e^{-m_M t}}{1+g_c}\left[F_{T_n}(t)+g_c F_{T_h}(t)\right] \tag{13}$$

For the following analysis the distribution of T_H will be approximated by a negative exponential distribution with mean $\overline{T}_H$ ($\underline{\underline{\Delta}}\ 1/\mu_H$) to calculate values of various system characteristics. From the family of negative exponential distribution functions, we will choose one function which best fits the distribution of T_H, by comparing the survivor function $F^C{}_{T_H}(t)$ and $exp(-m_H\ t)$. Because a negative exponential distribution function is determined by its mean value, we choose $\overline{T}_H$ ($\underline{\underline{\Delta}}\ 1/\mu_H$) which satisfies the following condition:

$$\int_0^\infty \left(F^C{}_{T_H}(t)-e^{-m_H t}\right)dt=0. \tag{14}$$

To prove the fairness, the "goodness of fit" for this approximation is measured by

$$G=\frac{\int_0^\infty \left|F^C{}_{T_H}(t)-e^{-m_H t}\right|dt}{2\int_0^\infty F^C{}_{T_H}(t)dt} \tag{15}$$

where G indicates the normalized difference between two functions and is on the interval [(0, 1)]. A value of G=0 specifies an exact fit and a value of G=1 indicates no correlation.

III. Probabilities and Performance Criteria

To clarify subsequent discussion, it is convenient to explain at this point the meaning of certain probabilities that arise in the development and calculation of appropriate system performance characteristics. For analytical tractability, we develop our initial model considering only the availability of radio links between one mobile party and the nearest base station. Blocking that is internal to the land system connecting base stations is ignored for the present. For mobile-to-mobile calls, blocking and forced terminations are considered only for links from one of the mobiles to the base station. Similar simplifying assumptions have appeared in the technical literature even for *simulation* studies of mobile communication systems [4]. Some aspects of the more general case will be discussed subsequently.

The probability that a new call does not enter service because of unavailability of channels is called the blocking probability P_B. A call that is not blocked, of course, enters service, but its ultimate fate has two possible outcomes. One is that the call is completed satisfactorily (when the message exchange is ended and the channel is no longer needed). The other is that the call is forced to terminate prematurely because the mobile experiences an unsuccessful handoff attempt prior to completion. We denote the probability that a call is ultimately forced into termination (though not blocked) by P_F. This represents *the average fraction of new calls which are not blocked but which are eventually uncompleted.*

To calculate P_F, it is convenient to define another probability P_{fh}. This denotes the probability that a given handoff attempt fails. It represents the *average fraction of handoff attempts* that are *unsuccessful*.
Not all calls that are initially assigned to a channel will require handoff. We characterize the handoff demand using two probabilities P_N and P_H that can be related to other system parameters.

The probability P_N that a new call that is not blocked will require at least one handoff before completion because of the mobile crossing the cell boundary is

$$P_N = Pr\{T_M > T_n\} = \int_0^\infty [1 - F_{T_M}(t)] f_{T_n}(t) dt = \int_0^\infty e^{-m_M t} f_{T_n}(t) dt. \tag{16}$$

The probability P_H that a call that has already been handed off successfully will require another handoff before completion is

$$P_H = P_r\{T_M > T_h\} = \int_0^\infty [1 - F_{T_M}(t)] f_{T_h}(t) dt = \int_0^\infty e^{-m_M t} f_{T_h}(t) dt. \tag{17}$$

Let us define the integer random variable K as the number of times that a nonblocked call is successfully handed off during its lifetime. Since the whole service area is much larger than the cell size, the event that a mobile moves out of the mobile service area during the call is very rare. A nonblocked call will have no successful handoffs if it is completed in the cell in which it was first originated or if it is forced to terminate on the first handoff attempt. It will have exactly k successful handoffs of all of the following events occur: 1) it is not completed in the cell in which it was first originated; 2) it succeeds in the first handoff attempt; 3) it requires and succeeds in k - 1 additional handoffs; 4) it is either completed before needing the next handoff or it is not completed but fails on the (k+1)st handoff attempt. The probability function for K is therefore given by

$$\begin{aligned} &P_r\{K = 0\} = (1 - P_N) + P_N P_{fh} \\ &P_r\{K = k\} = P_N (1 - P_{fh})(1 - P_H + P_H P_{fh}) \cdot \{P_H (1 - P_{fh})\}^{k-1}, \quad k = 1, 2, \cdots. \end{aligned} \tag{18}$$

From this, the mean value of K is found to be

$$\overline{K} = \sum_{k=0}^{\infty} k P_r\{K = k\} = \frac{P_N (1 - P_{fh})}{1 - P_H (1 - P_{fh})}. \tag{19}$$

If the entire service area has M cells, the total average new call attempt rate which is not blocked is ML_{Rc}, and the total average handoff call attempt rate is $\overline{K} ML_{Rc}$. Assuming that these traffic components are equally distributed among all cells, we find

$$g_c = \frac{\overline{K} ML_{Rc}}{ML_{Rc}} \equiv \overline{K}. \tag{20}$$

To proceed further, it is convenient at this point to specify in greater detail the mathematical analysis required to determine P_B (the fraction of new calls blocked) and P_{fh} (the fraction of handoff attempts that fail). These quantities depend on the scheme used to manage handoffs.

IV. Channel Assignment Schemes

When no priority is given to handoff call attempts over new call attempts, no difference exists between these call attempts; the probabilities of blocking and handoff attempt failure are the same. However, the occurrence of a call being forced to terminate is considerably less desirable from the user's viewpoint than is the occurrence of blocking. The probability of forced termination can be decreased by giving priority (for channels) to handoff attempts (over new call attempts). In this section, two priority schemes are described, and the expressions for P_B and P_{fh} are derived. A subset of the channels allocated to a cell is to be exclusively used for handoff calls in both priority schemes. In the first priority scheme, a handoff call is terminated if no channel is immediately available in the target cell. In the second priority scheme, the handoff call attempt is held in queue until either a channel becomes available for it, or the received signal power level becomes lower than the receiver threshold level.

A. Priority Scheme I

Priority is given to handoff attempts by assigning C_h channels exclusively for handoff calls among the C channels in a cell. The remaining $C - C_h$ channels are shared by both new calls and handoff calls. A new call is blocked if the number of available channels in the cell is less than or equal to C_h when the call is originated. A handoff attempt is unsuccessful if no channel is available in the target cell. We assume that both new and handoff call attempts are generated according to a Poisson point process with mean rates per cell of L_R and L_{Rh}, respectively. As discussed previously, the channel holding time T_H in a cell is approximated to have an exponential distribution with mean $\overline{T}_H$ ($\underline{\underline{D}}$ $1/m_H$). We define the state E_j of a cell such that a total of j calls is in the progress for the base station of that cell. Let P_j represent the steady-state probability that the base station is in state E_j; the probabilities can be determined in the usual way for birth-death processes [12]. The pertinent state-transition diagram is shown in Fig. 1. From Fig. 1, the "rate up = rate down" state equations are

$$P_j = \begin{cases} \dfrac{L_R + L_{Rh}}{m_H} P_{j-1}, & \text{for } j = 1,2,\cdots,C - C_h \\ \dfrac{L_{Rh}}{m_H} P_{j-1}, & \text{for } j = C - C_h + 1,\cdots,C \end{cases}. \tag{21}$$

Using (21) recursively, along with the normalization condition

$$\sum_{j=0}^{\infty} P_j = 1,$$

the probability distribution $\{P_j\}$ is easily found as follows:

$$P_0 = \left[\sum_{k=0}^{C-C_h} \frac{(L_R + L_{Rh})^k}{k!\, m_H{}^k} + \sum_{k=C-C_h+1}^{C} \frac{(L_R + L_{Rh})^{C-C_h} L_{Rh}{}^{k-(C-C_h)}}{k!\, m_H{}^k} \right]^{-1} \tag{22}$$

$$P_j = \begin{cases} \dfrac{(L_R + L_{Rh})^j}{j!\, m_H{}^j} P_0, & \text{for } j = 1,2,\cdots,C - C_h \\ \dfrac{(L_R + L_{Rh})^{C-C_h} L_{Rh}{}^{j-(C-C_h)}}{j!\, m_H{}^j} P_0, & \text{for } j = C - C_h + 1,\cdots,C \end{cases}. \tag{23}$$

The probability of blocking for a new call is the sum of the probabilities that the state number of the base station is larger than or equal to $C - C_h$. Hence

$$P_B = \sum_{j=C-C_h}^{C} P_j. \tag{24}$$

The probability of handoff attempt failure P_{fh} is the probability that the state number of the base station is equal to C. Thus

$$P_{fh} = P_c. \tag{25}$$

B. Priority Scheme II

In Priority Scheme II, we assume that the same channel-sharing method as that of Priority Scheme I is used, except that queuing of handoff attempts is allowed if necessary. No queuing of new call attempts take place. To analyze this scheme, it is necessary to consider the handoff the handoff procedure in more detail. When a mobile moves away from the base station, the received power generally decreases. When the received power gets lower than a handoff threshold level, the handoff procedure is initiated. The handoff area has been defined as the area in which the average received power level from the base station of a mobile receiver is between the handoff threshold level and the receiver threshold level [13]. If the handoff attempt finds all channels in the target cell occupied, we consider that it can be queued. If any channel is released while the mobile is in the handoff area, the next queued handoff attempt is accomplished successfully. If the received power level from the source cell's base station falls below the receiver threshold level prior to the mobile being assigned a channel in the target cell, the call is forced into termination. When a channel is released in the cell, it is assigned to the next handoff call attempt waiting in the queue (if any). If more than one handoff call attempt is in the queue, the first-come-first-served queuing discipline is used. We assume that the queue size at the base station is unlimited. Fig. 2 shows a schematic representation of the flow of call attempts through a base station.

The time for which a mobile is in the handoff area depends on system parameters such as the speed and direction of mobile travel and the cell size. We define this as the dwell time of a mobile in the handoff area and denote it by the random variable T_Q. For simplicity of analysis, we assume that this dwell time is exponentially distributed with mean $\bar{T}_Q$ ($\triangleq 1/m_Q$).

We define E_j as the state of the base station when j is the sum of the number of channels being used in the cell and the number of handoff call attempts in the queue. For those states whose state number j is less than equal to C, the state transition relation is the same as for scheme I.

We define the random variable X as the elapsed time from the instant a handoff attempt joins the queue (i.e., the mobile enters the handoff area toward a target cell in which all channels are occupied) to the first instant that a channel is released in the fully occupied target cell. For state numbers less than C, X is equal to zero. Succinctly, X is the minimum remaining holding time of those calls in progress in the fully occupied target cell. When a handoff attempt joins the queue for a given target cell, other handoff attempts may already be in queue (each is associated with a particular mobile). When any of these first joined the queue, the *time* that it could remain on the queue *without succeeding* is denoted by T_Q (according to our previous definition). We define the random variable T_i, to be the remaining dwell time for that attempt which is in the *i*th queue position when another handoff attempt joins the queue. Under the memoryless assumptions here, the distributions of all T_i and T_Q are identical. Let $N(t)$ be the state number of the system at time t. From the description of this scheme and the properties of the exponential distribution it follows that

$$\begin{aligned} P_r\{N(t+h)=C+k-1 \mid N(t)=C+k\} &= P_r\{X \le h \ \ or \ \ T_1 \le h \ \ or \ \cdots \ \ T_k \le h\} \\ &= 1-P_r\{X > h \ \ and \ \ T_1 > h \ \ or \ \cdots \ T_k > h\} \\ &= 1-P_r\{X>h\}P_r\{T_1>h\}\cdots P_r\{T_k>h\} \\ &= 1-exp\left[-\left(C_{m_H}+km_Q\right)h\right] \end{aligned} \tag{26}$$

since the random variables X, T_1, T_2, …, T_k are independent. From (26) we see that it follows the birth-and-death process and resulting state transition diagram is as shown in Fig. 3.

In the usual way for birth-death processes, the probability distribution $\{P_j\}$ is easily found to be

$$P_0 = \left[\sum_{k=0}^{C-C_h} \frac{(L_R+L_{Rh})^k}{k!\, m_H{}^k} + \sum_{k=C-C_h+1}^{C} \frac{(L_R+L_{Rh})^{C-C_h} L_{Rh}{}^{k-(C-C_h)}}{k!\, m_H{}^k} + \sum_{k=C+1}^{\infty} \frac{(L_R+L_{Rh})^{C-C_h} L_{Rh}{}^{k-(C-C_h)}}{C!\, m_H{}^C \prod_{i=1}^{k-C}\left(Cm_H+im_Q\right)} \right. \tag{27}$$

$$P_j = \begin{cases} \dfrac{(L_R + L_{Rh})^j}{j!\, m_H{}^j} P_0, & for \quad 1 \le j \le C - C_h \\ \dfrac{(L_R + L_{Rh})^{(C-C_h)} L_{Rh}{}^{j-(C-C_h)}}{j!\, m_H{}^j} P_0, & for \quad C - C_h + 1 \le j \le C \\ \dfrac{(L_R + L_{Rh})^{(C-C_h)} L_{Rh}{}^{j-(C-C_h)}}{C!\, m_H{}^C \prod_{i=1}^{j-C} (C m_H + i m_Q)} P_0, & for \quad j \ge C + 1 \end{cases} \tag{28}$$

The probability of blocking P_B is the sum of the probabilities that the state number of the base station is larger than or equal to $C - C_h$. Hence

$$P_B = \sum_{j=C-C_h}^{\infty} P_j \,. \tag{29}$$

A given handoff attempt that joins the queue will be successful if both of the following events occur before the mobile moves out of the handoff area: 1) all of the attempts that joined the queue earlier than the given attempt have been disposed; 2) a channel becomes available when the given attempt is at the first position in the queue.

Thus the probability of a handoff attempt failure can be calculated as the average fraction of handoff attempts whose mobiles leave the handoff area prior to their coming into the first queue position and getting a channel. Noting that arrivals that find k attempts in queue enter position k+1, this can be concisely stated mathematically as

$$P_{fh} = \sum_{k=0}^{\infty} P_{C+k} P_r \{ attempt\ \ fails\ given\ it\ enters\ the\ queue\ in\ position\ k + 1 \} \tag{30}$$

or

$$P_{fh} \underset{=}{D} \sum_{k=0}^{\infty} P_{C+k} P_{fh|k} \tag{31}$$

in which $P_{fh|k}$ in (31) is defined as the rightmost term in (30). Since handoff success for those attempts which enter the queue in position k+1 requires coming to the head of the queue and getting a channel, we have, under the memoryless conditions assumed in this development,

$$\left(1 - P_{fh|k}\right) = \left[\prod_{i=1}^{k} P(i|i+1) \right] \cdot P_r \{ get\ channel\ \ in\ \ first\ position \} \tag{32}$$

in which $P(\ i\ |\ i\ +\ 1\ )$ represents the probability that an attempt in position $i+1$ moves to position i before its mobile leaves the handoff area.

An attempt in position $i\ +\ 1$ will either be cleared from the system or will advance in queue to the next (lower) position. It will advance if the remaining dwell time of its mobile exceeds either 1) at least one of the remaining dwell times T_j , $j\ =\ 1,\ 2,\ \dots\ ,\ i$, for any attempt ahead of it in the queue, or 2) the minimum remaining holding time X of those calls in progress in the target cell. Thus

$$1 - P(i|i+1) = P_r\{T_{i+1} \le X ,\ T_{i+1} \le T_j , \quad j = 1,2,\cdots,i\} \qquad i = 1,2,\cdots. \tag{33}$$

$$\begin{aligned} 1 - P(i|i+1) &= P_r\{T_{i+1} \le X ,\ T_{i+1} \le T_1 ,\cdots ,T_{i+1} \le T_i\} \\ &= P_r\{T_{i+1} \le min(X ,T_1 ,T_2 ,\cdots ,T_i)\} \\ &= P_r\{T_{i+1} \le Y_i\} \qquad i = 1,2,\cdots \end{aligned} \tag{33 -1}$$

$$where$$

$$Y_i \equiv min(X ,T_1 ,T_2 ,\cdots ,T_i).$$

Since the mobiles move independently of each other and of the channel holding times, the random variables, X, T_j, $(j=1,2,...,i)$ are statistically independent. Therefore, the cumulative distribution of Y_i in (33) can be written as

$$F_{Y_i}(t) = 1 - \{1 - F_X(t)\}\{1 - F_{T_1}(t)\}\cdots\{1 - F_{T_i}(t)\} \tag{33-2}$$

Because of the exponentially distributed variates in the present discussion, this is just

$$\begin{aligned} F_{Y_i}(t) &= 1 - e^{-Cm_H t} e^{-m_Q t} \cdots e^{-m_Q t} \\ &= 1 - e^{-(Cm_H + im_Q)t} . \end{aligned} \tag{33-3}$$

Then (33) can be expressed as

$$\begin{aligned} 1 - P(i|i+1) &= P_r\{T_{i+1} \le Y_i\} \\ &= \int_0^\infty \{1 - F_{Y_i}(t)\} f_{T_{i+1}}(t)\,dt \\ &= \int_0^\infty e^{-(Cm_H + im_Q)t}\, m_Q e^{-m_Q t}\, dt \\ &= \frac{m_Q}{Cm_H + (i+1)m_Q} , \quad i = 1,2,\cdots. \end{aligned} \tag{34}$$

The handoff attempt at the head of the queue will get a channel (succeed) if its remaining dwell time T_1 exceeds X. Thus

$$\begin{aligned} &P_r\{get\ channel\ \ in\ \ first\ position\} = P_r\{T_1 > X\} \\ &and \\ &P_r\{does\ not\ get\ channel\ \ in\ \ first\ position\} = P_r\{T_1 \le X\} \\ &\qquad = \int_0^\infty e^{-C_{mH} t}\, m_Q e^{-m_Q t}\, dt \\ &\qquad = \frac{m_Q}{Cm_H + m_Q} . \end{aligned} \tag{35}$$

The probability (35) corresponds to letting $i = 0$ in (34). Then from (32),

$$
\begin{aligned}
1-P_{fh|k} &= \left[\prod_{i=1}^{k} P(i|i+1)\right] P_r\{get\ channel\ \ in\ \ first\ position\} \\
&= \frac{Cm_H + m_Q}{Cm_H + 2m_Q}\frac{Cm_H + 2m_Q}{Cm_H + 3m_Q}\cdots\frac{Cm_H + km_Q}{Cm_H + (k+1)m_Q}\frac{Cm_H}{Cm_H + m_Q} \\
&= \frac{Cm_H}{Cm_H + (k+1)m_Q}
\end{aligned}
\tag{35-1}
$$

and

$$
P_{fh|k} = \frac{(k+1)m_Q}{Cm_H + (k+1)m_Q}.
$$

The sequence of (27), (28) and (30) – (35) defines, for computational purpose, all quantities needed to calculate P_{fh} for Priority Scheme II.

Equations (1), (2), (5), (12) – (14), (16), (17), (19), (20), (22) – (25), (27) – (35) form a set of simultaneous nonlinear equations which can be solved for system variables when parameters are given. For example, given R, $\overline{T}_M$, V_{max}, C, C_h, L_a, the quantities P_B, P_{fh}, P_N, P_H, g_c, m_H, L_R, L_{Rh} can be considered unknowns. Beginning with an initial guess for the unknowns, the equations were solved numerically using the method of successive substitutions. The distributions and mean values of T_n, and T_h can also be determined from the calculation. Details of the procedure used to calculate the performance characteristics of Fig. 4 are given in APPENDIX B.

C. *Probabilities of Forced Termination, Blocking, and Noncompleted Calls*

Various performance characteristics can then be readily calculated for each priority scheme. Of particular interest are the fractions of new call attempts that are blocked, completed, and forced into termination (due to unsuccessful handoff). If the cell radius is large (compared with the product of the speed and mean holding time), the chance of a mobile crossing a cell boundary during a call duration is small. In this case the probability of blocking P_B is the major indication of system traffic performance. When the cell radius is small, however, a higher probability exists that a mobile crosses a cell boundary during the call duration. Also, the mean channel holding time of a call in a cell is smaller. Under these circumstances, nonblocked calls on the average experience more handoffs. The result is a greater chance of forced termination due to an unsuccessful handoff in a call's lifetime. So, for small cell radii, the probability, P_F, of forced termination, and the probability, P_{fh} , that a handoff attempt fails are also important performance measures.

From the user's point of view the probability P_F that a call which is not blocked is *eventually* forced into termination can be more significant than P_{fh}. A call which is not blocked will be eventually forced into termination if it succeeds in each of the first (l – 1) handoff attempts which it requires but fails on the l th. Therefore,

$$
P_F = \sum_{l=1}^{\infty} P_{fh}\left[P_N\left(1-P_{fh}\right)^{l-1} P_H{}^{l-1}\right] = \frac{P_{fh}P_N}{1-P_H\left(1-P_{fh}\right)}
\tag{36}
$$

where P_N and P_H are the probabilities of handoff demand of new and handoff calls, as defined previously.

Let P_{nc} denote the *fraction of new call attempts* that will *not* be *completed* because of either blocking or unsuccessful handoff. This is also a major system performance measure. This probability P_{nc} can be expressed as

$$
P_{nc} = P_B + P_F\left(1-P_B\right) = P_B + \frac{P_{fh}P_N\left(1-P_B\right)}{1-P_H\left(1-P_{fh}\right)}
\tag{37}
$$

where the first and second terms represent the effects of blocking and handoff attempt failure, respectively. In (37) we can guess roughly that when cell size is large, probabilities of cell crossing P_N and P_H will be small and the second term of (37) (i.e., effect of cell crossing) will be much smaller than the first term (i.e., effect of blocking). However, when the cell size is decreased, P_N and P_H will increase. The noncompleted call probability P_{nc} can be considered as a unified measure of both blocking and forced termination effects.

Another interesting measure of system performance is the weighted sum of P_B and P_F

$$CF = (1 - a)P_B + aP_F \tag{38}$$

where a is in the interval [(0, 1)] and indicates the relative importance of the blocking and forced termination effects. For some applications P_F may be more important than P_B from the user's point of view, and the relative cost α can be assigned using the system designer's judgment.

V. Performance Characteristics

Numerical results were obtained for all schemes discussed here. Generally, it has found that for given system parameters, Priority Scheme II allowed significantly smaller forced termination probabilities for given blocking probability. Most of the figures presented here therefore describe scheme II. For the calculations, the average unencumbered message duration was taken as $\overline{T}_M$ = 120 s and the maximum speed of a mobile of $V_{max} = 60$ mi/h was used.

The effect of cell radius on P_B and P_F can be seen in Fig. 4 which shows these as functions of (new) call origination rate per unit area L_a. A total of 20 channels per cell ($C = 20$) with (a margin of) one channel per cell for handoff priority, ($C_h = 1$) was assumed. Priority Scheme II was used for this figure, and the mean dwell time for a handoff attempt $\overline{T}_Q$ was assumed to be $\overline{T}_H$ /10. It was found that P_F is much smaller than P_B and that the difference between them decreases as cell size decreases. As expected for larger R the effect of handoff attempts and forced terminations on system performance is smaller.

Fig. 5 shows P_B and P_F as functions of L_a for different values of C_h with cell radius R = 2 mi and C = 20 channels. The effects of priority given to handoff calls over new calls by increasing C_h, P_F decreases by orders of magnitude with only small to moderate increase in P_B . This exchange is important because (as was mentioned previously) forced terminations are usually considered much less desirable than blocked calls.

In Fig. 6, a system with cell radius $R = 2$ mi, $C = 20$ channels/cell, and $L_a = 0.01$ (calls/sec)/mi^2 is considered. The cost function, CF, as a function of C_h for various values of the weighting factor, a, is displayed. A greater number of handoff channels C_h is required to minimize the cost function CF when a is large, (that is, when P_F is given more weight than P_B). For most of the range of a, the required values of C_h, that minimize the cost function CF are small because P_B has a predominant effect on CF.

The dependence of P_B and P_F on C_h for various values of C is shown in Fig. 7. The values of L_a were chosen to make P_B equal to 0.01 with $C_h = 0$ for each choice of C. It can be seen that if the total number of channels per cell C is increased, the loss (increase) in the blocking probability P_B is less as the number of handoff channels is increased; but the same order of magnitude reduction in forced termination probability P_F is attained. That is, the exchange of *increased blocking probability* for *decreased forced termination probability* becomes more favorable as C is increased.

Blocking and forced termination probabilities for the two priority schemes are shown in Fig. 8 as functions of call origination rate density L_a. The forced termination probability P_F is smaller for scheme II than for scheme I, but almost no difference exists in blocking probability P_B. We get this superiority of Priority Scheme II by queuing the delayed handoff attempts for the dwell time of the mobile in the handoff area.

The noncompleted call probability P_{nc} is shown as a function of call arrival rate density for various values of R in Fig. 9. For a system with fixed cell radius R, the noncompleted call probability increases rapidly with increasing new call origination rate density.

For various L_a, Fig. 10 shows the effect (on P_{nc}) of priority given to handoff calls by increasing C_h. The noncompleted call probability P_{nc} increases with increasing C_h. This shows that, with cell radius R = 2.0 mi, the major reason for the noncompletion of a call is blocking rather than forced termination.

Fig. 11 shows the required cell size as function of L_a such that P_{nc} = 0.02 for various values of C. When the D/R ratio and the number of channels per cell are determined from co-channel interference constraints, the spectrum bandwidth allocated to the system, and the modulation method; the required cell size can be determined for the required call arrival rate density L_a from this kind of graph.

The mean channel holding time in a cell $\overline{T}_H$ is expected to decrease with decreasing cell size. Fig. 12 shows this quantitatively. Notice that $\overline{T}_H$ becomes smaller with smaller cell size, but sensitivity to change in cell size is smaller for larger cells. As cell size increases the limiting factor is the unencumbered holding time of a call, that is, the holding time that a call would use if there were no forced termination.

Earlier in the paper we approximated the cumulative distribution function of the channel holding time in a cell (see (14)). The goodness-of-fit G of this approximation, defined as (15), is shown in Table I for various cell sizes. We see that G is very small for all ranges of cell radius R. These values support the use of the approximation in our calculations.

VI. Further Discussion

As mentioned earlier, we ignored blocking which is internal to the land network, and we considered only the availability of radio channels between one mobile party to a call, and the nearest base station. These assumptions are common in analyses and simulations of mobile systems [4], [11]. For mobile-to-land or land-to-mobile calls in systems whose internal blocking is negligible, the blocking of new calls and the failure of handoff attempts would indeed occur only at the mobile party to a call. Therefore, the performance characteristics obtained in the previous sections apply directly to those situations. However, for mobile-to-mobile calls, the call is blocked if either party to the call is blocked. The analysis is somewhat more complicated, but some rough extensions of the foregoing results can be obtained easily.

The mobiles of both parties generally move independently of each other. For cellular systems with small cell size, the case that the mobiles of both parties to a call are in the same cell is very small. If this case is ignored, then the blocking probability, P_B', of the mobile-to-mobile call is

$$P_B{}' = 1 - (1 - P_B)^2 = 2P_B - P_B{}^2 \tag{39}$$

where P_B is the blocking probability of one mobile party to a call.

Similarly, a nonblocked mobile-to-mobile call is forced into termination if either mobile party to the call fails in a handoff attempt at a cell boundary. Because of this, calls can be terminated when one of the mobile parties is in a cell (not at the cell boundary). Therefore, the average channel holding time in a cell $\overline{T}_H$ can be less than that obtained by our more thorough (but also more restrictive) analysis. However, if the handoff attempt failure probability is very small, those effects may be ignored. With this assumption, the handoff failures of both mobile parties to a call are considered independent of each other. Then the probability of forced termination, P_F', of the nonblocked mobile-to-mobile calls is found as

$$P_F{}' = 1 - (1 - P_F)^2 = 2P_F - P_F{}^2 \tag{40}$$

where P_F is the forced termination probability of one mobile party to a call.

VII. Conclusion

A traffic model for mobile radiotelephone systems with cellular structure, frequency reuse, and handoff has been considered. The probability of blocking P_B of new call attempts as well as the probability of forced termination P_F of nonblocked calls were calculated and plotted as functions of call origination rate density. As expected forced termination probability P_F for smaller cell systems is more significant. We found that P_F is decreased by a significantly larger order of magnitude than the increase of P_B when more priority is given to handoff calls by increasing the number of handoff channels.

Two prioritized handoff procedures were considered. In Priority Scheme I, a number of channels is used exclusively for handoff calls while the remaining channels are used for both new calls and handoff calls. Blocked calls are cleared from the system immediately. In Priority Scheme II, handoff call attempts can be queued for the time duration in which a mobile dwells in the handoff area between cells. Channels are shared in the same way as in Priority Scheme I. It was found that P_F is lower for Scheme II while there is essentially no difference for P_B over the interesting range of parameters.

The noncompletion probability P_{nc} that a new call attempt is not completed because of either blocking or forced termination was defined as one of the system performance measures. It was found that P_B is the major component of P_{nc} , even for small cell systems, with R = 2.0 mi. Because of this P_{nc} is increased when more priority is given to handoff calls. A weighted sum of P_B and P_F (cost function *CF*) was defined and used as another measure of system performance. The value of *CF* depends on the weighting factor α. As expected more priority is required to decrease *CF* when the weighting between P_F and P_B is shifted to the former. The required cell radius is shown as a function of call origination rate density for numbers of channels per cell *C* and values of P_{nc}. This graph is useful to determine the cell size from system parameters after the *D/R* ratio is chosen from co-channel constraints requirements. It is believed that the model and analysis in this paper can provide useful tools for designing and predicting the performance of cellular mobile radiotelephone systems.

APPENDIX A
PROBABILITY DISTRIBUTION OF RESIDING TIME IN A CELL

The probability distributions of the residing times T_n and T_h are to be investigated. The random variable T_n is defined as the time (duration) that a mobile resides in the cell *in which its call originated*. Also T_h is defined as the time a mobile resides in a cell to *which its call is handed off.*

In mobile radiotelephone systems the boundary between cells are determined by average received signal power levels from adjacent base stations. However, the received signal power levels vary from time to time because of shadowing and fading effects, even though the transmitting signal power is constant and distance from base station is fixed. Therefore, the actual cell boundary is not critically fixed, and the handoff area may be defined between the cells in which the received signal power level is lower than handoff threshold level and higher than receiver threshold level [13]. For this reason we approximate the hexagonal cell shape as a circle to simplify analysis.

For a hexagonal cell having radius R, the approximating circle with the same area has a radius, R_{eq}, which is given by

$$R_{eq} = \sqrt{\frac{3\sqrt{3}}{2p} R} \approx 0.91R. \tag{41}$$

The relation between R and R_{eq} is shown in Fig. 13. The base station is assumed to be at the center of a cell and is indicated by a letter B in the figure. The location of a mobile in a cell, which is indicated by a letter A in the figure, is represented by its distance r and direction f from the base station as shown. To find the distributions of T_n and T_h , we assume that the mobiles are spread evenly over the area of the cell. Then r and f are random variables with pdf's

$$f_r(r) = \begin{cases} \dfrac{2r}{R_{eq}{}^2}, & for\ \ 0 \le r \le R_{eq} \\ 0, & elsewhere \end{cases} \tag{42}$$

$$f_f(f) = \begin{cases} \dfrac{1}{2p}, & for\ \ 0 \le f \le 2p \\ 0, & elsewhere \end{cases}. \tag{43}$$

We assume that a mobile travels in any direction with equal probability and its direction remains constant during its travel in the cell. If we define the direction of mobile travel by the angle θ (with respect to a vector from the base station to the mobile), as shown in the figure, the distance Z from the mobile to boundary of approximating circle is

$$Z = \sqrt{R_{eq}{}^2 - (r \sin q)^2} - r \cos q . \tag{44}$$

Because ϕ is evenly distributed in a circle, Z is independent of ϕ and from the symmetry we can consider the random variable θ is in interval [0, π] with pdf

$$f_q(q) = \begin{cases} \dfrac{1}{p}, & \text{for } 0 \le q \le p \\ 0, & \text{elsewhere} \end{cases} . \tag{45}$$

If we define new random variables x, y as

$$x = r \cos q$$
$$y = r \sin q ,$$

then

$$Z = \sqrt{R_{eq}{}^2 - y^2} - x$$
$$W = x.$$

Since the mobile is assumed to be equally likely to be located anywhere in the approximating circle

$$f_{XY}(x,y) = \begin{cases} \dfrac{2}{pR_{eq}{}^2}, & \text{for } -R_{eq} \le x \le R_{eq},\ 0 \le x^2 + y^2 \le R_{eq}{}^2,\ 0 \le y \le R_{eq} \\ 0, & \text{elsewhere} \end{cases} .$$

From (42), (44), and (45), the joint density function of Z and W can be found by standard methods

$$f_{ZW}(z,w) = \frac{|z+w|}{\sqrt{R_{eq}{}^2 - (z+w)^2}} f_{XY}(x,y)$$
$$= \frac{2}{pR_{eq}{}^2} \frac{|z+w|}{\sqrt{R_{eq}{}^2 - (z+w)^2}}, \qquad \text{for } 0 \le z \le 2R_{eq},\quad -\frac{1}{2} z \le w \le -z + R_{eq} .$$

The pdf of the distance *Z* is then

$$f_Z(z) = \int_{-z/2}^{R_{eq}-z} \frac{2}{pR_{eq}{}^2} \frac{(z+w)}{\sqrt{R_{eq}{}^2 - (z+w)^2}} dw, \quad \text{for } 0 \le z \le 2R_{eq}$$
$$= \begin{cases} \dfrac{2}{pR_{eq}{}^2} \sqrt{R_{eq}{}^2 - \left(\dfrac{z}{2}\right)^2}, & \text{for } 0 \le z \le 2R_{eq} \\ 0, & \text{elsewhere} \end{cases} . \tag{46}$$

We assume that the speed *V* of a mobile is constant during its travel in the cell and random variable which is uniformly distributed on the interval [$0, V_{max}$] with pdf

$$f_V(v) = \begin{cases} \dfrac{1}{V_{max}}, & for\ \ 0 \le v \le V_{max} \\ 0, & elsewhere \end{cases}.$$

Then the time T_n is expressed by

$$T_n = \frac{Z}{V}$$

with pdf

$$\begin{aligned} f_{T_n}(t) &= \int_{-\infty}^{\infty} |w| f_Z(tw) f_V(w) dw \\ &= \begin{cases} \dfrac{2}{V_{max} p R_{eq}{}^2} \displaystyle\int_0^{V_{max}} w\sqrt{R_{eq}{}^2 - \left(\frac{tw}{2}\right)^2}\, dw, & for\ \ 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ \dfrac{2}{V_{max} p R_{eq}{}^2} \displaystyle\int_0^{2R_{eq}/t} w\sqrt{R_{eq}{}^2 - \left(\frac{tw}{2}\right)^2}\, dw, & for\ \ t \ge \dfrac{2R_{eq}}{V_{max}} \end{cases} \\ &= \begin{cases} \dfrac{8R_{eq}}{3V_{max} p t^2}\left[1 - \sqrt{\left\{1 - \left(\dfrac{tV_{max}}{2R_{eq}}\right)^2\right\}^3}\,\right], & for\ \ 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ \dfrac{8R_{eq}}{3V_{max} p t^2}, & for\ \ t \ge \dfrac{2R_{eq}}{V_{max}} \end{cases}. \end{aligned} \qquad (47)$$

The cdf of T_n is

$$\begin{aligned} F_{T_n}(t) &= \int_{-\infty}^{t} f_{T_n}(x) dx \\ &= \begin{cases} \dfrac{2}{p} arcsin\left(\dfrac{V_{max} t}{2R_{eq}}\right) - \dfrac{4}{3p} tan\left[\dfrac{1}{2} arcsin\left(\dfrac{V_{max} t}{2R_{eq}}\right)\right] + \dfrac{1}{3p} sin\left[2\, arcsin\left(\dfrac{V_{max} t}{2R_{eq}}\right)\right], & for\ \ 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ 1 - \dfrac{8R_{eq}}{3pV_{max}} \dfrac{1}{t}, & for\ \ t \ge \dfrac{2R_{eq}}{V_{max}} \end{cases}. \end{aligned} \qquad (48)$$

To find the distribution of T_h, we note that when a handoff call is attempted, it is always generated at the cell boundary, which is taken as the boundary of the approximating circle. Therefore, to find T_h one must recognize that the mobile will move from one point on the boundary to another. The direction of a mobile when it crosses the boundary is indicated by the angle θ between the direction of the mobile and the direction from the mobile to the center of a cell as shown in Fig. 14. If we assume that the mobile moves with any direction with equal probability, the random variable θ has pdf given by

$$f_q(q) = \begin{cases} \dfrac{1}{p}, & for\ -\dfrac{p}{2} \le q \le \dfrac{p}{2}. \\ 0, & elsewhere \end{cases}$$

The distance *Z* is as shown in Fig. 14

$$Z = 2R_{eq} \cos q \tag{49}$$

which has a cdf given by

$$\begin{aligned} F_Z(z) &= P_r\{Z \le z\} \\ &= \begin{cases} 0, & for\ \ z < 0 \\ 1 - \dfrac{2}{p} \arccos\left(\dfrac{z}{2R_{eq}}\right), & for\ \ 0 \le z \le 2R_{eq}\,. \\ 1, & for\ \ z > 2R_{eq} \end{cases} \end{aligned} \tag{50}$$

The pdf of *Z* is

$$\begin{aligned} f_Z(z) &= \frac{d}{dz} F_Z(z) \\ &= \begin{cases} \dfrac{1}{p} \dfrac{1}{\sqrt{R_{eq}{}^2 - \left(\dfrac{z}{2}\right)^2}}, & for\ \ 0 \le z \le 2R_{eq} \\ 0, & elsewhere \end{cases}. \end{aligned} \tag{51}$$

The time in the cell T_h is the time that a mobile travels the distance *Z* with speed V, then

$$T_h = \frac{Z}{V}. \tag{52}$$

With the same assumption about *V*, the pdf of T_h is

$$
\begin{aligned}
f_{T_h}(t) &= \int_0^{\infty} |w| f_Z(tw) f_V(w) dw \\
&= \begin{cases} \dfrac{1}{pV_{max}} \displaystyle\int_0^{V_{max}} \dfrac{w}{\sqrt{R_{eq}^2 - \left(\dfrac{tw}{2}\right)^2}} dw, & for \;\; 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ \dfrac{1}{pV_{max}} \displaystyle\int_0^{2R_{eq}/t} \dfrac{w}{\sqrt{R_{eq}^2 - \left(\dfrac{tw}{2}\right)^2}} dw, & for \;\; t \ge \dfrac{2R_{eq}}{V_{max}} \end{cases} \\
&= \begin{cases} \dfrac{4R_{eq}}{pV_{max}} \dfrac{1}{t^2} \left[1 - \sqrt{1 - \left(\dfrac{V_{max} t}{2R_{eq}} \right)^2} \right], & for \;\; 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ \dfrac{4R_{eq}}{pV_{max}} \dfrac{1}{t^2}, & for \;\; t \ge \dfrac{2R_{eq}}{V_{max}} \end{cases}
\end{aligned}
\tag{53}
$$

and the cdf of T_h is

$$
\begin{aligned}
F_{T_h}(t) &= \int_{-\infty}^{t} f_{T_h}(x) dx \\
&= \begin{cases} 0, & for \;\; t < 0 \\ \dfrac{2}{p} \arcsin\left(\dfrac{V_{max} t}{2R_{eq}} \right) - \dfrac{2}{p} \tan\left[\dfrac{1}{2} \arcsin\left(\dfrac{V_{max} t}{2R_{eq}} \right) \right], & for \;\; 0 \le t \le \dfrac{2R_{eq}}{V_{max}}. \\ 1 - \dfrac{4R_{eq}}{pV_{max}} \dfrac{1}{t}, & for \;\; t > \dfrac{2R_{eq}}{V_{max}} \end{cases}
\end{aligned}
\tag{54}
$$

APPENDIX B
EXAMPLE CALCULATION PROCEDURE FOR FIGURE 4

The following procedure was used to generate the performance characteristics shown in Fig. 4.

(Step 1) Set the values of the system parameters such as R, V_{max} , $\overline{T}_M$ $(=1/ m_M)$, C and C_h.
(Step 2) Calculate R_{eq} using equation (41).
(Step 3) Calculate P_N using equations (16) and (47).
(Step 4) Calculate P_H using equations (17) and (53).
(Step 5) Set the value of L_a.
(Step 6) Calculate L_R using equation (1).
(Step 7) Initialize values of P_B and P_{fh}.
(Step 8) Calculate g_c using equations (19) and (20).
(Step 9) Calculate L_{Rh} using equations (2) and (5).
(Step 10) Calculate $\overline{T}_H$ $(=1/ m_H)$ using equations (14), (12), (48), and (54).
(Step 11) Calculate P_j using equations (27) and (28).
(Step 12) Calculate new P_B using equation (29).
(Step 13) Calculate new P_{fh} using equations (31) - (35).
(Step 14) Repeat (Step 8) through (Step 13) until values of the new P_B and new P_{fh} of (Step 12) and (Step 13) converge to the values of P_B and P_{fh} used at (Step 9) for equation (5). Replace values of P_B and P_{fh} of (Step 9) with values of the new P_B and new P_{fh} of (Step 12) and (Step 13) respectively at each iteration.
(Step 15) Calculate P_F using equation (36).
(Step 16) Output data of L_a, P_B and P_F.
(Step 17) Repeat (Step 5) through (Step 16) for values of L_a.
(Step 18) Repeat (Step 1) through (Step 17) for values of R.

REFERENCES

[1] V. H. MacDonald, "Advanced mobile phone service: The cellular concept", Bell Syst. Tech. J., vol. 58, pp. 15-41, Jan. 1979.
[2] R. H. Frenkiel, "A high-capacity mobile radiotelephone system model using a coordinated small-zone approach", IEEE Trans. Veh. Technpl., vol. VT-19, pp. 173-177, May 1970.
[3] L. Schiff, "Traffic capacity of three types of common-user mobile radio communication systems", IEEE Trans. Commun. Technol., vol. COM-18, pp 12-21, Feb. 1970.
[4] W. C. Jakes, Ed., Microwave Mobile Communications. New York: Wiley, 1974.
[5] D. C. Cox and D. O. Reudink, "Dynamic channel assignment in high-capacity mobile communications systems", Bell Syst. Tech. J., vol. 50, pp. 1833-1857, 1971.
[6] D. C. Cox and D. O. Reudink, "Dynamic channel assignment in two-dimensional large-scale mobile radio systems", Bell Syst. Tech. J. .vol. 51, pp. 1611-1629, 1972.
[7] D. C. Cox and D. O. Reudink, "A comparison of some channel assignment strategies in large-scale mobile communication systems", IEEE Trans. Commun., vol. COM-20, pp. 190-195, 1972.
[8] L. G. Anderson, "A simulation study of some dynamic channel assignment algorithms in a high capacity mobile telecommunications system", IEEE Trans. Veh. Technol., vol. VT-22, pp. 210-217, 1973.
[9] J. S. Engel and M. M. Peritsky, "Statistically-optimum dynamic server assignment in systems with interfering servers", IEEE Trans. Veh. Technol., vol. VT-22, pp. 203-209, 1973.
[10] D. C. Cox and D. O. Reudink' "Increasing channel occupancy in large-scale mobile radio systems: Dynamic channel reassignment", IEEE Trans. Veh. Technol., vol. VT-22, pp. 218-222, 1973.
[11] T. J. Kahwa and N. D. Georganas, "A hybrid channel assignment scheme in large-scale cellular-structure mobile communication systems", IEEE Trans. Commun., vol. COM-26, pp. 432-438, 1978.
[12] R. B. Cooper, Introduction to Queuing Theory, 2nd ED. New York: Elsevier North Holland, 1981.
[13] J. D. Wells, "Cellular system design using the expansion cell layout method", IEEE Trans. Veh. Technol., vol. VT-33, May 1984.

Daehyoung Hong received his B.S. degree in electronics engineering from the Seoul National University, Korea, in 1977, and the M.S. and Ph.D. degrees in electrical engineering from the State University of New York at Stony Brook, in 1982 and 1986, respectively. He was a faculty member of the Electrical and Electronics Engineering Department of ROK Air Force Academy and an air force officer from 1977 to 1981. He joined Motorola Communication Systems Research Lab. in 1986 where he was a senior staff research engineer and participated in the research and development of digital trunked radio systems (TRS) as well as CDMA digital cellular systems. Since 1992 he has been a faculty member and now is an associate professor in the Electronic Engineering Department of Sogang University in Seoul, Korea. His research interests include design, performance analysis, control algorithms, and operations of wireless communication systems. He has published numerous technical papers and filed several patents in the areas of wireless communication systems. He is a consultant for a number of industrial firms. Dr. Hong is a member of IEEE, Korea Institute of Communication Sciences (KICS) and Institute of Electronic Engineers Korea (IEEK). He is also a member of Tau Beta Pi and Eta Kappa Nu. He has been active in a number of professional societies. His service includes: Chairman, Korea Chapter, IEEE Communications Society; Chairman, Mobile Communications Technical Activity Group, KICS; Chairman, Communications Society, IEEK; and Technical Program Committees for several major conferences. He is a Division Editor for Wireless Communications of the Journal of Communications and Networks.

Stephen S. Rappaport (IEEE M'65-SM'76-F'87) received the B.E.E. degree from the Cooper Union, New York City, in 1960; the M.S.E.E. degree from the University of Southern California, Los Angeles, in 1962; and the Ph.D. in Electrical Engineering from New York University, New York City, in 1965. He is a Fellow of IEEE and Leading Professor of Electrical & Computer Engineering at the State University of New York at Stony Brook. Dr. Rappaport has numerous technical publications on communications systems and techniques, multiple access, cellular and non-cellular mobile radio networks and systems, queuing, communications traffic, and spread spectrum. In 1995 he received the MOUNTBATTEN PREMIUM from the Institution of Electrical Engineers (UK). He has been on the Editorial Board of IEEE Communications Magazine, the IEEE Transactions on Communications, and the Wireless Networks Journal. He was Guest Editor of the IEEE Journal on Selected Areas in Communications for a special issue on Portable and Mobile Communications and Guest Editor of WINET for a special issue on Performance Evaluation Methods for Personal and Mobile Communications and Technical Program Vice-Chair of IEEE 1998 International Conference on Universal Personal Communications. From 1994 to 1996 he was Chairman of IEEE Communications Society's Technical Committee on Personal Communications. Listings include : *American Men & Women of Science, Who's Who in America, Who's Who In the East, Who's Who In Technology Today,* and *Who's Who in Science and Engineering.*

Prof. Rappaport's experience includes Technical Staff positions at Hughes Aircraft Company and at Bell Telephone Laboratories as well as consulting for industrial firms. He is an active member of the IEEE Communications Society and the Long Island Section. His service includes: IEEE Communications Society's Board of Governors (elected member); Chairman, Technical Committee on Data Communications Systems; Nominations and Elections Board; Awards Board; Fellow Evaluation Committee; National Chairman for Universities on the Member Activities Council; Associate Editor for the IEEE Transactions on Communications; Communications Society Conference Board; elected member of Advisory Council; Technical Affairs Council; Chairman, Long Island Section Award Nominations Committee; First Vice-Chair, Long Island Section; Treasurer, Long Island Section; Chairman, L.I. Communications Society Chapter; Associate Editor, Communications Magazine; and Technical Program Committees for several major conferences and workshops.

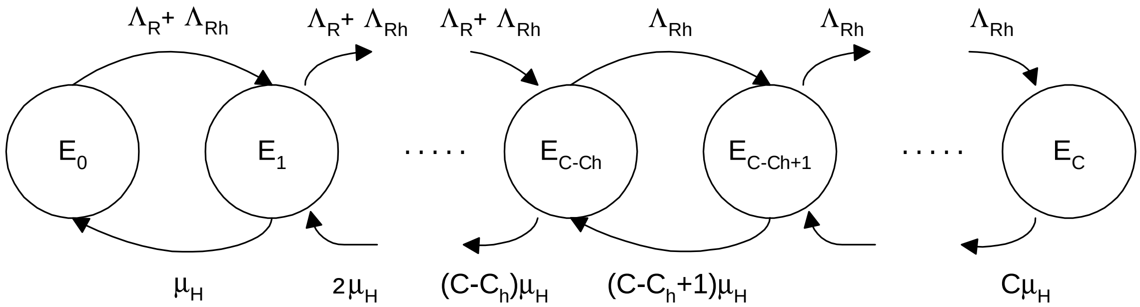

Fig. 1. State-transition diagram for Priority Scheme I.

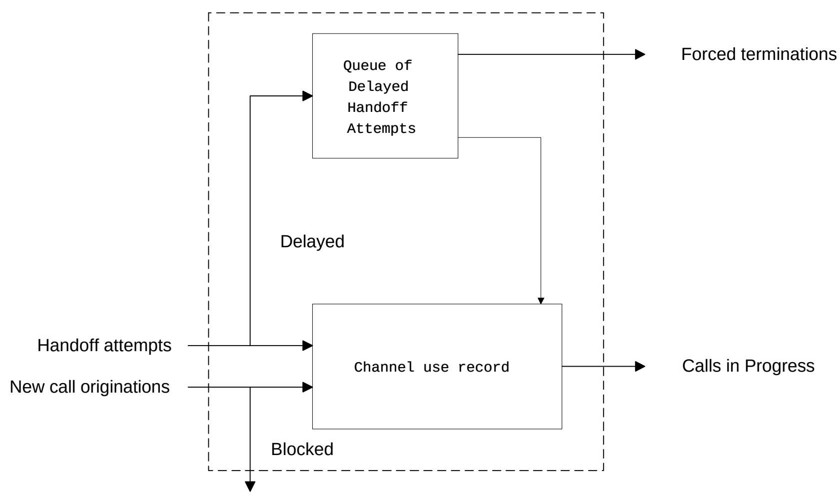

Fig. 2. Call flow diagram for Priority Scheme I.

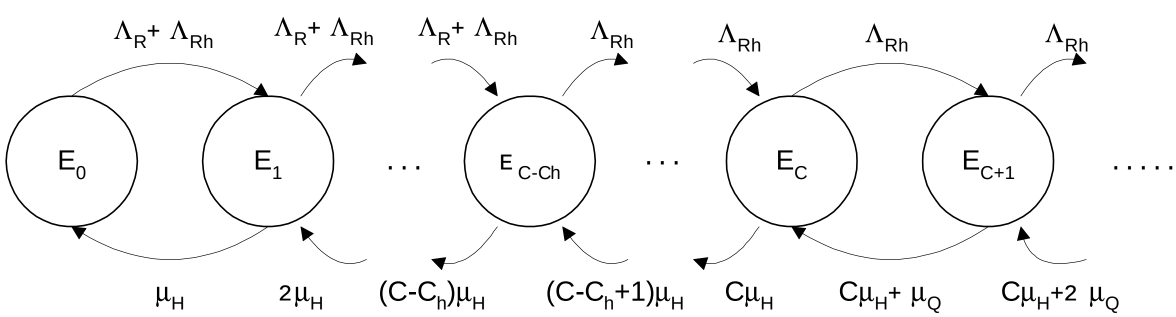

Fig. 3. State-transition diagram for Priority Scheme II.

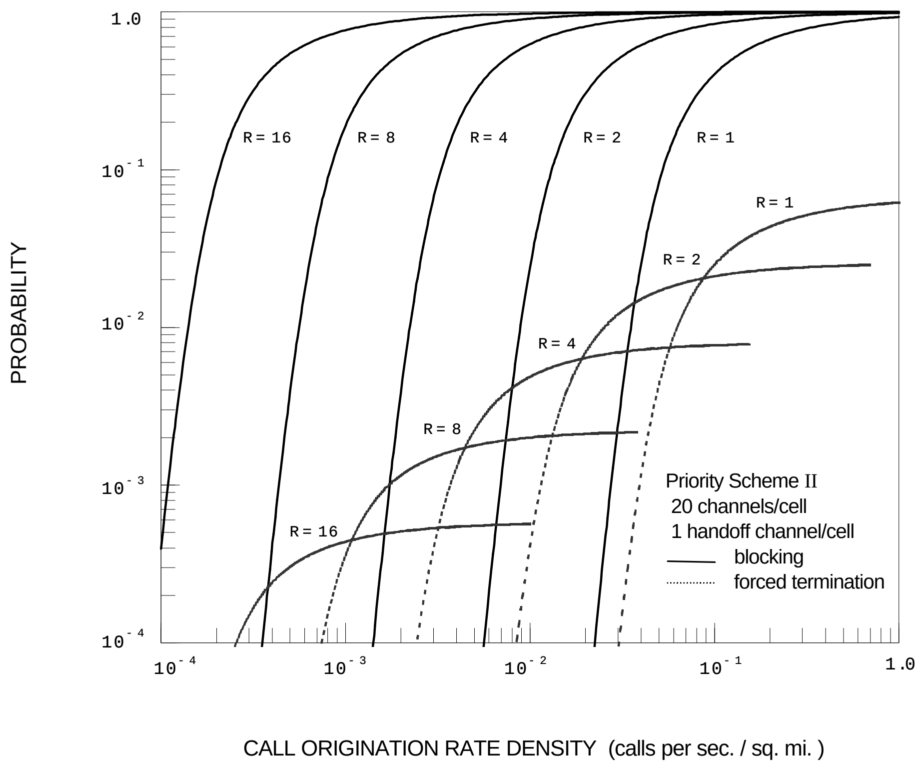

Fig. 4. Blocking and forced termination probabilities (Priority Scheme II, 20 channels/cell, 1 handoff channel/cell).

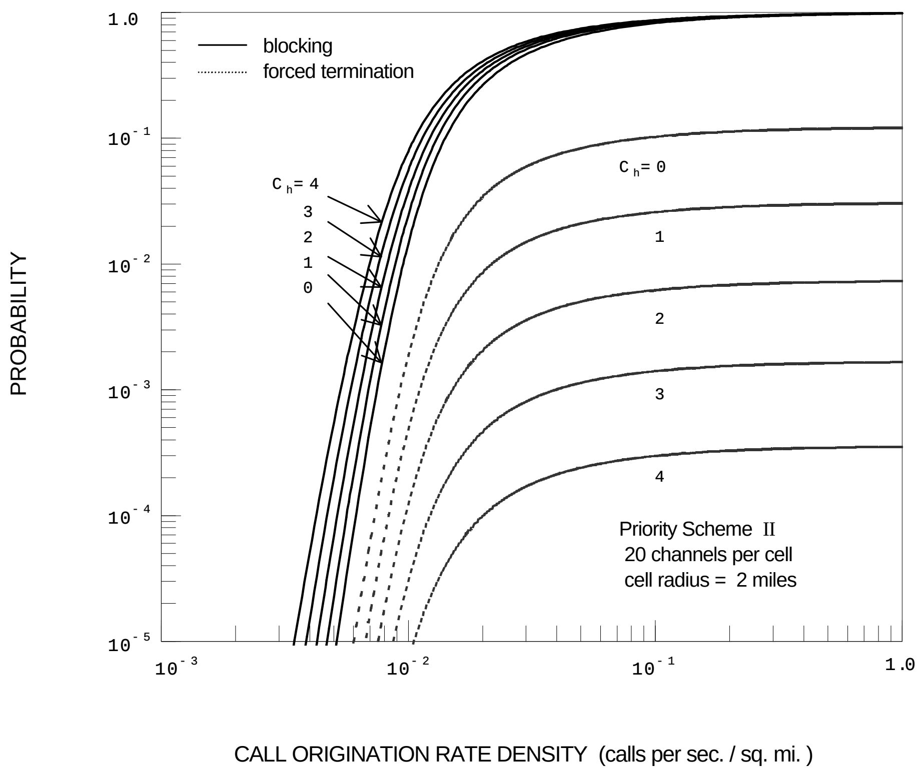

Fig. 5. Blocking and forced termination probabilities for systems with different priority (Priority Scheme II, 20 channels/cell, cell radius = 2 mi).

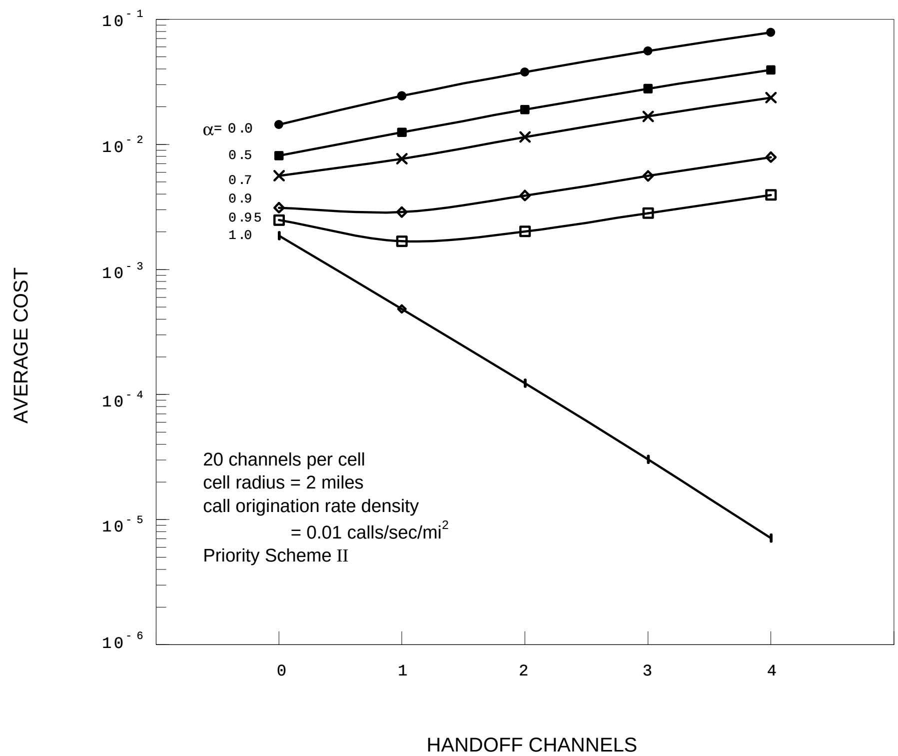

Fig. 6. Average cost of noncompletion (Priority Scheme II, 20 channels/cell, cell radius = 2 mi, call origination rate density = 0.01 (calls/s)/mi^2).

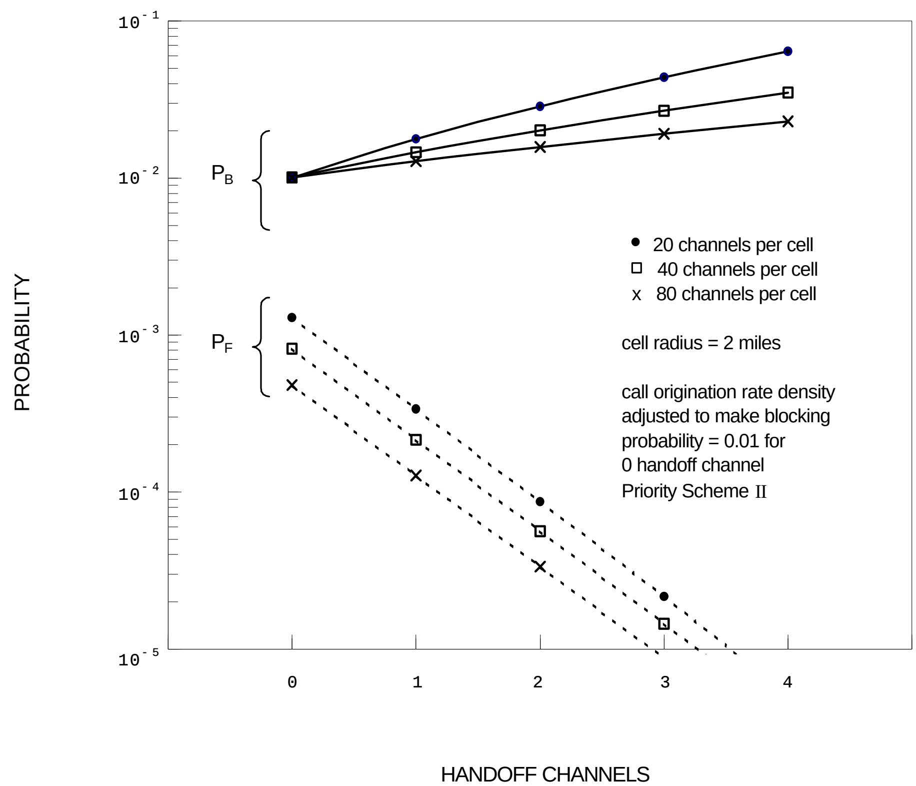

Fig. 7. Blocking and forced termination probabilities (Priority Scheme ii, cell radius = 2 mi, call origination rate density adjusted to make blocking probability = 0.01 for 0 handoff channel).

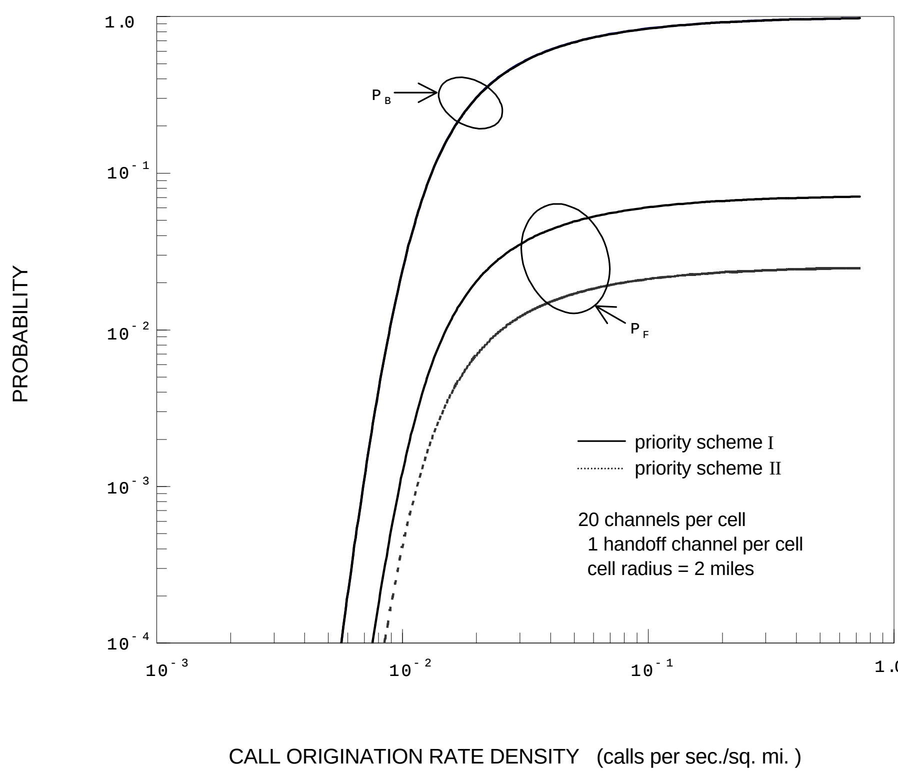

Fig. 8. Blocking and forced terminations for priority schemes (20 channels/cell, 1 handoff channel/cell, cell radius = 2 mi).

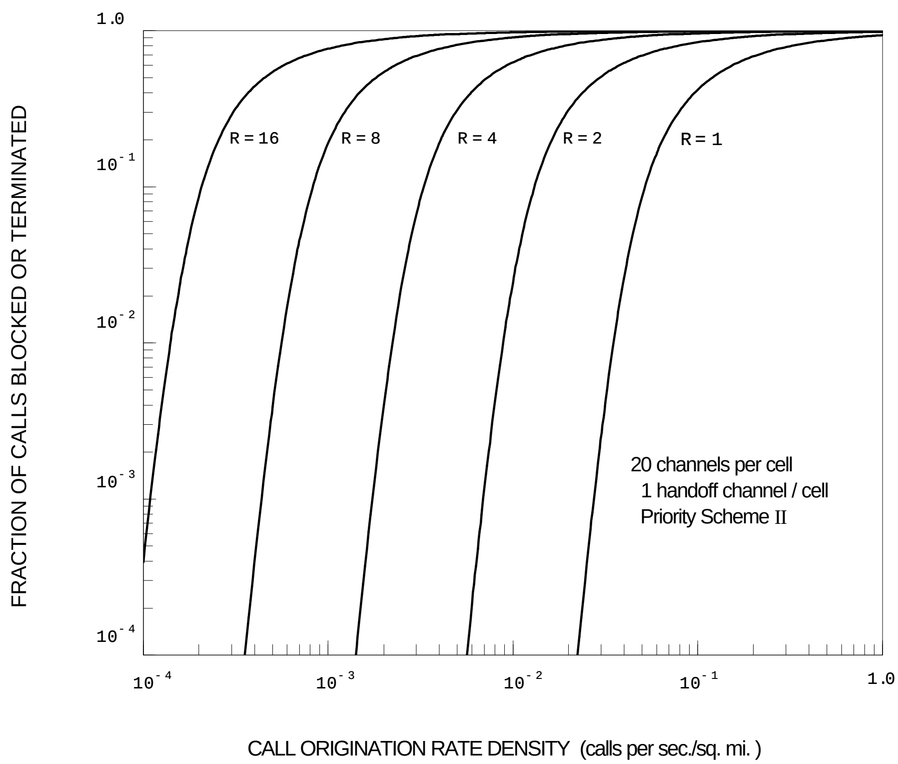

Fig. 9. Fraction of new reserved channels on uncompleted calls (20 channels/cell, cell radius = 2 mi).

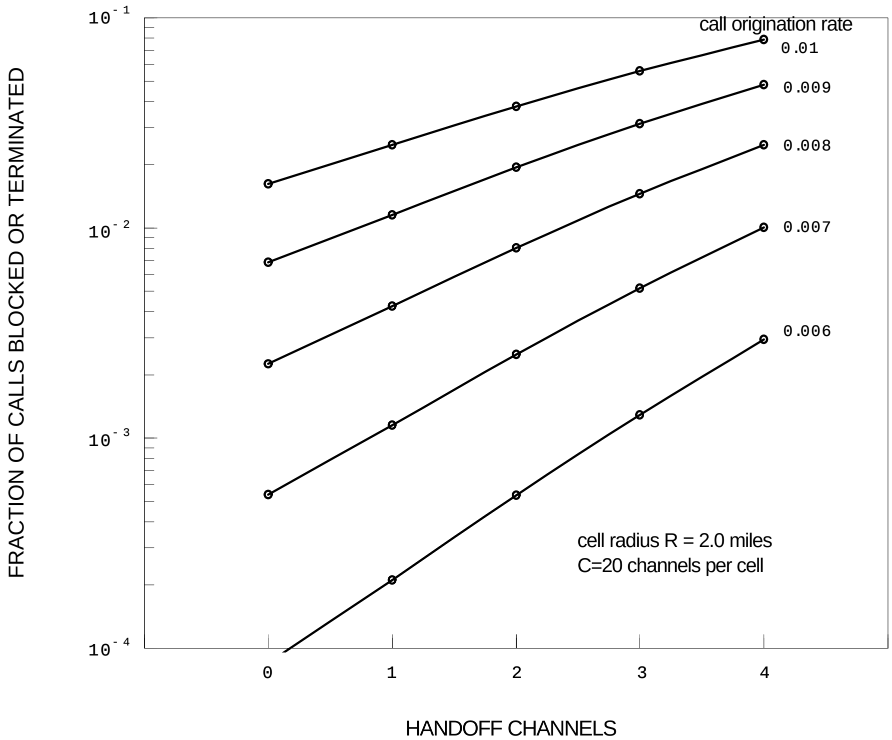

Fig. 10. Effect of reserved channels on uncompleted calls (20 channels/cell, cell radius = 2 mi).

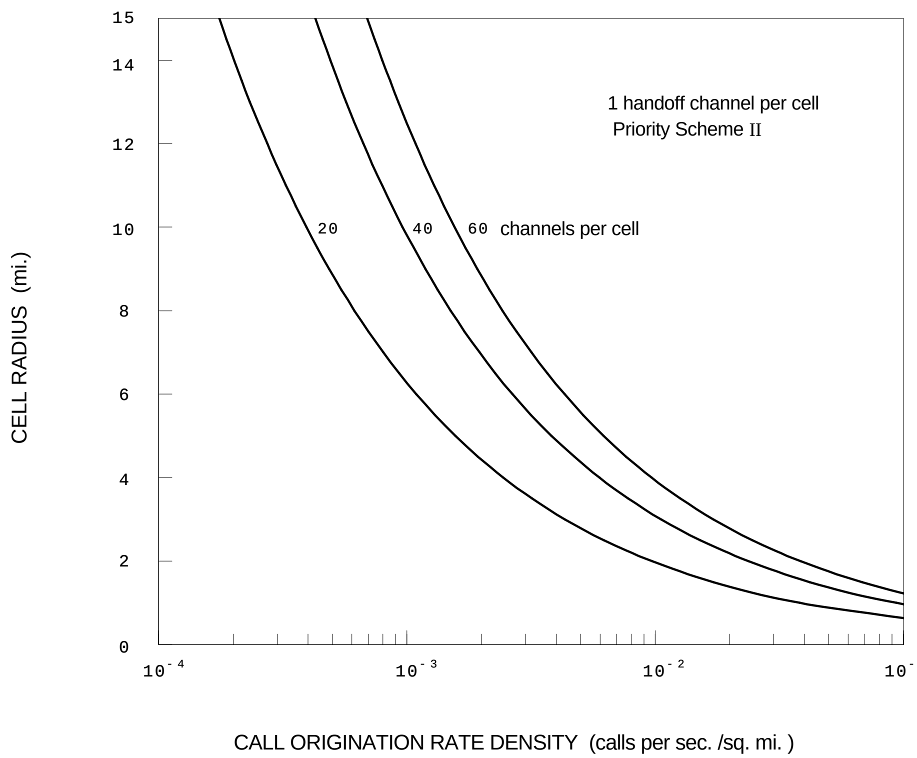

Fig. 11. Cell size for call completion probability = 0.98 (Priority Scheme II, 1 handoff channel/cell).

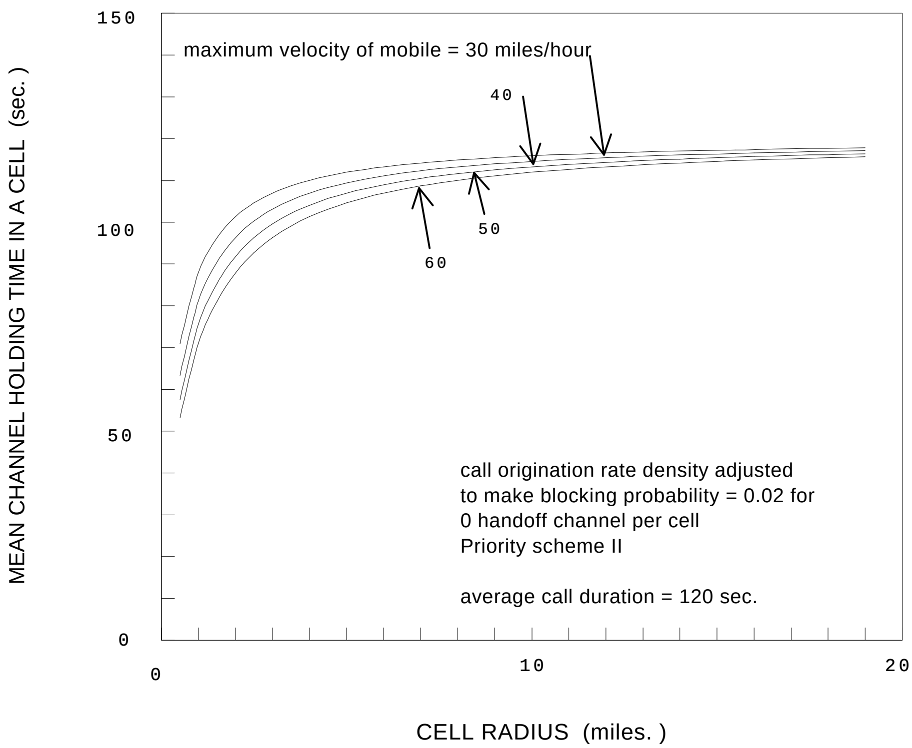

Fig. 12. Effect of cell size on channel holding time (average call duration = 120 s).

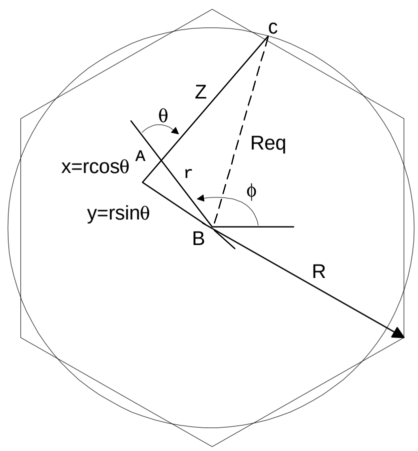

Fig. 13. Illustration of distance from point A in cell (where call is originated), to point C on cell boundary (where mobile exits from cell).

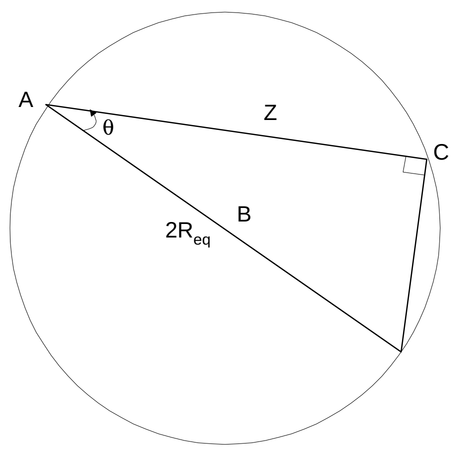

Fig. 14. Illustration of distance from point A on cell boundary (where mobile enters cell), to point C on cell boundary (where mobile exits from cell).

TABLE I
GOODNESS-OF-FIT OF FUNCTION APPROXIMATION

Cell Radius, R	G
1.0	0.020220
2.0	0.000120
4.0	0.000003
6.0	0.000094
8.0	0.000121
10.0	0.000107
12.0	0.000086
14.0	0.000066
16.0	0.000053