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BanyanNet: A Multihop WDM-Based Lightwave Network

by

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ABSTRACT

A multihop, wavelength division multiplex (WDM)-based network, BanyanNet, is proposed for the realization of terabit lightwave networks. BanyanNet can be considered as the bi-directional equivalence of the popular ShuffleNet. Exploiting its represenation, we developed a fast, decentralized, bi-directional routing algorithm for BanyanNet. The performance of BanyanNet is compared with that of the ShuffleNet. A better performance in channel efficiency, total and user throughput is reported for BanyanNet.

Key Words: LAN, MAN, lightwave network, ShuffleNet, and wavelength division multiplex.

1   Introduction

With the recent advances in fiber optics, *lightwave networks* composed of optical fibers have embarked on an important role in telecommunications. The strength of the photonic technology include an enormous bandwidth, noise immunity, and high security. The bandwidth offered by optical fibers is in the order of terahertz (THz) whereas those by conventional coaxial cable and twisted pair is only in the order of gigahertz or even megahertz. Furthermore, optical fibers are almost immune to noise and have excellent security. They are not affected by electromagnetic interference and are nearly impossible to wiretap without detection. These superior qualities over conventional methods made lightwave networks attractive candidates for large local and metropolitan area networks.

However, exploitation of the vast bandwidth in optical networks has been hindered by the speed of the electro-optic converter, a device converting electrical signals to optical signals and vice versa. These electronic devices can only operate in gigabit per second. Such mismatch in bandwidth between the electronic components and the optical fibers is the main obstacle in the realization of *terabit lightwave networks*. Much research effort has been directed to resolve this dilemma [1, 2, 3]. This include multiple users sharing an optical fiber via time and wavelength (frequency) multiplexing.

While time division multiplexing is limited by electronic speeds, wavelength division multiplexing (WDM) is preferred for large-scale concurrency on a single fiber [4]. There are two classes of WDM-based systems, *single-hop* and multi-hop [5, 6]. Single hop systems imply nodes communicate in one hop. Typically, a node has a small number of

optical transmitters (lasers) and optical receivers (filters). These optical devices must be wavelength-agile. That is, they are capable of rapidly tuning to the transmission frequency. Also, pre-transmission communication must exist to coordinate the transmission time and frequency between transmitters and receivers [7].

The multihop approach, on the other hand, assigns fixed transmission frequency to each communication link, and therefore, eliminates the need for pre-transmission communications and rapidly tunable transmitters and receivers [6]. Again, each node has a small number of transmitters, transmitting and receiving signals in an assigned and fixed wavelength. This arrangement allows simultaneous transmission among multiple users and thus attaining the terabit capacity of the network. For example, the typical bandwidth for the low-loss region in a single mode optical fiber is about $25 - 30$ THz and the electronic processing speed is a few Gb/s. In other words, a single fiber can accommodate up to 10^4 electronic-grade channels [6]. As the word *multihop* suggested, a message may be required to route through intermediate nodes, each retransmits the message on a different wavelength until it reaches the destination.

The establishment of an efficient multihop lightwave network relies heavily on the proper assignment of wavelengths to communication links of each node. The goals are to ensure that there is at least one path between any pair of nodes and that the average and maximum number of hops for a message to reach its destination should be small. Such assignments are based on an interconnection topology. Since this topology is not directly related to the physical connection of nodes, it is referred to as a *virtual topology*. A number of virtual topologies have been proposed [8]. These include: ShuffleNet [9, 10, 7], Hypercube [11], Generalized Hypercube [8], DeBruijn [12], and MSN (Manhattan Street Network) [13]. There are advantages and disadvantages for the different options. A review can be found in [6, 8]. Of the many options, ShuffleNet is one of the most popular topology [6]. It has been shown that a 64-node ShuffleNet has better performance than the corresponding MSN [14].

The ShuffleNet was first proposed by Acampora et al. [9] and later extended and generalized by Hluchyi and Karol [10, 7]. Conceptually, it is a uni-directional, cylindrically connected Omega network [15]. In general, there are $N = p^m \times m$ nodes arranged in p^m rows and m columns. Interconnection between adjacent columns is a perfect shuffle [16]. Figure 1 shows an $N = 2^2 \times 2 = 8$ ShuffleNet. In this case, each node has two transmitters and two receivers, each with a fixed and assigned frequency ($\lambda_{ij}, i, j = 0, \ldots, 7$). A single transmitter and receiver can also be used at each node if p users in each column are allowed to share the same transmission frequency. In that case, however, multiple access problems and inefficiency are possible [9, 7]. Physically, the network topology can be arbitrary, provided that direct transmission exists between adjacent node in the ShuffleNet. Popular topologies in local and metropolitan area networks such as the bus, star or tree networks are sufficient. As indicated in [8], Figure 1 (b) shows a star implementation of the 8-node ShuffleNet.

Figure 1: *An 8-Node ShuffleNet and Its Implementation.*
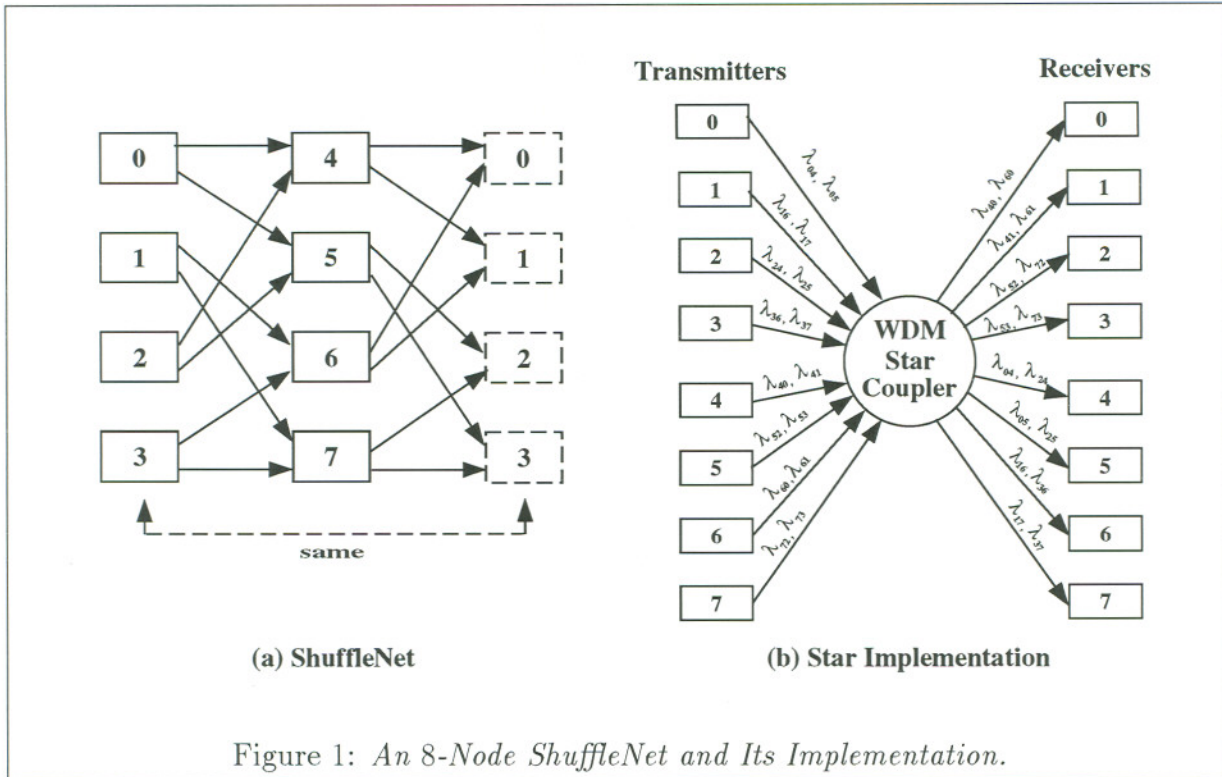
One advantage of this uni-directional ShuffleNet is its simple routing algorithm. Since messages usually require multiple hops to get to destinations, the goal of routing is to determine an appropriate outgoing links for each incoming message. A simple, *distributed, self-routing* algorithm that can identify shortest paths based only on address of the destination exists for the uni-directional ShuffleNet [7]. With this algorithm, the maximum distance (in hops) for a message to get to its destination is $2m - 1$ for $N = p^m \times m$ nodes [9]. Using graph terminology, this distance is called the *diameter* of the network [17]. Obviously, a small diameter implies a small communication delay.

With the growing number of computer users and network size, it is desirable to reduce the diameter and the number of hops by considering bi-directional ShuffleNet. However, extension of the uni-directional ShuffleNet to the bi-directional case is not trivial. There is no simple, bi-directional, *self-routing algorithm* that allow fast, decentralized routing decisions at a node. In this paper, we propose an interconnection graph corresponding to the bi-directional alternative of ShuffleNet. Analogous to the ShuffleNet, the resultant network is called the *BanyanNet*.

BanyanNet is basically a bi-directional, cylindrically connected *SW-Banyan* network used in multiprocessor interconnection [18]. It is known that SW-Banyan networks are topologically equivalent to the ShuffleNet [19], but possess different representations. For an $N = p^m \times k$ network, a cylindrically connected SW-Banyan network is composed of p^m interconnected rings of k nodes. We observed that these ring connections allow a simple

self-routing algorithm in the bi-directional case. Furthermore, we remove the constraint $k = m$ as in the ShuffleNet. Instead, k can be any multiples of m. This additional flexibility allows more users on the network to exploit the immense bandwidth of fiber optics. This paper is organized as follows: in section 2, we present the BanyanNet as a bi-directional multihop lightwave network. Diameter analysis and the routing algorithm are discussed in section 3. Section 4 evaluates and compares the performance of BanyanNet and ShuffleNet. Finally, in section 5 we present a summary and conclusions.

2   BanyanNet

Goke and Lipovski proposed a general class of dynamic networks for multiprocessor interconnection, called *Banyan networks* [18, 20]. These networks are essentially made up of superimposed trees. ("Banyan" is the name of a multiply rooted tree in India.) Of the general class of Banyan networks, a sub-class called *regular SW-Banyan* is of special interest to us. Mathematically these networks can be defined as:

For an $N = p^m \times k$ network, any node (x, y), $x \in \{0, \ldots, m-1\}$,
$y = (y_0, \ldots, y_{m-1}) = y_{m-1}p^{m-1} + \ldots + y_0 p^0 \in \{0, \ldots, p^m - 1\}$,

$$(x,\ y) \sim \begin{cases} ([x+1]_k,\ y_0, \ldots, y_{r-1}, 0, y_{r+1}, \ldots, y_{m-1}) \\ ([x+1]_k,\ y_0, \ldots, y_{r-1}, 1, y_{r+1}, \ldots, y_{m-1}) \\ \qquad\qquad \vdots \\ ([x+1]_k,\ y_0, \ldots, y_{r-1}, p, y_{r+1}, \ldots, y_{m-1}) \\ ([x-1]_k,\ y_0, \ldots, y_{r-2}, 0, y_r, \ldots, y_{m-1}) \\ ([x-1]_k,\ y_0, \ldots, y_{r-2}, 1, y_r, \ldots, y_{m-1}) \\ \qquad\qquad \vdots \\ ([x+1]_k,\ y_0, \ldots, y_{r-2}, p, y_r, \ldots, y_{m-1}) \end{cases} \tag{1}$$

where $y_i \in \{0, 1, \ldots, p-1\}$, the symbols $\sim$ signifies connections, and $[x+1]_k$ denotes $x + 1 \pmod{k}$ and $r = x \pmod{m}$. Similar to the ShuffleNet, we consider modular wrap around connections exist between stage 0 and $k - 1$. But unlike the ShuffleNet, we consider bi-directional networks and the number of stages k is not limited to m. Instead, k can be any multiples of m. (Obviously, when k is much larger than m, average number of hops, diameter and hence propagation delay becomes unacceptably large. However, our performance analysis in section 4 shows that when k is moderately large, the network has a good total throughput.) The result is a cylindrically and multiply cascaded version of the SW-Banyan network by Goke and Lipovski [18, 20]. We called this network, an $N = p^m \times k$ BanyanNet. These additional flexibilities reduce the diameter and increase network performance such as channel efficiency, network and user throughputs.

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Figure 2: *An $N = 2^2 \times 4$ BanyanNet and Its Implementation.*
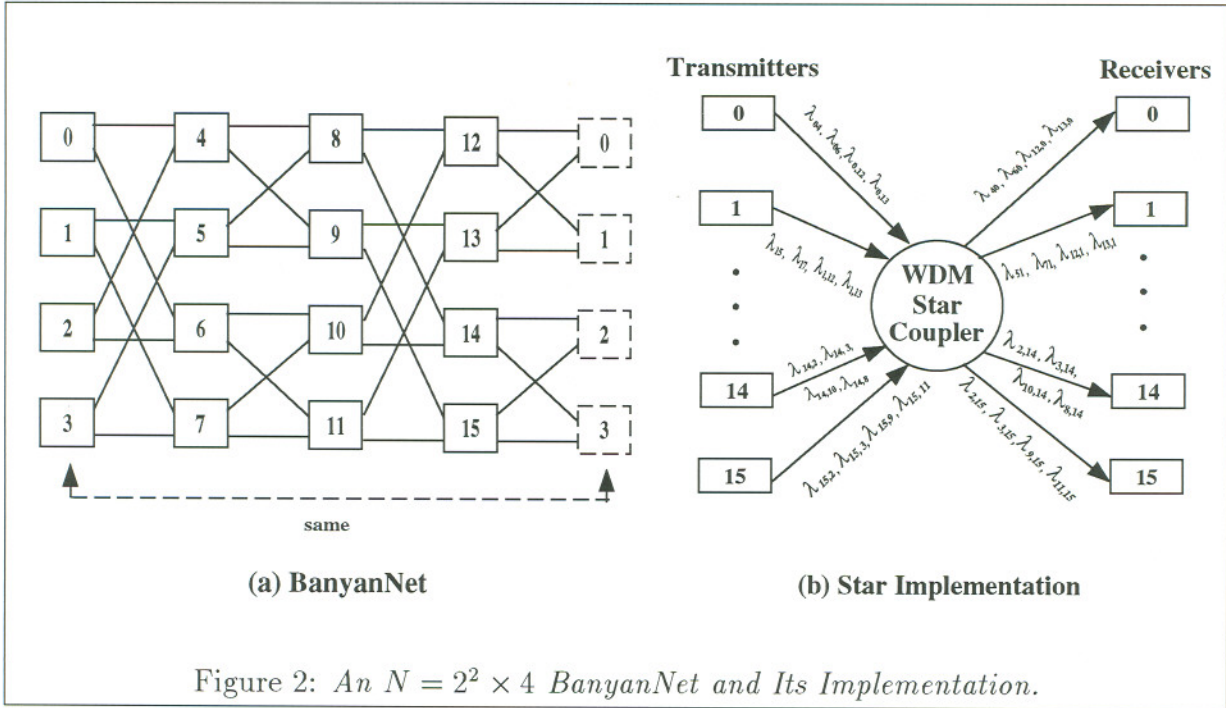
As obvious from the above definition, there are $2p$ communication links at each node. In this paper, we focus on the binary case $p = 2$, although most of our results can be easily extended to the general case. For $p = 2$, there are four connections at each node: *forward straight* and *exchange*, and *reverse straight* and *exchange*. They are defined as:

$$(x, \ y) \sim \begin{cases} ([x+1]_k, \ y) & \text{(forward straight)}; \\ ([x+1]_k, \ y_0, \ldots, \overline{y}_r, \ldots, y_{m-1}) & \text{(forward exchange)}; \\ ([x-1]_k, \ y) & \text{(reverse straight)}; \\ ([x-1]_k, \ y_0, \ldots, \overline{y}_{r-1}, \ldots, y_{m-1}) & \text{(reverse exchange)}; \end{cases} \qquad (2)$$

Figure 2 (a) shows an $N = 2^2 \times 4$-node BanyanNet and Figure 2 (b) shows the star implementation of the network. Like the ShuffleNet, the particular topology is of no significance. The BanyanNet topology is used for wavelength assignment. Each user has four bi-directional links capable of transmitting and receiving signals at four different but fixed wavelengths. These wavelengths are labelled as λ_{ij}, $i, j = 0, \ldots, 15$, corresponding to transmitter i and receiver j. For example, consider node 0 is sending a message to node 9. Based on the routing algorithm introduced in the next section, the router at node 0 sends this message out with wavelength λ_{04}. Since node 4 is the only node that can receive signals at this wavelength, the message is transmitted to node 4, which then retransmit the signal with wavelength λ_{49} and node 9 will be the only user capable of receiving the message. Consequently, the message takes two hops to get to its destination.

As illustrated in this example, the performance of the network is affected by (1) the assignment of wavelengths to the communication links and (2) the routing algorithm that

determines the transmitting wavelength of a message. Inappropriate assignment may result in unnecessary long communication delay or even non-existing paths among users. An ideal assignment ensures that all nodes are connected with the minimum number of hops. Furthermore, to fully exploit a high-speed lightwave network, the routing algorithm needs to provide fast, decentralized decisions. A distributed, self-routing scheme based only on addresses of the source and destination is, therefore, highly desirable.

As mentioned earlier, BanyanNet can be considered as a bi-directional equivalence of the ShuffleNet. Being a bi-directional network, the BanyanNet is capable of providing a wavelength assignment with a smaller average number of hops and hence a smaller propagation delay. More importantly, this representation possesses the advantage that all nodes on the same row are connected. In other words, an $N = 2^m \times k$ BanyanNet can be viewed as composed of 2^m interconnected rings of k nodes. This observation allows us to develop a relatively simple self-routing algorithm based on addresses of the users in the network. The details of this algorithm and the diameter analysis are discussed in next section.

3   Diameter Analysis and Routing

As a direct consequence of bi-directional communications, the diameter of a BanyanNet is much smaller than that of the ShuffleNet. More specifically, the diameter of an $N = 2^m \times k$ (k is a multiple of m) BanyanNet is:

$$D_B = \begin{cases} m + \lfloor m/2 \rfloor, & \text{if } k = m; \\ \max(2m, \lfloor k/2 \rfloor), & \text{if } k > m, \end{cases} \qquad (3)$$

where $\lfloor k/2 \rfloor$ denotes the largest integer smaller than $k/2$. For $k = m$, the BanyanNet is a bi-directional and cylindrical version of the original SW-Banyan network. In this case, using the original routing algorithm, the distance between nodes on the same column is at most m. The distance between nodes separated by i columns, where $1 \leq i \leq \lfloor m/2 \rfloor$ is at most $m + i$ because it takes at most m steps to get to a node on the same column as the source and the same ring as the destination, and finally at most another i hops through the ring to the destination. Hence the diameter is $D_B = m + \lfloor m/2 \rfloor$ for $k = m$ stages.

For $k > m$, again it takes m steps to get to a node m stages from the source. For nodes separated by less than m stage, at most $2m$ hops are needed because m steps are required to traverse to a node on the same ring as the destination but m stages from the source, then at most another m hops through the ring are necessary to arrive at the destination. As for nodes separated by i stages where $m < i \leq \lfloor k/2 \rfloor$, their distance is i because $m < i$ hops are required to get to the correct ring m stages from the source and then $i - m$ steps over the ring to the final destination. Hence the diameter is $D_B = \max(2m, \lfloor k/2 \rfloor)$.
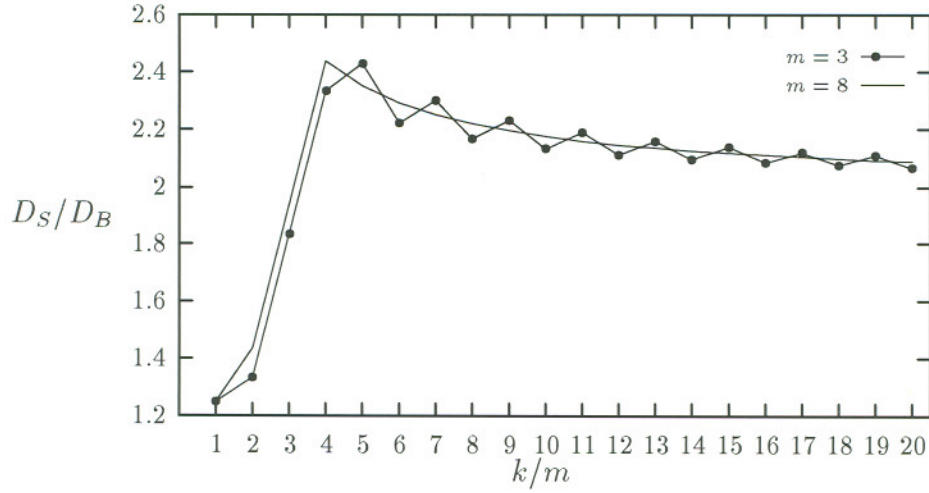
Figure 3: *Diameter Comparison of $2^m \times k$ BanyanNet and ShuffleNet*

Routing between (x^s, y^s) and (x^d, y^d) in a BanyanNet
with $N = n \times k$ nodes, $n = 2^m$ and k is a multiple of m.
Assume $y^s = (y_0^s, y_1^s, \ldots, y_{m-1}^s)$ and $y^d = (y_0^d, y_1^d, \ldots, y_{m-1}^d)$.

Evaluate $x = <x^d - x^s>_k$, where $<x>_k = \begin{cases} x, & \text{if } |x| \leq \lfloor \frac{k}{2} \rfloor; \\ x - k & \text{if } x > \lfloor \frac{k}{2} \rfloor; \\ x + k & \text{if } x < -\lfloor \frac{k}{2} \rfloor. \end{cases}$

Case 1: $|x| > m$.

If $x > m$, both forward straight and exchange are optimal.

If $x < -m$, both reverse straight and exchange are optimal.

Case 2: $|x| \leq m$. Let $r = x^s \bmod m$.

Subcase 1: $y^s = y^d$.

If $x > 0$ use forward straight; use reverse straight, otherwise.

Subcase 2: $y^s \neq y^d$ and $x >= 0$.

If $(y_r^s = y_r^d)$, use forward straight; use forward exchange, otherwise.

Subcase 3: $y^s \neq y^d$ and $x <= 0$.

If $(y_{r-1}^s = y_{r-1}^d)$, use reverse straight; use reverse exchange, otherwise.

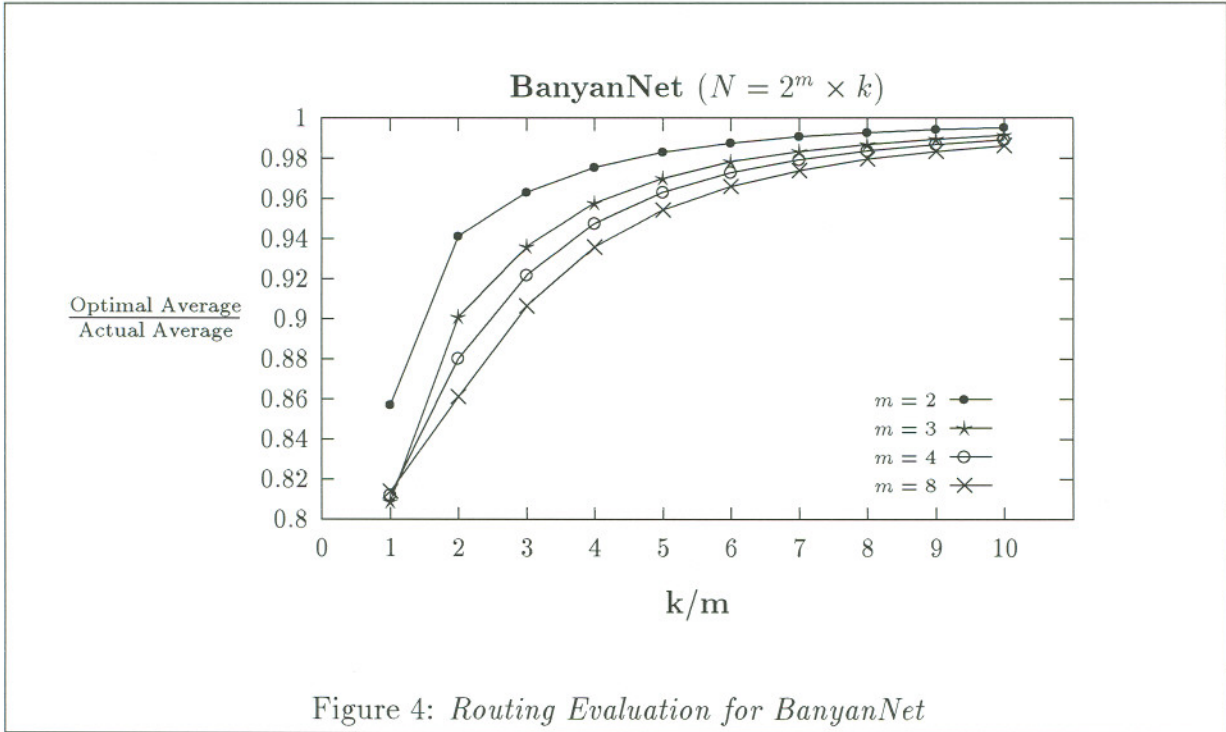Table 1: *A Routing Algorithm for $N = 2^m \times k$ BanyanNet.*

Figure 4: *Routing Evaluation for BanyanNet*

Since Omega networks are topologically equivalent to SW-Banyan networks, the diameter of a uni-directional ShuffleNet is the same as that of a uni-directional BanyanNet. Using a similar argument for the bi-directional BanyanNet, the diameter of an $N = 2^m \times k$ (k is a multiple of m) uni-directional ShuffleNet is

$$D_S = k + m - 1. \tag{4}$$

This is, it takes m hops to traverse to any node m stages from the source. For nodes, separated by i stages, where $m < i \leq k - 1$, a total of i hops are needed because the first m hops will get to the destination ring and then another $i - m$ hops to the destination column. For nodes i stages apart, where $0 \leq i < m$, a total of $k + i$ hops are needed because it takes k hops to any node in the same column as the source and then another i hops to the final destination. Hence $D_S = k + m - 1$. Figure 3 compares the diameter for the BanyanNet and the ShuffleNet for $m = 3, 8$. From Equations 3 and 4 and the Figure 3, it is clear that $D_B < D_S$ except for the trivial case $k = m = 2$. Furthermore, the optimal ratio occurs when $k/m \approx 4$ or 5.

From the definition of BanyanNet in Equation 2, we observe that for any node $(x, y_0, \ldots, y_{m-1})$, the forward exchange connection changes the rth bit and the reverse exchange connection changes the $r - 1^{\text{th}}$ bit, where $r = x \pmod{m}$. This observation coupled with our argument in the diameter analysis facilitated a simple self-routing algorithm. This algorithm is summarized in Table 1.

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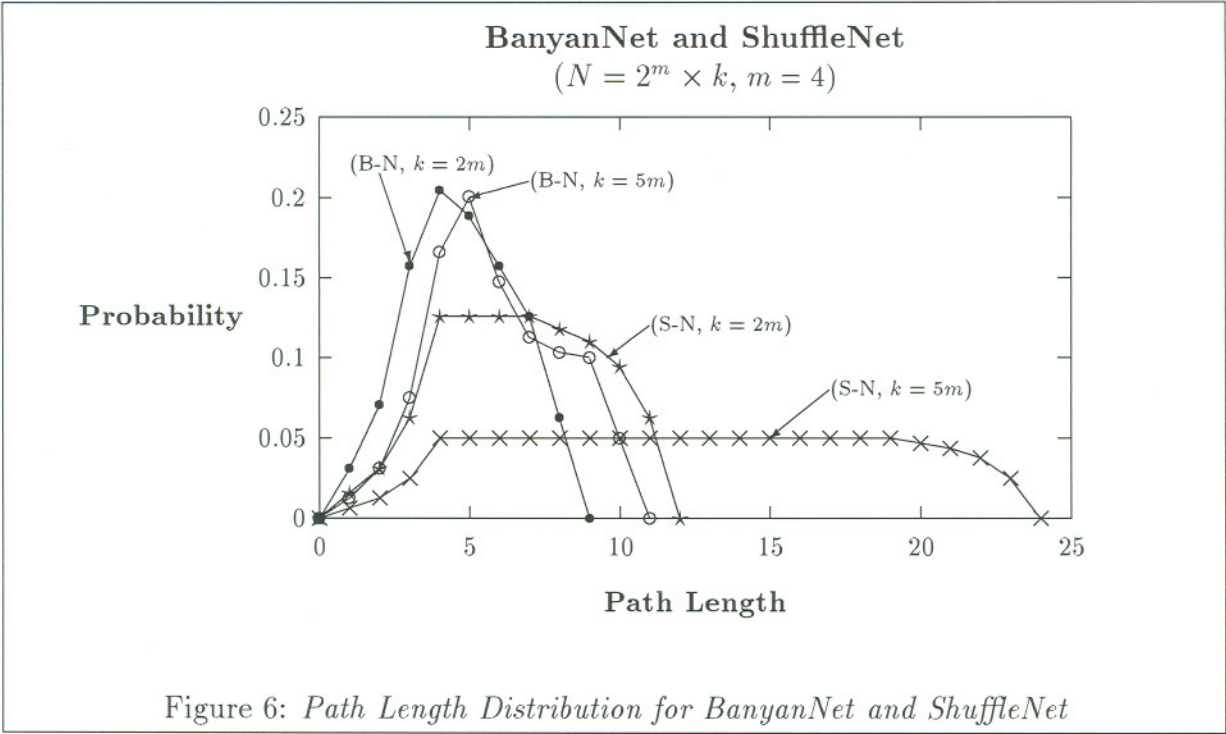Figure 5: *Path Length Distribution for BanyanNet*
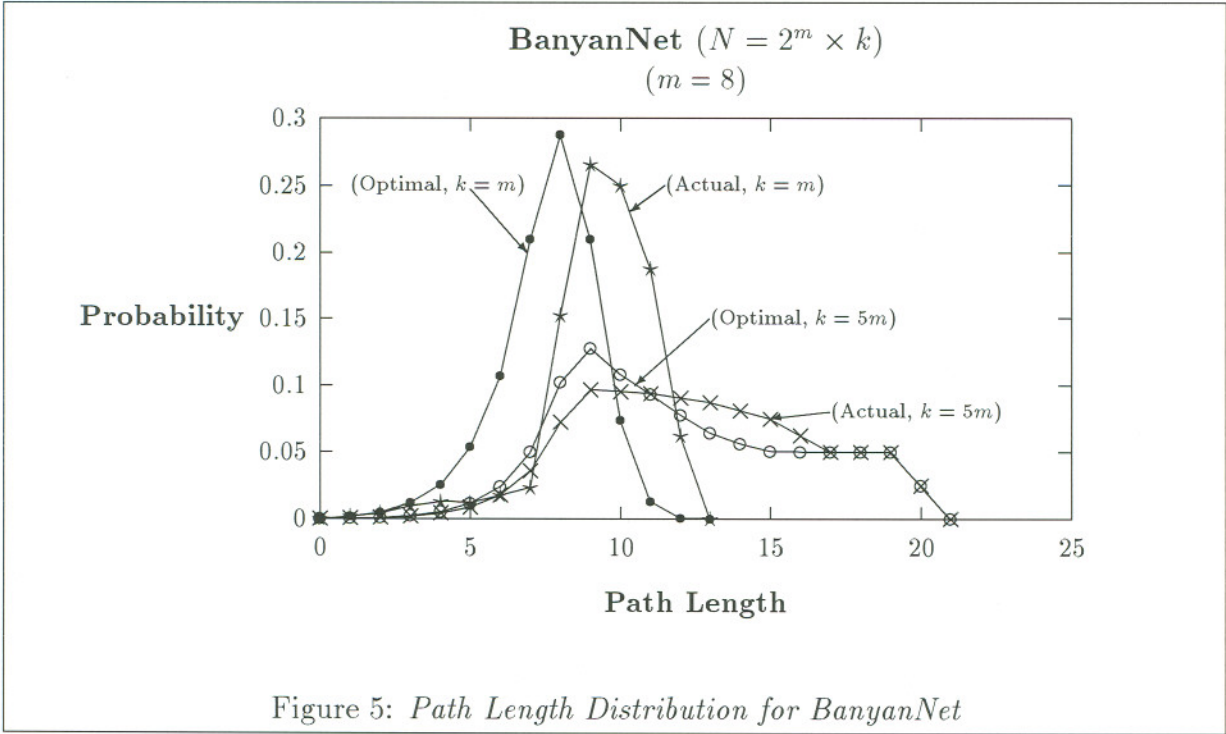
Routing for nodes separated by more than m stages (Case 1) is achieved by first sending the message to any node m stages from the destination. The nature of the original topology guarantees a path of length m between any two nodes separated by m stages. Hence in this case, both the straight and exchange connections in the appropriate direction (forward or reverse) can be used. For nodes within m stages apart, we differentiate three subcases. Subcase 1 corresponds to source on the same ring as the destination, and therefore straight connections in the appropriate directions can be used. For Subcases 2 and 3, our goal is to route to an intermediate nodes (x', y') with $y' = y^d$. This is achieved by comparing the r-th and the $(r-1)$-th bit of y^s and y^d, where r is the stage number modulo m of the source node. If the r-th or $(r-1)$-th bit is different, forward or reverse exchange is used respectively. Otherwise, straight through connections are used.

This algorithm provides fast decentralized routing decision. However, it is not optimal in the sense that the shortest path (in hops) between any two nodes is not guaranteed for nodes separated by $\leq m$ stages. To evaluate the performance of the algorithm, we implement the algorithm with a computer program. A message is sent from an arbitrary source node, say node $(0,0)$ to all other nodes in the network. The path length for each message is recorded. We found that the maximum path length from the algorithm equals to the diameter D_B, the optimal upperbound.

Figure 4 shows the ratio of the *optimal* average to the *actual* average path length. Here optimal average refers to the average length if shortest paths are obtained, whereas actual average refers to the average path length obtained through the algorithm in Table 1.

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BanyanNet and ShuffleNet
$(N = 2^m \times k,\ m = 4)$

Figure 6: *Path Length Distribution for BanyanNet and ShuffleNet*

Obviously, this ratio is upperbounded by one. When it approaches unity, the algorithm provides path length close to optimal. From this figure, the routing performance increases with decreasing m and increasing k. Figure 5 depicts the path length distribution for $N = 2^m \times k$ BanyanNet with $m = 8$ and $k = m, 5m$. The y-axis shows the probability of a path with length x. This value is calculated by dividing the total number of path with length x by $N - 1$, where N is the number of nodes. The curve labelled with *"Actual"* refers to the result from the routing algorithm; whereas the label *"Optimal"* refers to shortest path distribution. When $k = m$, the actual distribution is shifted towards the right of the optimal distribution, accounting for its higher average length. But when k increases, the actual distribution approaches that of the optimal as in the case of $k = 5m$.

However, despite the non-optimal routing algorithm, BanyanNet still possesses superior path length distribution than the ShuffleNet. Figure 6 illustrates the path length distribution of the two networks for $m = 4$ and $k = 2m, 5m$. In the case of the Banyan-Net, the distribution from the routing algorithm is used. We have also investigated the case for $m = 8$ and found that it is similar to $m = 4$, except for a bigger difference between the two networks. We conclude that the superiority of BanyanNet increases with the network size. Performance comparison between the two networks is further discussed in the next section.

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4 Performance Evaluation and Comparison

In this section, we compare the performance of uni-directional ShuffleNet with bi-directional BanyanNet. Analogous to the work in [9] and [10], the performance attributes considered are channel efficiency, η; network throughput, C; and user throughput, c. Assuming the traffic load is uniformly distributed, these attributes are defined as:

$$
\begin{aligned}
\eta &= \frac{1}{\text{E[number of hops]}} \\
C &= \eta\, W \\
c &= \eta\, \omega
\end{aligned}
$$

where W is the total number of channels in the network and ω is the number of channels per user. For an $N = p^m \times k$ ShuffleNet, the expected number of hops can be calculated by considering the number of users, N_h, h hops from the source node, where $h = 1, \ldots, k + m - 1$. More specifically,

$$
N_h = \begin{cases} p^h & \text{if } h = 1, \ldots, m - 1; \\ p^m & \text{if } h = m, \ldots, k - 1; \\ p^m - p^{h-k} & \text{if } h = k, \ldots, k + m - 1. \end{cases}
$$

$$
\begin{aligned}
\text{Expected number of hops} &= \frac{1}{p^m\,k-1}\left[\sum_{h=1}^{m-1} h p^h + \sum_{h=m}^{k-1} p^m h + \sum_{h=k}^{k+m-1} h(p^m - p^{h-k})\right] \\
&= \frac{k\,p^m\,(p-1)\,(k+2m-1) - 2k\,(p^m-1)}{2(p-1)\,(p^m k-1)}
\end{aligned}
$$

Hence channel efficiency for ShuffleNet, η_S

$$
\eta_S = \frac{2(p-1)\,(p^m k - 1)}{k\,p^m\,(p-1)\,(k+2m-1) - 2k\,(p^m-1)}
$$

For an $N = p^m \times k$ ShuffleNet, the total number of channels, the number of channels per user, the total and the user throughput are respectively,

$$
\begin{aligned}
W_S &= k\,p^{m+1} \\
\omega_S &= p \\
C_S &= \eta_S\,k\,p^{m+1} \\
c_S &= \eta_S p
\end{aligned}
\tag{5}
$$

For an $N = p^m \times k$ bi-directional BanyanNet, we do not have a closed form solution for the expected number of hops. Instead, we use the average path length obtained by our computer implementation of the routing algorithm to determine channel efficiency, η_B. For the total number of channels, the number of channels per user, the total and the user throughput, we have

$$
\begin{aligned}
W_B &= 2k\,p^{m+1} \\
\omega_B &= 2p \\
C_B &= \eta_B\,2k\,p^{m+1} \\
c_B &= \eta_B\,2p
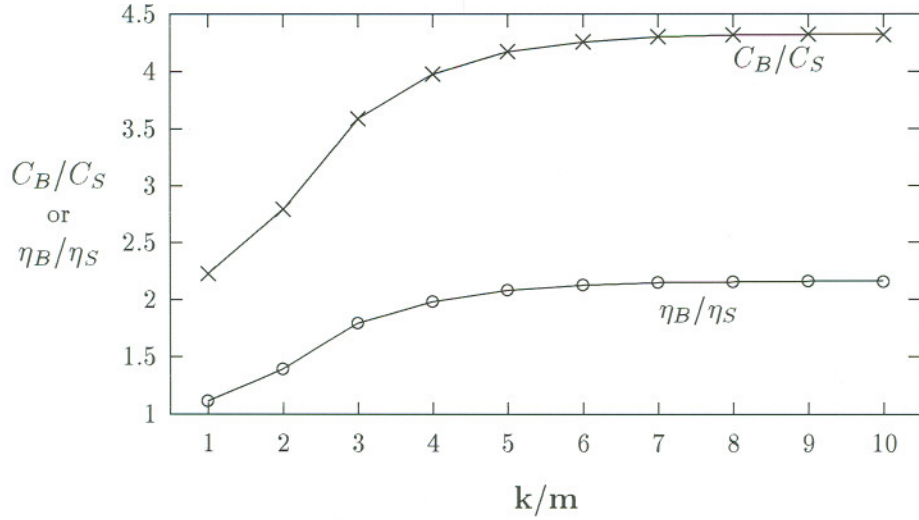\end{aligned}
\tag{6}
$$

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Figure 7: *Performance Comparison of ShuffleNet and BanyanNet*

Assuming a 1-Gb/s user transmission rate, these performance attributes for $N = 2^m \times k$ ShuffleNet and BanyanNet with different values of m and k are plotted in Figures 8 to 13. As a result of a larger number of users, channel efficiency and throughput per user decrease with increasing m and k. The total network throughput, on the other hand, increases with m and k. However, network throughput tends to saturate when $k/m > 5$, suggesting that for maximum throughput, k should not be more than $5m$. This observation agrees with intuition because when k is much larger than m, average number of hops between users increases and messages are routed through a large number of intermediate users.

Obviously, with bi-directional channels, BanyanNet has a better performance than its counterpart. For illustration, we plot the ratios η_B/η_S and C_B/C_S in Figure 7. We observe that the superiority of BanyanNet over ShuffleNet increases with k/m but tends to saturate for $k/m > 5$. For $k/m \geq 5$, $\eta_B \approx 2.25\eta_S$ and $C_B \approx 4.5C_S$. Figure 7 does not consider user throughput because its result resembles that of the the network throughput. In particular, according to equations 5 and 6,

$$\frac{C_B}{C_S} = \frac{c_B}{c_S} = \frac{2\eta_B}{\eta_S}.$$

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5 Conclusions

Wavelength division multiplexed (WDM) systems are useful in the realization of terabit lightwave networks. For WDM systems, there are the *single-hop* and multi-hop approaches. By eliminating the need for pre-transmission communication and wavelength-agile transmitters, the multihop approach is readily implementable [9]. In a multihop network, a user has a small number of transmitters and receivers, each transmitting and receiving signals in a fixed and assigned wavelength. The wavelength assignment is based on a virtual topology. Existing examples include the ShuffleNet, MSN, and hypercube.

An efficient virtual topology implies that a large number of users are connected through a small number of hops. Of the many topologies, the uni-directional ShuffleNet is one of the most popular option [6]. However, its limitations and disadvantages include a restricted number of nodes, $p^m \times m$, and an asymmetric transmission distance between two nodes. For example, if node i connects to node j in one hop, a message from node j to node i take $D_S + 1$ steps, where D_S is the diameter of the network.

To alleviate these problems, our initial approach is to consider a bi-directional ShuffleNet with $p^m \times k$ nodes, where k is a multiple of m. However, a decentralized, bi-directional routing algorithm for ShuffleNet is not obvious. We therefore, turned our attention to the SW-Banyan network, a topology proved to be equivalent to the ShuffleNet but possesses a representation that facilitates bi-directional routing. Analogous to the uni-directional ShuffleNet, we called the resultant network, *BanyanNet*.

In general, a BanyanNet has $p^m \times k$ nodes and each user has p transmitters and receivers. Because of its specific representation, it can be considered as p^m interconnected rings of k nodes. Exploiting this observation, we developed a decentralized, bi-directional routing algorithm for BanyanNet with $p = 2$. This algorithm can be generalized to any value of p. Furthermore, for $p > 2$, multiple users can be assigned to one channel and thus reducing the number of required transmitters and channels [7]. However, in this case, multiple access problems and possible inefficiencies may arise.

For $p = 2$, we show that a bidirectional $N = 2^m \times k$ (k is a multiple of m) BanyanNet has diameter $D_B = m + \lfloor m/2 \rfloor$ for $k = m$ and $D_B = \max(2m, \lfloor k/2 \rfloor)$ for $k > m$. On the other hand, for an $N = 2^m \times k$ (k is a multiple of m) unidirectional ShuffleNet, its diameter $D_S = k + m - 1$. Plotting the ratio of D_S/D_B versus k/m, we found that $D_S > D_B$ and $D_S \approx 2.4 D_B$ when $k = 4$ or 5 is the optimal ratio.

We further evaluate performance of the two networks in terms of *channel efficiency* η_S, η_B, network and user *throughput* C_S, c_S, C_B, c_B. We found that both have similar behavior. In particular, channel efficiency and user throughput decrease with increasing m and k/m. Network throughput, on the other hand, increases with m and k/m but tends to saturate for $k/m > 5$. For comparison we also plot η_B/η_S and C_B/C_S versus

k/m. Obviously, $\eta_B > \eta_S$ and $C_B > C_S$ for all cases, and the superiority of BanyanNet increases with k/m but tends to saturates for $k/m > 5$.

In conclusion, BanyanNet has superior performance compared to the ShuffleNet. However, this is achieved at the higher cost of bi-directional links and more complex routing algorithm. Finally, as it has been noted in [8], we emphasize that there is no ideal universal topology. An efficient topology is application dependent and is subject to various physical and economical constraints. One of the purposes of this paper is to suggest BanyanNet as an alternate design in pursuit of terabit lightwave networks.

References

[1] P.S. Henry. "Very-High Capacity Lightwave Networks". In *Proceedings, International Communication Conference*, pages 1206–1209, June 1988.

[2] J. Stone and L.W. Stulz. "Pigtailed High-Finess Tunable Fibre Fabry-Perot Interferometers with Large, Medium and Small Free Spectral Ranges". *Electronics Letter*, 23:781–783, July 1987.

[3] P.R. Prucnal, M.A. Santoro, and S.K. Sehgal. "Ultrafast All-Optical Synchronous Multiple Access Fiber Networks". *IEEE Journal on Selected Areas of Communications*, pages 1484–1493, December 1986.

[4] C.A. Brackett. "Dense Wavelength Division Multiplexing Networks: Principles and Applications". *IEEE Journal of Selected Areas on Communications*, 8:948–964, August 1990.

[5] B. Mukherejee. "WDM-Based Local Lightwave Networks–Part I: Single-Hop Systems". *IEEE Network*, 6:12–26, May 1992.

[6] B. Mukherejee. "WDM-Based Local Lightwave Networks–Part II: Multi-Hop Systems". *IEEE Network*, 6:20–32, July 1992.

[7] M.G. Hluchyj and M.J. Karol. "ShuffleNet: An Application of Generalized Perfect Shuffles to Multihop Lightwave Networks". *Journal of Lightwave Technology*, 9(10), October 1991.

[8] B.Li and A. Ganz. "Virtual Topologies for WDM Star LANs - The Regular Structures Approach". In *Proceedings of the IEEE INFOCOM'92*, pages 2134–2143, May 1992.

[9] A.S. Acampora, M.J. Karol, and M.G. Hluchyj. "Terabit Lightwave Networks: The Multihop Approach". *AT & T Technical Journal*, 66(4):21–34, November 1987.

[10] M.G. Hluchyj and M.J. Karol. "ShuffleNet: An Application of Generalized Perfect Shuffles to Multihop Lightwave Networks". In *INFOCOM 1988*, New Orleans, Louisiana, March 1988.

[11] P.W. Dowd. "Wavelength Division Multiple Access Channel Hypercube Processor Interconnection'. *IEEE Transactions on Computers*, 41(10):1223–1241, October 1992.

[12] K. Sivarajan and R. Ramaswami. "Multihop Lightwave Networks Based on de Bruijn Graphs". In *Proceedings of the IEEE INFOCOM'91*, pages 1001–1011, April 1991.

[13] N.F. Maxemchuk. "Regular Mesh Topologies in Local and Metropolitan Area Networks. *AT & T Technical Journal*, 64(7):1659–1685, September 1985.

[14] E. Ayanoglu. "Signal Flow Graphs for Path Enumeration and Deflection Routing Analysis in Multihop Networks". In *Proceedings of the IEEE GLOBECOM'89*, November 1989.

[15] D.H. Lawrie. "Access and Alignment of Data in an Array Processor". *IEEE Transactions on Computers*, 24(12):1145–1155, 1975.

[16] H.S. Stone. "Parallel Processing with the Perfect Shuffle". *IEEE Transactions on Computers*, 20:153–161, February 1971.

[17] C. Berge. *The Theory of Graphs*. John Wiley & Sons, New York, 1972.

[18] L.R. Goke and G.J. Lipovski. "Banyan Networks For Partitioning Multiprocessor Systems". In *Proceedings of the First Annual Computer Architecture Conference*, pages 21–28, 1973.

[19] C.L. Wu and T.Y. Feng. "On a Class of Multistage Interconnection Networks". *IEEE Transactions on Computers*, 29(8):694–702, August 1980.

[20] L.R. Goke. *Banyan Networks For Partitioning Multiprocessor Systems*. PhD thesis, University of Florida, Gainesville, Florida, 1976.

[21] Howard Jay Siegel. *Interconnection Networks for Large-Scale Parallel Processing*. Lexington Books, Massachusetts, 1985.

[22] Abdou Youssef and Bruce W. Arden. "Equivalence Between Functionality and Topology of Log N-Stage Banyan Networks". *IEEE Transactions on Computers*, 39(6):829–832, June 1990.

[23] Abdou Youssef. *Properties of Multistage Interconnection Networks*. PhD thesis, Computer Science Department, Princeton University, Princeton, New Jersey, 1988.

Figure 8: *Channel Efficiency for ShuffleNet*

Figure 9: *Network Throughput for ShuffleNet*

ShuffleNet

Figure 10: *User Throughput for ShuffleNet*

BanyanNet ($N = 2^m \times k$)

Figure 11: *Channel Efficiency for BanyanNet*

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Figure 12: *Network Throughput for BanyanNet*

Figure 13: *User Throughput for BanyanNet*