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Performance Analysis of Micro-Cellular Communication Systems with Hierarchically Overlaying Macro-Cells

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Abstract

A hierarchical overlaying scheme suitable for high-capacity micro-cellular communications systems is considered as a strategy to achieve high system performance and broad coverage. High teletraffic areas are covered by microcells while overlaying macrocells cover low teletraffic areas and provide overflow groups of channels for clusters of microcells. In both microcell and macrocell hierarchies, hand-off calls are given priority access to channels. The layout has inherent load-balancing capability, so spatial teletraffic variations are accommodated without the need for elaborate coordination of base stations (wireless gateways). The structure is that of a hierarchical overflow system, in which the microcells receive Poisson input streams, whereas overlaying macrocells receive a Poisson input stream in addition to non-Poisson overflow traffic components from subordinate microcells. An analytical model for teletraffic performance (including hand-off) in such an environment is developed. Theoretical performance characteristics are calculated and presented. These show the carried traffic, blocking, hand-off failure, and forced termination probabilities. Effects of nonuniform teletraffic demand profiles and channel allocation strategies on system performance are discussed.

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I Introduction

As cellular communications mature the use of alternative layout schemes to enhance coverage, capacity, or performance becomes increasingly important [1]. In this paper, a hierarchically overlaying cell layout is considered. The layout provides coverage for high teletraffic areas with as many microcells as required. Overlaying macrocells cover low teletraffic areas and provide *overflow* channels for overlaid microcells. In both microcell and macrocell hierarchies, hand-off calls are given priority access to channels. Unlike the DCA scheme [2]- [3], *call overflow* to the next higher layer of the system is used instead of *channel borrowing* from a channel pool. Since channel overflow does not require the Carrier-to-Interference Ratio (CIR) calculation (or estimation) nor complicated gateway coordinations, the scheme can efficiently accommodate teletraffic fluctuations and is easy to implement. Related work includes: [4], which considers an overlaid architecture for an early proposed personal communication system; [5], which considers techniques to reduce the forced termination of calls in progress in microcellular personal communication networks, including deploying a macrocell to overlay a microcellular cluster; [6], which considers the teletraffic performance of highway microcellular systems with an overlay macrocell that spans many microcells; and [7], which considers the multiple access options for the support of personal communication systems with hierarchical cell structure.

Initially we consider the system to be operated in a hierarchical fashion so that a communicating platform served by a cell that is higher in hierarchy will not request hand-off from a cell that is lower in hierarchy. The structure is that of a hierarchical overflow system, in which the microcells receive Poisson input streams, whereas overlaying macrocells receive a Poisson input stream in addition to non-Poisson overflow traffic components from subordinate microcells. The architecture and mathematical structure suggest a traffic model and analysis using *equivalent random method*, which has been widely used in teletraffic engineering studies of systems where overflow trunks are employed [8]-[9]. However, this analytical approach is not directly applicable for the architecture proposed here because the presence of hand-off traffic components, render the available (equivalent random method) formulation inapplicable. As an alternative, we develop an analytical model for teletraffic performance of the proposed architecture using multi-dimensional birth-death processes [10]. We define a suitable set of state variables to characterize the system and develop a set of simultaneous equations for the (statistical equilibrium) state probabilities. The set of resultant simultaneous state equations are solved for the *exact* equilibrium state probabilities, rather than for an *approximate* fitting distribution of combined overflow traffic. In this way we can consider the hand-off traffic with cut-off priority, hierarchical control mechanisms, and important performance measures in addition to blocking and carried traffic.

The hand-off problem arises in a cellular communications system when a communicating platform moves from a region served by one wireless gateway to a region served by another. Analytical models that characterize the hand-off problem in cellular communications system were presented in [11]. Some subsequent work include [12]-[13], which consider the multiple-call hand-off problems in micro-cellular communications systems and [13]-[15], which consider the hand-off problems for cellular systems with mixed platform types. A general methodology for teletraffic performance analysis of cellular communications networks with hand-off was presented in [13]. The approach uses multi-dimensional birth-death processes to characterize system states.

In this paper, the framework is extended to cellular architectures with hierarchically overlaying macrocells. Theoretical performance indices that show carried traffic, blocking probability, hand-off failure probability, and forced termination probability are derived. Effects of nonuniform teletraffic demand distributions and channel allocation strategies on system performance are discussed. Extensions to non-homogeneous systems and to cellular hierarchical architectures with multiple levels of overlaying cell are also discussed.

II System Description

To the system under discussion, we adopt the terminology put forth in [12] but continue in a style which allows the present paper to be self-contained. Figure 1 depicts a large geographical region tessellated by cells, refered to as *macrocells*, each of which overlays several *microcells*. The microcells in turn cover areas of denser telecommunication traffic. The region is traversed randomly by a large number of mobile platforms. The term *communicating platform* refers to a mobile platform that has at least one call in progress.

When a communicating platform crosses a cell boundary, the cell in which it is currently served is termed the *source cell*, and the cell into which it moves is the *target cell*. The system can sense the occurrence of platform cell boundary crossing by utilizing signal power measurements from nearby base stations (gateways). Cell boundaries are not sharply defined but system operation and our models implicitly assume that a decision directed procedure for hand-off is in place. The problem of hand-off initiation has received some attention in the literatures [16]- [18] and is another aspect of the hand-off problem. The focus here is on the assignment and availability of communication resources. An approach to combine these aspects of hand-off is discussed in [15]. On the basis of positive hand-off initiation decision, a *hand-off attempt* will be generated. That is, the system would attempt to provide a link in the target cell. If this can not be achieved, the call will be *forced* into *termination*. The effect perceived by a user in this circumstance is a *call interruption*.

Since this effect is very obtrusive, we assume that hand-off calls are given priority access to channels in the following way. Each cell reserves a certain number, C_h, of the total channels, C, allocated to it. Specific channels are not reserved, just a given number. A new call that originates in the cell can use any idle channel if fewer than $C - C_h$ channels are in use in the cell at the time of origination. If $C - C_h$ or more channels are in use, a new call is blocked. A hand-off call, on the other hand, can use any idle channel of the C channels in the cell. In this way, hand-off calls have potential access to more channels than do new calls. Of course, other priority schemes

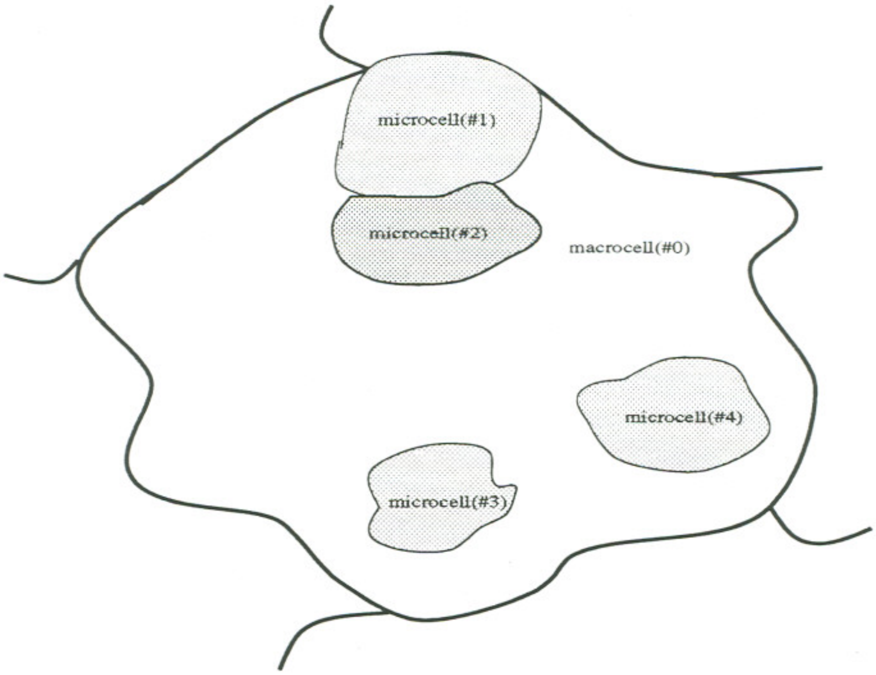

Figure 1: A typical *macro-area*: microcells within an overlaying macrocell.

are also posssible.

To illustrate system operation with hierarchically overlaying architecture, we consider a typical overlaying macrocell as shown in Fig. 1. The overlaying macrocell, denoted cell 0 for convenience, has the coverage area bounded by the outermost closed contour. The overlaid microcells, denoted respectively as cell 1, cell 2, ..., cell N, are within the coverage area of the overlaying macrocell and have their respective coverage areas shaded as shown in Fig. 1. For clarity, the *gateway* of overlaying macrocell (cell 0) is called *macrocell* throughout this paper, whereas the coverage area of an overlaying macrocell, *including cell 0, ..., cell* N, is termed *macro-area.* Within the macro-area, the area outside microcells (cell 1, 2, ..., N) is the region that is served only by the macrocell (cell 0). Hence, it is called the *macrocell-only region.* Note that a *macrocell channel* is accessible by the macrocell (cell 0) and microcells (cell 1, 2, ..., N) throughout the *macro-area.*

We consider a fixed channel assignment scheme. Cell i is allocated a number of channels, C_i, of which C_{hi} channels are reserved for priority access of hand-off calls as previous described. The gateway serving each cell (macrocells and microcells) are linked to the fixed network.

We now focus attention on a typical *macro-area* as shown in Fig. 1 and consider the system operation.

(i) A new call that originates in a microcell, say the i_{th} microcell, $(i = 1, 2, \ldots, N)$, will be served by the i_{th} microcell if the number of channels in use in the i_{th} microcell is fewer than $C_i - C_{hi}$ at the time of origination; otherwise, it will overflow to its overlaying macrocell, cell 0. The overflowed new call will be accommodated by the macrocell if the channels in use in the macrocell is fewer than $C_0 - C_{h0}$ at the time of overflow; otherwise, it will be blocked and cleared from the system.

(ii) A new call that originates in a region served only by the macrocell, *macrocell-only region*, will be served by the macrocell if the number of channels in use in the macrocell is fewer than $C_0 - C_{h0}$ at the time of origination; otherwise, it will be blocked and cleared from the system.

(iii) If a hand-off from microcell i to another microcell j occurs, it generates a handoff attempt to target cell j. This hand-off call will be accommodated by microcell j if there is any idle channel in microcell j at the time of hand-off call arrival; otherwise, it will overflow to the macrocell. The overflowed hand-off attempt will be accommodated by the macrocell if there is any idle channel in the macrocell at the time of overflow; otherwise, this hand-off attempt will fail and the call will be forced to terminate.

(iv) If a hand-off from a microcell to the macrocell-only region arises, the hand-off call will be served by the target macrocell if it has any idle channel at the time of hand-off call arrival. Otherwise, this hand-off attempt will fail and the call will be forced to terminate.

(v) In the present paper it is assumed that if a hand-off arrival from the macrocell-only region to a microcell is sensed, no action will be taken for such an event. This is due to the hierarchical control mechanism we envision, in which once a communicating platform is served at a level that is higher in the hierarchy, it will not return the higher level channels and be served at a lower level, if any. It should be noted that since a *macrocell channel* can be used throughout the *macro-area*, there is no hand-off attempt generated with this event. However, this simple operation may be at the expense of bandwidth utilization.

(vi) If a hand-off from an adjacent *macro-area* arises, regardless of the specific region on which the communicating platform impinges, the hand-off attempt is directed to the target macrocell. The hand-off attempt will be accommodated by the target macrocell if it has any idle channel at the time of arrival. Otherwise, the hand-off attempt will fail and the call will be forced to terminate.

We summarize the main principles. The macrocell provides the *primary* group of channels for new calls that originate in regions covered only by the macrocell and the hand-off attempts from its overlaid microcells and from adjacent macro-areas. It also provides the *secondary* group of channels for new calls that can not be served in microcells and for the hand-off attempts between any two overlaid microcells that are in the same macro-area. Priority access for hand-off calls is employed at both the microcell and macrocell levels.

It is important to note that since mobile platforms traverse cells of different sizes, there must be an appropriate bi-directional power control so that the minimum required Carrier-to-Interference ratio (CIR) can be maintained regardless of the gateway which serves the platform. In principle, this is not difficult since channels used by macrocells level are not used at other levels in the hierarchy. So an appropriate channel reuse pattern can be used at each level in a manner similar to that described in [4].

III Mathematical Model

III.1 Preliminaries and Assumptions

The overall cellular system consists of many *macro-areas* each of which contains an overlaying macrocell and N overlaid microcells. We assume that the overall cellular system is *homogeneous* in the sense that every *macro-area* has the same parameters for the underlying processes. That is, all *macro-areas* are statistically identical. Thus, as in [12], analyze the overall system by focusing on a given *macro-area*, and consider the statistical behavior of this given *macro-area* under the conditions that the neighboring *macro-areas* exhibit their typical random behavior independently. In addition, all mobile platforms are assumed to be statistically identical and that each can support at most one call.

The number of noncommunicating platforms in cell i, that are equipped to access the cellular system, is assumed to be much larger than the number of channels, C_i, allocated to the cell. The amount of time that a communicating platform remains *within the communications range* of a given gateway is characterized by a *dwell time*. Clearly, the dwell time is a random variable whose parameters depend on many factors such as cell radius, platform mobility, and the path a platform follows, etc. [15]

The following assumptions regarding memoryless properties allow the problem to be cast in the framework of multi-dimensional birth-death processes. Similar assumptions have served telecommunications traffic engineering well for many years.

(1) The new call arrival processes in any cell follow Poisson point processes. The parameters of the processes are determined by the number of noncommunicating platforms and average user activity. The mean new call arrival rates for cell 0, cell 1, ..., cell N, are denoted Λ_0, Λ_1, Λ_2, ..., and Λ_N, respectively.

(2) The hand-off call arrival process from neighboring *macro-areas* is a Poisson point process.

(3) The dwell time, T_{Di}, of a communicating platform in a cell i, is a random variable having a negative exponential probability density function. The mean of T_{Di}

is $\overline{T}_{Di} = \mu_{Di}^{-1}$.

(4) The *unencumbered session duration* of a call is the amount of time that the call would remain in progress if it could continue to completion *without forced termination by hand-off failure*. The unencumbered session duration, T, is a random variable having a negative exponential probability density function. The mean of T is $\overline{T} = \mu^{-1}$.

III.2 Identification of System States and Driving Processes

We now focus attention on a given *macro-area*. The macro-area under observation can be characterized as being, at any given instant, in any one of a finite number of states. A state is defined by a sequence of nonnegative integers; $v_0, v_1, v_2, \ldots, v_N$, where $v_i, (i = 0, 1, 2, \ldots, N)$, is the number of *communicating platforms* in the i_{th} cell, and N is the number of *microcells* overlaid by the *macrocell* (cell 0). It was found convenient to enumerate and order the states. Each state is assigned an integer index, s, ranging from 0 to s_{max}, where s_{max} is determined so that all possible states are accommodated. Thus, state s corresponds to a distinct sequence of nonnegative integers

$$v_0(s), v_1(s), v_2(s), \ldots, v_N(s) \ ,$$

where $v_i(s)$ is the number of communicating platforms in the i_{th} cell when the macro-area under observation is in state s. Therefore, the total number of communicating platforms in a given *macro-area* in state s is

$$v(s) = \sum_{i=0}^{N} v_i(s) \ . \tag{1}$$

Because in the present case all platforms are assumed to be able to support at most one call, the number of channels in use in the i_{th} cell is also $v_i(s)$. Thus, *permissible* states correspond to those sequences of integers

$$v_0(s), v_1(s), v_2(s), \ldots, v_N(s) \ ,$$

for which, for $i = 0,\ 1, \ldots,\ N$,

$$0 \ \leq \ v_i(s) \ \leq \ C_i.$$

Other constraints can also be included [12].

As time progresses, the *macro-area* under observation changes state at random instants which result from the driving processes and the system dynamics. The underlying random processes that drive the system are:

(1) Generation of new calls in the *macrocell-only region.*

(2) Generation of new calls in overlaid microcells.

(3) Completion of calls in the *macro-area* under observation.

(4) Hand-off arrivals to the *macro-area* from neighboring macro-areas.

(5) Hand-off arrivals to the *macrocell-only region* from overlaid microcells.

(6) Hand-off departures from the macro-area under observation.

(7) Hand-off arrivals to a *microcell* from adjacent microcells in the same macro-area.

Any state transition, must be caused by one of the events listed above. In particular, the hand-off departure events affect the system dynamics in a way depending on the geographic configuration. To characterize the *teletraffic geography* among the cells within a macro-area, a *teletraffic flow matrix* is defined by:

$$\mathcal{A} = \begin{pmatrix} \alpha_{10} & \alpha_{11} & \alpha_{12} & \dots & \alpha_{1N} & \alpha_{1D'} \\ \alpha_{20} & \alpha_{21} & \alpha_{22} & \dots & \alpha_{2N} & \alpha_{2D'} \\ \alpha_{30} & \alpha_{31} & \alpha_{32} & \dots & \alpha_{3N} & \alpha_{3D'} \\ \vdots & \vdots & & \ddots & & \vdots \\ \vdots & \vdots & & & \ddots & \vdots \\ \alpha_{N0} & \alpha_{N1} & \alpha_{N2} & \dots & \alpha_{NN} & \alpha_{ND'} \end{pmatrix}, \tag{2}$$

in which the array element, $\alpha_{ij}, i \neq j$, represents the average fraction of hand-off departures from microcell i that *flow* into (towards) the service area of cell j. For $i = j$, the α_{ij} is defined as zero. Note that the macrocell is numbered '0' and the neighboring macro-areas is denoted as D'.

Note that, since a communicating platform served by the macrocell will not request a hand-off within its macro-area, a platform served by the macrocell requires a hand-off only if it is departing towards adjacent macro-areas. Thus, no row in $\mathcal{A}$ is dedicated to the flow of hand-off departures from the macrocell, cell 0. The teletraffic flow matrix can be reasonably determined based on the geographic configuration, highway distribution, and some other related survey data. For the present purpose, these parameters are assumed to be known and given.

III.3 Flow Balance Equations

After defining the *system states* and identifying the *driving processes*, the statistical equilibrium state probabilities remain to be determined. This can be done by writing a flow balance equation for each state, and then solve a set of resultant $s_{max}+1$ simultaneous equations for the unknown state probabilities, $p(s)$. The $s_{max}+1$ simultaneous equations are of the form:

$$\sum_{j=0}^{s_{max}} q(i,j)p(j) = 0 , \qquad i = 0, 1, 2, \ldots, s_{max} \tag{3}$$

$$\sum_{j=0}^{s_{max}} p(j) = 1 , \tag{4}$$

in which, for $i \neq j$, coefficients $q(i,j)$ represents the net transition rate into state i from predecessor state j, and $q(i,i)$ is the total transition rate out of state i. The rate into a state is assumed positive in this paper. Solution of the equations follows the approach outlined in [12], [13], and [14].

III.3.1 Probability Flow Into a State

The coefficients of flow balance equations, $q(i,j)$, are determined by the underlying driving processes as in [12]-[14]. Transitions into a given state s arise from several other states j, depending on the driving process that causes the transition. Figure 2 illustrates the transitions into state s, due to each driving process, for a hierarchically overlaying system that has 3 microcells in a typical macro-area. States embraced with an arc are *possible predecessors* of state s due to the driving process(es) shown. However, a *predecessor* state must also be a *permissible* state. That is, all channel, platform limit and quota constraints must be met [12]. Because of hand-offs there is coupling between some of the driving processes. The determination of permissible predecessors and transition rates for a given state s is to be discussed next.

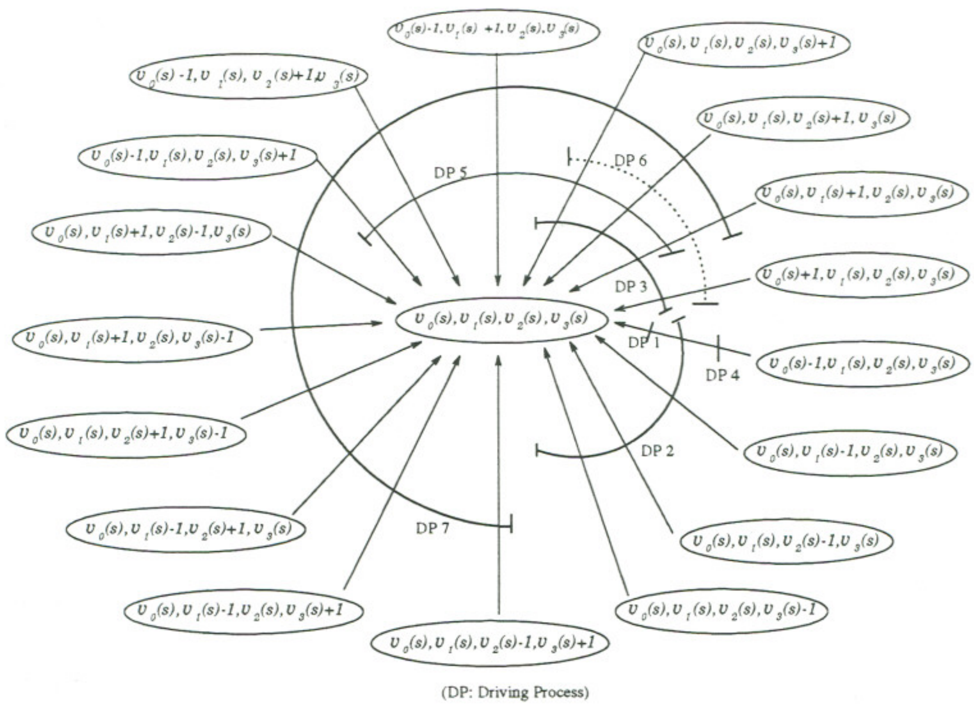

Figure 2: The state transition diagram for a hierarchically overlaying system whose typical macro-area has 3 overlaid microcells. The driving processes $DP1$, $DP2$, ..., $DP7$ are defined in Sec. III.2.

(1) *Flow In Due to New Call Originations in the Macrocell-Only Region*

Consider the given *macro-area* is in state s, corresponding to $v_0(s), v_1(s), \ldots, v_N(s)$. The transitions into state s due to the origination of new calls in the macrocell-only region are possible only from certain other permitted states. Specifically, let $q_1(s,j)$ denote the transition rate from state j to state s *due to new calls originating in the macrocell-only region* when the macro-area is in state j. The predecessor due to this event is identified as a state corresponding to the following sequence:

$$j: \quad \underbrace{v_0(s)-1}, \; v_1(s), \; v_2(s), \ldots, \; v_{N-1}(s), \; v_N(s) \quad , \tag{5}$$

if the condition $0 < v_0(s) \leq (C_0 - C_{h0})$ holds. The transition rate for this predecessor is the new call origination rate in this macrocell-only region, Λ_0. Therefore, the rate of transition into state s, due to new call originations in macrocell-only region, can be written as

$$q_1(s,j) = \begin{cases} \Lambda_0, & \text{if } 0 < v_0(s) \leq C_0 - C_{h0} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

(2) *Flow In Due to New Call Originations in Microcells*

A new call which originates in the i_{th} microcell will be accommodated in the i_{th} microcell if the number of channels in use in the microcell is strictly less than $(C_i - C_{hi})$. It will be accommodated by the macrocell if the number of channels in use in the microcell is $(C_i - C_{hi})$ or more *and* that in the macrocell is strictly less than $(C_0 - C_{h0})$. Let $q_{2i}(s,j)$ denote the rate of transition from state j to state s, *due to new call originations in the i_{th} microcell* when the macro-area is in state j. Thus, given a current state s and a new call origination in microcell i, a permissible j, $(j = 0, 1, \ldots, s_{max})$, is a predecessor state of s if the state variables of j are

$$j: \quad v_0(s), \; v_1(s), \ldots, \; v_{i-1}(s), \; \underbrace{v_i(s)-1}, \; v_{i+1}(s), \ldots, \; v_N(s) \tag{7}$$

and the condition $0 < v_i(s) \leq (C_i - C_{hi})$ holds. On the conditions $v_i(s) > (C_i - C_{hi})$ *and* $0 < v_0(s) \leq (C_0 - C_{h0})$, a different permissible predecessor state is identified as

$$j: \quad \underbrace{v_0(s)-1}, \; v_1(s), \; \ldots, \; v_i(s), \ldots, \; v_{N-1}(s), \; v_N(s) \tag{8}$$

The rate of transition into state s, due to new call originations in the i_{th} microcell, is summarized as follows:

$$q_{2i}(s,j) = \begin{cases} \Lambda_i, & \text{if } 0 < v_i(s) \leq (C_i - C_{hi}) \\ \Lambda_i, & \text{if } v_i(s) > (C_i - C_{hi}) \text{ and } 0 < v_0(s) \leq (C_0 - C_{h0}) \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $i = 1, 2, 3, \ldots, N$, and j corresponds to either (7) or (8), depending on the channels usage in the i_{th} microcell and the macrocell.

(3) *Flow In Due to Call Completions in the Macro-Area Under Observation*

Let $q_{3i}(s,j)$ denote the rate of transition from state j to state s, *due to completion of calls being served by the i_{th} cell* when the macro-area is in state j. Thus, a permissible j, $(j = 0, 1, \ldots, s_{max})$, is a predecessor state of s if one state variable of j has *exactly* one more communicating platform than in the state s. That is,

$$j: \quad v_0(s),\ v_1(s), \ldots,\ v_{i-1}(s),\ \underbrace{v_i(s)+1},\ v_{i+1}(s), \ldots,\ v_N(s)\ , \tag{10}$$

in which $v_i(s) < C_i$. The corresponding transition rate, $q_{3i}(s,j)$, is the call completion rate in the i_{th} cell while in state j. In state j, there are exactly $v_i(s)+1$ calls served by channels assigned to the i_{th} cell. Since the unencumbered session duration, T, has a negative exponential probability density function with a mean of $\overline{T} = \mu^{-1}$, the completion rate, and thus the transition rate from state j to state s is given by

$$q_{3i}(s,j) = \begin{cases} \mu \cdot (v_i(s)+1), & \text{if } v_i(s) < C_i \\ 0, & \text{if } v_i(s) = C_i \end{cases} \tag{11}$$

where $i = 0, 1, 2, 3, \ldots, N$.

(4) *Flow In Due to Hand-Off Arrivals to the Macro-Area From Adjacent Macro-Areas*

Let $q_4(s,j)$ denote the rate of transition from state j to state s, due to communicating platforms served by adjacent macro-areas entering the macro-area under observation. Since it was assumed that the overall system is homogeneous in the sense that all *macro-areas* are statistically identical, in statistical equilibrium, the average rate of hand-off arrivals from adjacent macro-areas, Λ_h, must be equal to the average rate of hand-off departures towards adjacent macro-areas, Δ_h. For the formulation of flow balance equations here, it is temporarily assumed that Λ_h is known and given. We will subsequently show how to determine this parameter.

Recall that the hand-off arrivals from adjacent macro-areas, impinging on the macro-area under observation, will be accommodated by the macrocell if it has any idle channel. Thus, only those states with one less communicating platform in the macrocell than in state s can cause a transition into state s. Therefore, the predecessor j, due to this driving process, is a state corresponding to the sequence:

$$j: \quad \underbrace{v_0(s)-1}, \; v_1(s), \; v_2(s), \ldots, \; v_{N-1}(s), \; v_N(s) \quad , \tag{12}$$

if $v_0(s) > 0$. The transition rate is the average rate of hand-off arrivals from adjacent macro-areas, Λ_h, written as

$$q_4(s,j) = \begin{cases} \Lambda_h, & \text{if } v_0(s) > 0 \\ 0, & \text{if } v_0(s) = 0 \end{cases} \tag{13}$$

(5) *Flow In Due to Hand-Off Arrivals to the Macrocell-Only Region From Overlaid Microcells*

The hand-off arrivals to the *macrocell-only region* from the i_{th} microcell will be accommodated by the *macrocell* if the macrocell has any idle channel. Otherwise, the hand-off attempt will fail. For the successful hand-off scenario, the number of communicating platforms in the macrocell, $v_0(s)$, is increased by one whereas the $v_i(s)$ is decreased by one. Thus, given a state s and under current driving process, a permissible predecessor j is a state with one more communicating platform in microcell i and one less communicating platform in the macrocell than in state s, written as

$$j: \quad \underbrace{v_0(s)-1}, \; v_1(s), \ldots, \; v_{i-1}(s), \; \underbrace{v_i(s)+1}, \; v_{i+1}(s), \ldots, \; v_N(s) \quad , \tag{14}$$

if $v_i(s) < C_i$ and $0 < v_0(s) \leq C_0$. The transition rate, denoted $q_{5i}(s,j)$, is the rate of hand-off departures from the i_{th} microcell towards the macrocell. Recall that the dwell time of a communicating platform in the i_{th} microcell is assumed to have a negative exponential probability density function with a mean of μ_{Di}^{-1}, and the fraction of departing platforms from the i_{th} microcell towards the macrocell is α_{i0}. Thus, the transition rates are given as

$$q_{5i}(s,j) = \begin{cases} \mu_{Di} \cdot (v_i(s)+1) \cdot \alpha_{i0}, & \text{if } v_i(s) < C_i \text{ and } 0 < v_0(s) \leq C_0 \\ 0, & \text{if } v_i(s) = C_i \text{ or } v_0(s) = 0 \end{cases} \tag{15}$$

If the given state s has $v_0(s) = C_0$, the number of communicating platforms in the macrocell stay unchanged, and a failed hand-off results. Another predecessor j for such a boundary state s is thus identified as a state corresponding to

$$j: \quad v_0(s),\ v_1(s), \ldots,\ v_{i-1}(s),\ \underbrace{v_i(s)+1},\ v_{i+1}(s), \ldots,\ v_N(s)\ , \tag{16}$$

if $v_i(s) < C_i$ and $v_0(s) = C_0$. The transition rate for this predecessor state j is the same as (15) but subject to different conditions as shown below.

$$q_{5i}(s,j) = \begin{cases} \mu_{Di} \cdot (v_i(s)+1) \cdot \alpha_{i0}, & \text{if } v_i(s) < C_i \text{ and } v_0(s) = C_0 \\ 0, & \text{if } v_i(s) = C_i \text{ or } v_0(s) = 0 \end{cases} \tag{17}$$

where $i = 1,\ 2,\ 3, \ldots,\ N$.

Note that there will be two different predecessors for the *boundary* state s with $v_0(s) = C_0$, since both scenarios are valid predecessors for such a state. These two identical transition rates can not be simply added up since they correspond to two *different* predecessors.

(6) *Flow In Due to Hand-Off Departure From the Macro-Area Under Observation Towards Adjacent Macro-Areas*

Let $q_{6i}(s,j)$ denote the rate of transition from state j to state s, due to the communicating platforms served by the channels assigned to the i_{th} cell, $(i = 0,\ 1,\ 2, \ldots,\ N)$, entering adjacent macro-areas. Since a hand-off departure from this macro-area towards adjacent macro-areas implies that the predecessor, due to this driving process, must have *exactly* one more communicating platform in $v_i(s)$ than that in state s.

Therefore, given a state s, if the state j corresponds to the sequence

$$j: \quad v_0(s),\ v_1(s), \ldots,\ v_{i-1}(s),\ \underbrace{v_i(s)+1},\ v_{i+1}(s), \ldots,\ v_N(s) \tag{18}$$

and $v_i(s) < C_i$, a transition from j to state s will take place when a hand-off departure from the i_{th} cell towards adjacent macro-areas occurs. Since *all* the hand-off departures from the macrocell are towards adjacent macro-areas, the transition rate $q_{60}(s,j)$ is given by

$$q_{60}(s,j) = \begin{cases} \mu_{D0} \cdot (v_0(s)+1), & \text{if } v_0(s) < C_0 \\ 0, & \text{if } v_0(s) = C_0 \end{cases} \tag{19}$$

For $q_{6i}(s,j),\ \ i = 1,\ 2, \ldots,\ N$, the transition rate is only *the fraction* of hand-off departures from the i_{th} microcell that are towards adjacent macro-areas. Hence,

$$q_{6i}(s,j) = \begin{cases} \mu_{Di} \cdot (v_i(s)+1) \cdot \alpha_{iD'}, & \text{if } v_i(s) < C_i \\ 0, & \text{if } v_i(s) = C_i \end{cases} \tag{20}$$

where $i = 1,\ 2,\ 3, \ldots,\ N$.

(7) *Flow In Due to the Hand-Off Arrival in a Microcell from Adjacent Microcells in the Same Macro-Area*

A communicating platform served by the i_{th} microcell entering one of its adjacent microcells that are in the same macro-area, microcell k, would be accommodated by the k_{th} microcell immediately if it has any idle channel. If all the channels in the k_{th} microcell are used and there is any idle channel in the macrocell, the hand-off arrival will be accommodated by the macrocell. Otherwise, this hand-off will be failed. Given a state s and under this driving process, if $v_i(s) < C_i$ and $0 < v_k(s) \leq C_k$, a predecessor j is identified as:

$$j: \quad v_0(s),\ v_1(s), \ldots,\ \underbrace{v_i(s)+1}, \ldots,\ \underbrace{v_k(s)-1}, \ldots,\ v_N(s)\ . \tag{21}$$

That is, a transition of state from j into s is caused due to the immediate accomodation (by microcell k) of a hand-off arrival from microcell i to microcell k. The transition rate is the fraction of hand-off departures from the i_{th} microcell that are towards the k_{th} microcell, given as

$$q_{7ik}(s,j) = \begin{cases} \mu_{Di} \cdot v_i(s) \cdot \alpha_{ik}, & \text{if } v_i(s) < C_i \text{ and } 0 < v_k(s) \leq C_k \\ 0, & \text{otherwise,} \end{cases} \tag{22}$$

where $i, k = 1, 2, \ldots, N$ and $i \neq k$.

Additional predecessors are further identified if the given state s is a boundary state. Specifically, if $v_i(s) < C_i$, $v_k(s) = C_k$, and $0 < v_0(s) \leq C_0$, a transition from state j, given as (23), to state s occurs when a communicating platform served by microcell i enters the k_{th} microcell.

$$j: \quad \underbrace{v_0(s) - 1}, v_1(s), \ldots, \underbrace{v_i(s) + 1}, \ldots, v_k(s), \ldots, v_N(s) \tag{23}$$

If the given state s has $v_i(s) < C_i$, $v_k(s) = C_k$, and $v_0(s) = C_0$, one more predecessor state is found as:

$$j: \quad v_0(s), v_1(s), \ldots, \underbrace{v_i(s) + 1}, \ldots, v_k(s), \ldots, v_N(s) \tag{24}$$

The transition rates from predecessors (23) or (24) into state s are the same as that described in (22), but subject to their respective channel usage condition. Note that for a given state s, it may have one, two, or even three different predecessors, depending on the channel usages in $v_0(s)$, $v_i(s)$, and $v_k(s)$ of state s. Meticulous consideration of boundary states is essential for the correct formulation of flow balance equations, and is paticularly important for the derivation of performance indices.

In summary, the probability flow into a state s is the sum of the probability flow due to each individual driving process that characterizes the scheme, given as

$$\text{Probability flow into state } s = \sum_{j=0, j\neq s}^{s_{max}} q(s,j)p(j)$$

III.3.2 Probability Flow Out of a State

In previous section, the probability flow into a state s, $\sum_{j=0,j\neq s}^{Smax} q(s,j) \cdot p(j)$, is determined by first identifying the predecessors of state s, and then finding the corresponding transition rates for each predecessor. Since $q(k,s)$ is the total transition rate from state s to state k, and because any transition out of a state is a transition into some other state, we must have

$$q(s,s) = -\sum_{k=0,k\neq s}^{s_{max}} q(k,s) \quad . \tag{25}$$

That is, the total transition rate *out* of state s is the sum of the transition rates from s *towards* any other of the states, with reversed direction of flow. Since $q(k,s), k \neq s$, have been determined previously, the calculation of $q(s,s)$ turn out to be a careful pick and add up.

Thus far, the coefficients needed in equation (3) and (4) to form a set of $s_{max}+1$ simultaneous equations for the unknown state probabilities, $p(s)$, are determined. These equations can be solved using any suitable numerical algorithm. Once the equilibrium state probabilities, $p(s), s = 0, 1, \ldots, s_{max}$, are found, the system performance measures cab be determined.

III.4 Determination of the Hand-Off Parameters

In the foregoing analysis, it was assumed that the cellular system is homogeneous in the sense that all *macro-areas* are statistically identical. Thus in statistical equilibrium, the average rate of hand-off arrivals towards a given macro-area from adjacent macro-areas, Λ_h, must be equal to the average rate of hand-off departures from the macro-area under observation towards adjacent macro-areas, Δ_h. The Λ_h was assumed given during the derivation of flow balance equations. We now derive an expression for Δ_h from the system dynamics.

If one observes *the system (a given macro-area)* over a long time, the system under observation changes state at random instants which result from the driving processes and the system dynamics. Among those events that cause a change of state in the system, only a fraction of which are the hand-off departure events, arising from the observing macro-area towards adjacent macro-areas, denoted Δ_h. We denote the *average rate* at which the transition events occur in this given macro-area, as R, and the *fraction* of transition events which are hand-off departures towards adjacent macro-areas, as d. Thus, we have

$$\Delta_h = d \cdot R \ . \tag{26}$$

The average rate at which transition events occur in a given macro-area, R, is given by

$$R = \sum_{s=0}^{s_{max}} |q(s,s)| \cdot p(s), \tag{27}$$

where the $|q(s,s)|$ is the total transition rate *out* of state s when the system is in state s, and $p(s)$ is the equilibrium state probability of state s.

Now, consider the probability that a hand-off departure towards adjacent macro-areas occurs *when the system is in state* s, denoted $d(s)$. This is the probability that the state changes due to a hand-off departure towards adjacent macro-areas when the system is in state s. Recall that when the system is in state s, the total rate of state changes (transitions) is $|q(s,s)|$, from which the components contributed by the hand-off departures towards adjacent macro-areas are $\mu_{D0} \cdot v_0(s) + \sum_{i=1}^{N}(\mu_{Di} \cdot v_i(s) \cdot \alpha_{iD'})$. Therefore,

$$d(s) = \frac{\mu_{D0} \cdot v_0(s) + \sum_{i=1}^{N}(\mu_{Di} \cdot v_i(s) \cdot \alpha_{iD'})}{|q(s,s)|} \ , \tag{28}$$

and the fraction of transition events that are hand-off departures towards adjacent macro-areas regardless of the system state, d, is given by

$$d = \sum_{s=0}^{s_{max}} d(s) \cdot \tilde{p}(s) \ , \tag{29}$$

where $d(s)$ denotes the probability that a hand-off departure towards adjacent macro-areas occurs *when the system is in state* s, and $\tilde{p}(s)$ is the probability that the *system visits state* s.

Note that the $p(s)$, the equilibrium state probabilities found previously from the multi-dimensional birth-death process, physically describe *the fraction of time* that the system spends in state s. The probability that the system *visits* state s, $\tilde{p}(s)$, is however the equilibrium solution of the *"jump"* Markov chain, whose states are those given in multi-dimensional birth-death process [19]. It can be shown that $\tilde{p}(s)$ and

IV Performance Measures

IV.1 Blocking Probability

The blocking probability is the probability that *new call* arrivals are denied access to a channel. Blocking events occur only when the system is in certain states and a new call originates in the system. Since the blocking events may occur in the macrocell and overlaid microcells, the following subsets of states, which together include all states where a blocking event can occur, are identified:

$$\begin{aligned}
B_0 &= \{\, s: \ v_0(s) \geq C_0 - C_{h0} \ \,\} \\
B_1 &= \{\, s: \ v_0(s) \geq C_0 - C_{h0}, \ v_1(s) \geq C_1 - C_{h1} \,\} \\
&\vdots \\
B_i &= \{\, s: \ v_0(s) \geq C_0 - C_{h0}, \ v_i(s) \geq C_i - C_{hi} \,\}, \ i = 1, \ 2, \ldots, N. \\
&\vdots \\
B_N &= \{\, s: \ v_0(s) \geq C_0 - C_{h0}, \ v_N(s) \geq C_N - C_{hN} \,\}\,. \qquad (33)
\end{aligned}$$

If the system is in a state s which belongs to B_0, then a blocking event *must* occur if a new call originates *in the macrocell*, i.e. *macrocell-only region.* Thus, the blocking probability in the macrocell is given by

$$P_{B_0} = \sum_{s \in B_0} p(s). \qquad (34)$$

For *microcell* i, $(i = 1, \ 2, \ldots, N)$, consider the system to be in a state s that belongs to B_i. A blocking event occurs if a new call arises either in *the* i_{th} *microcell* or in *the macrocell.* Note that B_i is a subset of B_0, thus the blocking probability in the macrocell is always greater than that in the i_{th} microcell. The blocking probability in the i_{th} microcell is given as

$$P_{B_i} = \sum_{s \in B_i} p(s), \quad i = 1, \ 2, \ldots, N. \qquad (35)$$

IV.2 Hand-Off Failure Probability

The hand-off failure probability of cell i, P_{H_i}, is the probability that a *hand-off call attempt* impinging on cell i is denied access to a channel. Hand-off failure can occur only when the system is in certain states *and* a hand-off arrival event to cell i occurs. Those states in which hand-off call failures can occur can be grouped into $N+1$ subsets as follows:

$$\begin{aligned}
H_0 &= \{ s: \ v_0(s) = C_0 \ \} \\
H_1 &= \{ s: \ v_0(s) = C_0, \ v_1(s) = C_1 \} \\
\vdots \quad & \quad \vdots \\
H_i &= \{ s: \ v_0(s) = C_0, \ v_i(s) = C_i \} \\
\vdots \quad & \quad i = 1, \ 2, \ldots, N. \\
H_N &= \{ s: \ v_0(s) = C_0, \ v_N(s) = C_N \}. \qquad (36)
\end{aligned}$$

If the system is in a state s that belongs to H_0, any hand-off arrival attempt that impinges on the macrocell will fail because no idle channels are available. If the system is in a state s that belongs to H_i, $i = 1, \ 2, \ldots, \ N$, any hand-off call attempt that impinges on the i_{th} microcell will fail because it can not be served by the i_{th} microcell or the overlaying macrocell.

The probability that the system visits state s is $\tilde{p}(s)$. While the system is in state s, the total transition rate out of state s is $|q(s,s)|$, of which only a fraction of state transitions are caused by hand-off arrivals to the i_{th} cell. Let $\Lambda_{Ti}(s)$ be the hand-off call attempts rate impinging on the i_{th} cell when the system is in state s, in which $i = 0, \ 1, \ldots, \ N$. For a target cell i, since the mean dwell time of a source *microcell* j is μ_{Dj}^{-1} and the number of communicating platforms in cell j is $v_j(s)$ when the system is in state s, thus we obtain, for $i = 1, \ 2, \ldots, \ N$,

$$\Lambda_{Ti}(s) = \sum_{j=1, j \neq i}^{N} \mu_{Dj} \cdot v_j(s) \cdot \alpha_{ji} \ , \qquad (37)$$

where α_{ji} is the fraction of departing platforms from cell j that towards cell i.

For the macrocell ($i = 0$), the hand-off arrivals consists of the hand-offs from microcells, adjacent macro-areas, and the overflowed hand-offs from subordinate microcells. Let $\Lambda_{i0}(s)$ be the hand-off call attempts rate impinging on the macrocell *contributed* by the i_{th} microcell. Therefore, while the system is in state s,

$$\begin{aligned}\Lambda_{i0}(s) &= \mu_{Di} \cdot v_i(s) \cdot \alpha_{i0} + \mu_{Di} \cdot v_i(s) \cdot \alpha_{i1} \cdot b_1(s) + \cdots + \mu_{Di} \cdot v_i(s) \cdot \alpha_{iN} \cdot b_N(s) \\ &= \mu_{Di} \cdot v_i(s) \cdot \alpha_{i0} + \sum_{j=1}^{N} (\mu_{Di} \cdot v_i(s) \cdot \alpha_{ij} \cdot b_j(s)) \end{aligned} \tag{38}$$

in which,

$$b_j(s) = \begin{cases} 1, & \text{if } v_j(s) = C_j \\ 0, & \text{otherwise.} \end{cases} \tag{39}$$

Thus the rate of hand-off call attempts that impinge on the macrocell, $\Lambda_{T0}(s)$, is

$$\Lambda_{T0}(s) = \left(\sum_{i=1}^{N} \Lambda_{i0}(s) \right) + \Lambda_h . \tag{40}$$

The probability that a hand-off arrival event to cell i occurs while the system visits state s can be written as

$$\frac{\Lambda_{Ti}(s)}{|q(s,s)|} .$$

Since the probability that the system visits state s is $\tilde{p}(s)$, therefore,

$$P_{H_i} = \sum_{s \in H_i} \tilde{p}(s) \cdot \frac{\Lambda_{Ti}(s)}{|q(s,s)|} , \tag{41}$$

for $i = 0, 1, 2, \ldots, N$. Using (30), we obtain

$$\begin{aligned} P_{H_i} &= \sum_{s \in H_i} \left(p(s) \cdot \frac{|q(s,s)|}{R} \right) \cdot \frac{\Lambda_{Ti}(s)}{|q(s,s)|} \\ &= \sum_{s \in H_i} p(s) \cdot \frac{\Lambda_{Ti}(s)}{R} . \end{aligned} \tag{42}$$

IV.3 Forced Termination Probability

While the hand-off failure probability, P_{H_i}, gives the probability that a hand-off attempt impinging on cell i will fail, this performance measure does *not* describe the practical impact of alternative schemes and systems on an *individual user*. Suitable performance measures from the users' viewpoint should reflect the possibility that a new call is initially blocked, and the probability that a call, which is not initially blocked, will be maintained by the system to a satisfactory completion. The first performance measure is the *blocking probability* discussed earlier. The *forced termination probability*, denoted P_{FT}, is defined accordingly, as the probability that a call which is not blocked is interrupted due to hand-off failure during its lifetime.

Obviously, the forced termination probability for the system architecture under discussion is strongly dependent on the paths a communicating platform would follow during its lifetime. For the communicating platforms that are driving through several high teletraffic areas, such as city centers or shopping malls, it is reasonable to expect a higher forced termination probability than in rural areas. Thus, instead of using an *average* quantity for the forced termination probability in this macro-area, we consider the forced termination probability for a *given path.* A *path* is defined as a sequence of ordered cell boundary crossing events during the lifetime of a call. Different paths can be chosen to give an indication of forced termination probability in this macro-area.

For a call in progress in the i_{th} cell, a hand-off *attempt* is generated if the call duration is longer than the platform dwell time in the i_{th} cell. Recall that the dwell time in the i_{th} cell is a random variable, T_{Di}, exponentially distributed with mean μ_{Di}^{-1}, that is,

$$f_{T_{Di}}(t) = \mu_{Di} \cdot e^{-\mu_{Di} t} \,, \quad i = 0,\ 1, \ldots,\ N, \tag{43}$$

and the unencumbered session duration, T, is

$$f_T(t) = \mu \cdot e^{-\mu t} \,. \tag{44}$$

Since random variables T and T_{Di} are independent each other, the probability that a call, *currently served by the* i_{th} *cell*, will make another hand-off attempt is

$$\text{Prob}\{\text{ a call, served by the } i_{th} \text{ cell, will make another hand-off attempt }\}$$

$$= P\{T > T_{Di}\}$$

$$= \int_0^\infty dt \int_0^t \mu \cdot e^{-\mu t} \cdot \mu_{Di} \cdot e^{-\mu_{Di} t_{Di}} \, dt_{Di}$$

$$= \frac{\mu_{Di}}{\mu + \mu_{Di}} . \tag{45}$$

Note that equation (45) represents the probability that a communicating platform, served by the i_{th} cell, will make a hand-off attempt (departure), whereas the elements of $\mathcal{A}$, α_{ij}, denote the average fraction of hand-off departures from microcell i that flow into the service area of cell j (or into adjacent macro-areas when $j = D'$).

A hand-off attempt from *source cell* i towards *target cell* j, will fail if there are no idle channels in target cell and the macrocell. Since the *hand-off failure probability* for hand-off attempts onto cell j is P_{H_j}, thus the probability that a call served by a microcell i will fail in a hand-off attempt onto cell j, denoted as $P_{H_{i,j}}$, is

$$P_{H_{i,j}} = \frac{\mu_{Di}}{\mu + \mu_{Di}} \cdot P_{H_j} , \tag{46}$$

$$i = 1,\ 2, \ldots,\ N, \quad j = 0,\ 1, \ldots,\ N.$$

If the target cell is adjacent macro-areas (D'), since the overall cellular system was assumed that all *macro-areas* are statistically identical, thus

$$P_{H_{i,D'}} = \frac{\mu_{Di}}{\mu + \mu_{Di}} \cdot P_{H_0} , \tag{47}$$

$$i = 0,\ 1,\ 2, \ldots,\ N.$$

Note that calls served by the macrocell will not generate hand-off attempts within the macro-area under observation.

Figure 3 shows several arbitrarily chosen example paths in a macro-area. Path 1, labeled $P1$, indicates the route (order of cell boundary crossings) that a communicating platform initially served by cell 1, could possibly follow during its lifetime. Observe that once a communicating platform is successfully handed off to the macrocell, there will be *no* subsequent hand-off attempts within this macro-area. Thus, the forced termination probability along $P1$, denoted $P_{FT}(P1)$, is given by

$$P_{FT}(P1) = P_{H_{1,0}} , \tag{48}$$

where $P_{H_{1,0}}$ can be found from (46). Obviously,

$$P_{FT}(P2) = P_{FT}(P1) . \tag{49}$$

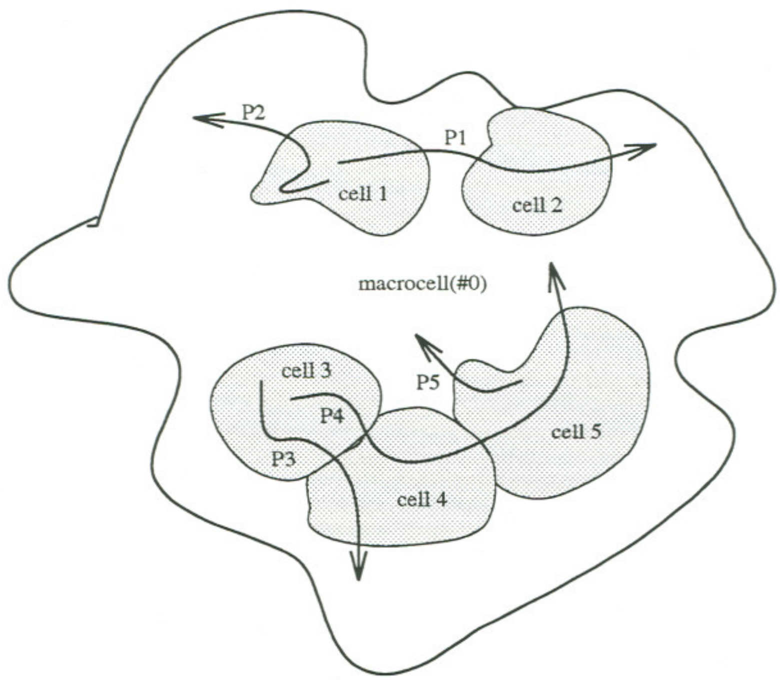

Figure 3: Several arbitrarily chosen example paths in a macro-area.

We now consider the forced termination probability along $P3$, $P_{FT}(P3)$. Forced termination may occur during any cell boundary crossings along $P3$. For calls served by cell 3 and following path $P3$, the probability that a communicating platform will make a hand-off attempt to cell 4 and fail in that attempt is

$$\frac{\mu_{D3}}{\mu + \mu_{D3}} \cdot P_{H_4} \ .$$

A successful hand-off attempt from cell 3 to cell 4 is accommodated in cell 4 if it has any idle channel. Otherwise, it will be accommodated by the macrocell. If a macrocell channel is used, no subsequent hand-off attempts in this macro-area will be generated. If the hand-off attempt is served by a channel of microcell 4, another hand-off attempt may be generated in the future. Let F_0 be a set of states in which the hand-off attempts can be served by microcell 4. That is,

$$F_0 = \{ \ s : \ \ v_3(s) \ > \ 0, \ v_4(s) \ < \ C_4 \ \} \ . \tag{50}$$

Then, the probability that a communicating platform, served by cell 3 and following path $P3$, will make a hand-off attempt to cell 4 and *successfully accommodated by cell* 4 is

$$\frac{\mu_{D3}}{\mu + \mu_{D3}} \cdot \sum_{s \in F_0} \tilde{p}(s) \ .$$

The probability that this communicating platform will make another hand-off attempt to the macrocell and *fail* in that attempt is

$$\frac{\mu_{D4}}{\mu + \mu_{D4}} \cdot P_{H_0} \ .$$

Therefore, the forced termination probability along $P3$, $P_{FT}(P3)$, is given by

$$P_{FT}(P3) = \left(\frac{\mu_{D3}}{\mu + \mu_{D3}} \cdot P_{H_4} \right) + \left(\frac{\mu_{D3}}{\mu + \mu_{D3}} \cdot \sum_{s \in F_0} \tilde{p}(s) \right) \left(\frac{\mu_{D4}}{\mu + \mu_{D4}} \cdot P_{H_0} \right) \ . \tag{51}$$

If the route $P3$ is extended to adjacent macro-areas, called $P3'$, the forced termination probability along $P3'$ can be written as

$$P_{FT}(P3') = P_{FT}(P3) + \left(\frac{\mu_{D3}}{\mu + \mu_{D3}} \cdot \sum_{s \in F_0} \tilde{p}(s) \right) \left(\frac{\mu_{D4}}{\mu + \mu_{D4}} \right) \left(\frac{\mu_{D0}}{\mu + \mu_{D0}} \cdot P_{H_{0,D'}} \right) \ , \tag{52}$$

where $P_{H_{0,D'}}$ is the hand-off failure probability for the calls served by the macrocell.

The forced termination probability along arbitrary path can be found using similar developement.

IV.4 Carried Traffic

The carried traffic in cell i, $A_c(i)$, can be easily calculated once the state probabilities are determined. It is simply, the average number of occupied channels in cell i, and is given by

$$A_c(i) = \sum_{s=0}^{s_{max}} v_i(s) \cdot p(s), \quad i = 0, 1, \ldots, N. \,, \tag{53}$$

where $v_i(s)$ is the number of communicating platforms in cell i when the system is in state s.

V Example and Discussion

A hierarchically overlaying micro-cellular system is considered as a numerical example to illustrate the system operation and performance. An example *macro-area* of the cellular communications systems under observation, as shown in Fig. 4, consists of two microcells and an overlaying macrocell (all roughly circular in shape). In this example, the new call demand in each microcell and in the macrocell are equal. The number of noncommunicating platforms per cell was taken as 550, and the new call origination rate from any noncommunicating platform, regardless of its position in a macro-area, was varied from $6.94\text{x}10^{-5}$ to $2.78\text{x}10^{-4}$ calls/s. The value $2.78\text{x}10^{-4}$ corresponds to a demand rate such that each platform originates one call during an hour. The mean unencumbered call duration is 100 s, thus the *offered load* to each cell ranges from 3.82 erlangs to 15.28 erlangs. The mean dwell times in the macrocell and microcell are 225 s and 150 s, respectively.

The *teletraffic flow matrix*, characterizing the geographic configuration in a macro-area, was taken as

$$\mathcal{A} = \begin{pmatrix} 0.2 & 0.0 & 0.5 & 0.3 \\ 0.2 & 0.5 & 0.0 & 0.3 \end{pmatrix} \tag{54}$$

throughout the discussion. That is, 20% of the hand-off departures from a microcell will be toward its overlaying macrocell, 50% towards neighboring microcell, and 30% towards the adjacent macro-areas. Statistically, both microcells have the same teletraffic flow parameters. These parameters chosen here are solely for the purpose of illustraion. They can be properly modified to reflect other situations.

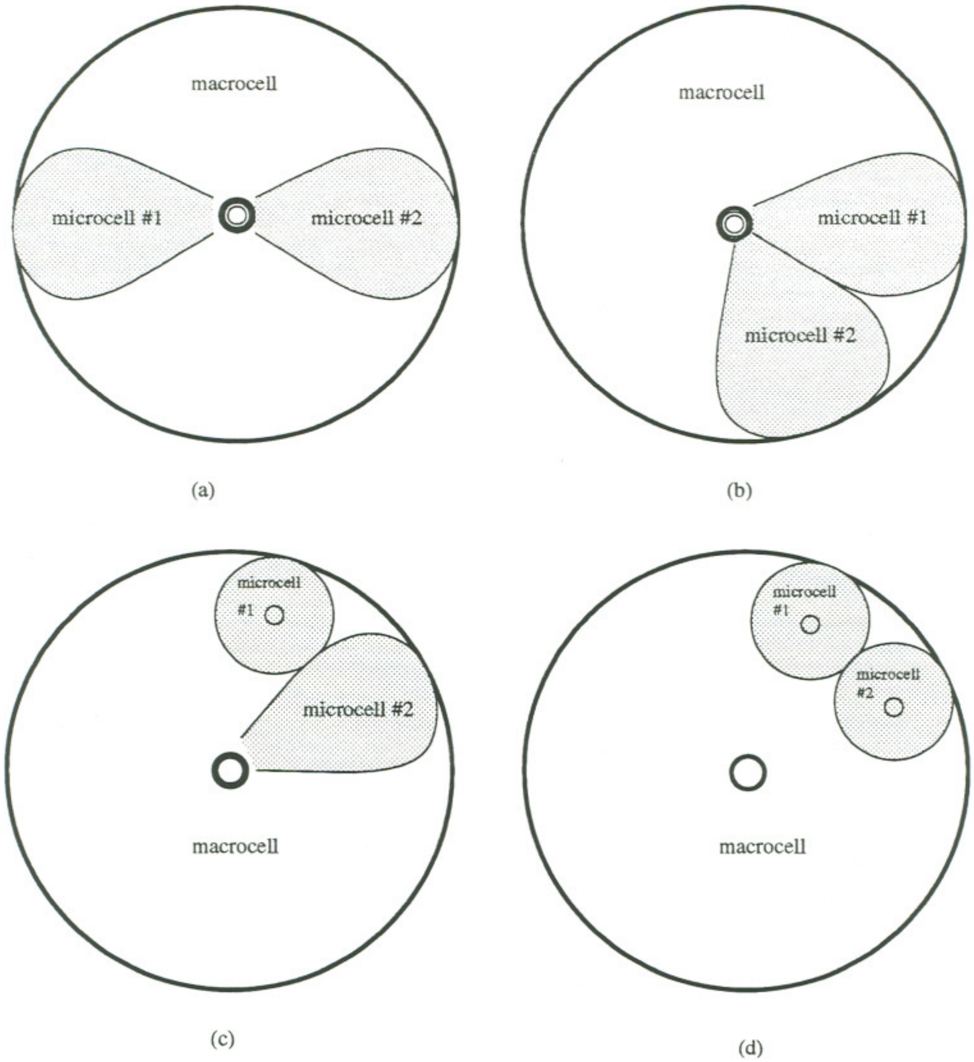

Figure 4: Example macro-areas of a cellular system with two microcells and one macrocell per macro-area. Macrocell gateways, represented by small dark circles in the center of macro-areas, are omnidirectional. Microcell gateways, represented by small light circles, are either directional or omnidirectional depending on the shapes of microcell coverage areas.

Fig. 5 shows the blocking probabilities for three different channel allocation patterns. Pattern 1 is an uniform allocation of 20 channels to each cell, denoted as {20, 20, 20}. Pattern 2 allocates 26 channels to the macrocell and 17 channels to each microcell, denoted as {26, 17, 17}. Pattern 3 further increases the number of channels in the macrocell up to 32, and allocates 14 channels to each microcell, that is {32, 14, 14}. No channels are reserved for the exclusive use of hand-off calls. The blocking probabilities in the macrocell and microcells are denoted as P_{B_0}, P_{B_1}, and P_{B_2} respectively. Since both microcells have the same offered load and teletraffic flow parameters, P_{B_1} is equal to P_{B_2}.

From Fig. 5, the curves of pattern 1 and pattern 3 shows that P_{B_0} is significantly decreased as the number of channels allocated to the macrocell is increased from 20 to 32. However, the blocking probabilities in microcells, P_{B_1} (or P_{B_2}), of pattern 3 is lower than those of pattern 1 only when the system is in low offered load. Once the offered load is moderate or high, the overflow traffic from microcells is greatly increased because of less allocated channels, and meanwhile the overlaying macrocell is less capable of providing backup channels for microcells. Thus pattern 3 has higher blocking probabilities in microcells when the offered load is high. Similar trends also appear in the curves of corresponding hand-off failure probabilities, as shown in Fig. 6.

Let the system be operated with channel allocation pattern {32, 14, 14}, the effects of reserved channels on blocking and hand-off failure probabilities are shown in Fig. 7 and Fig. 8, respectively. The reserved channels are for the exclusive use of hand-off calls. At first, the number of reserved channels in the macrocell and each microcell is 2 and 1, respectively, denoted as C_h={2, 1, 1}. Curves correspond to another parameter, C_h={4, 2, 2}, are also shown in the figures. It is seen that the reserved channels have the effects of increasing blocking probabilities while decreasing hand-off failure probabilities. That is, the reserved channels have the effects of trading the blocking performance for the hand-off success. Determination of a suitable number of reserved channels for system operation is always an engineering issue.

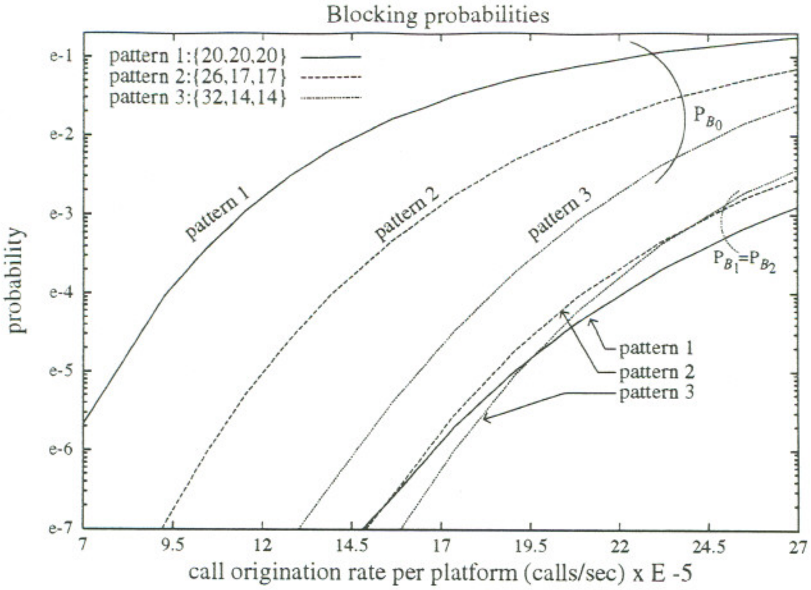

Figure 5: Blocking probabilities depend on call demand for three different channel allocation patterns. The number of noncommunicating platforms per cell is 550. The dwell times are $\overline{T}_{D0} = 225$ sec and $\overline{T}_{D1} = \overline{T}_{D2} = 150$ sec.

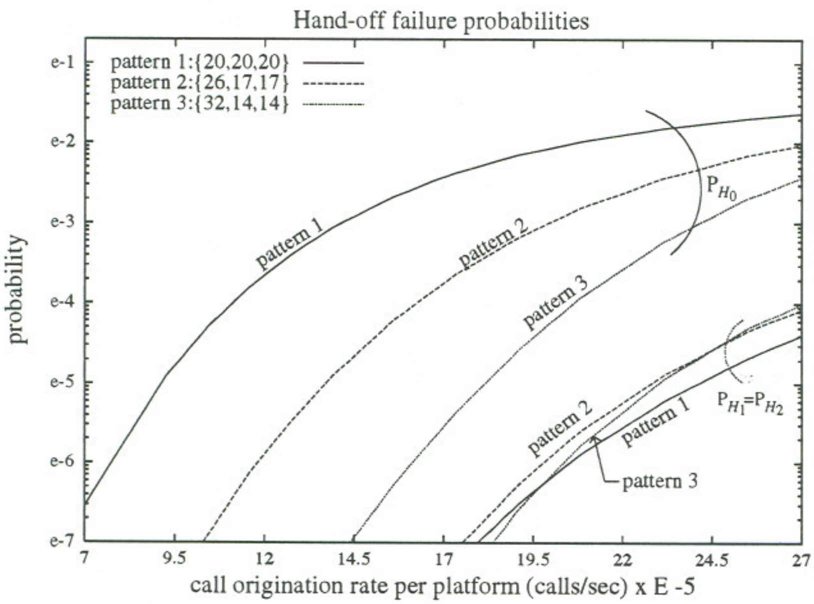

Figure 6: Hand-off probabilities depend on call demand for three different channel allocation patterns. The number of noncommunicating platforms per cell is 550. The dwell times are $\overline{T}_{D0} = 225$ sec and $\overline{T}_{D1} = \overline{T}_{D2} = 150$ sec.

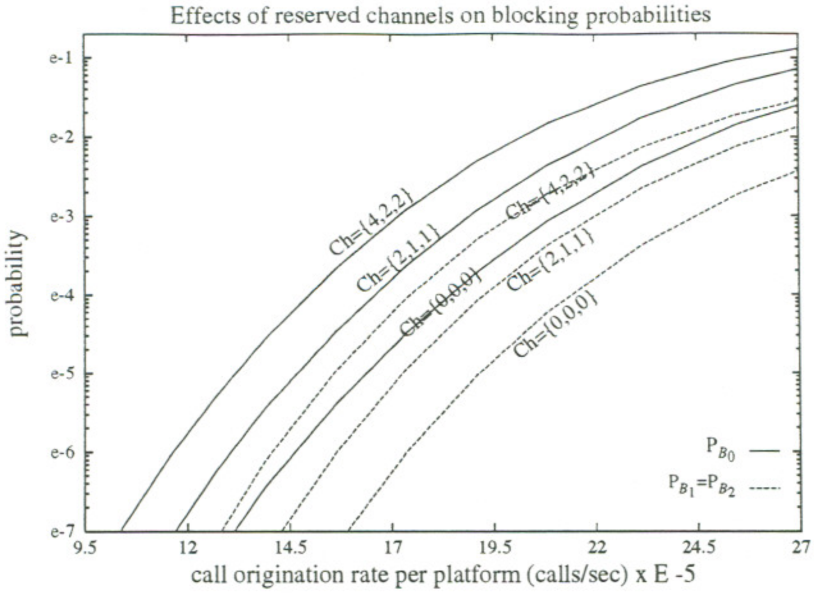

Figure 7: Effects of reserved channels on blocking probabilities. The channel allocation pattern is {32, 14, 14}, and teletraffic demand is uniformly distributed. The parameters used are the same as Fig. 5.

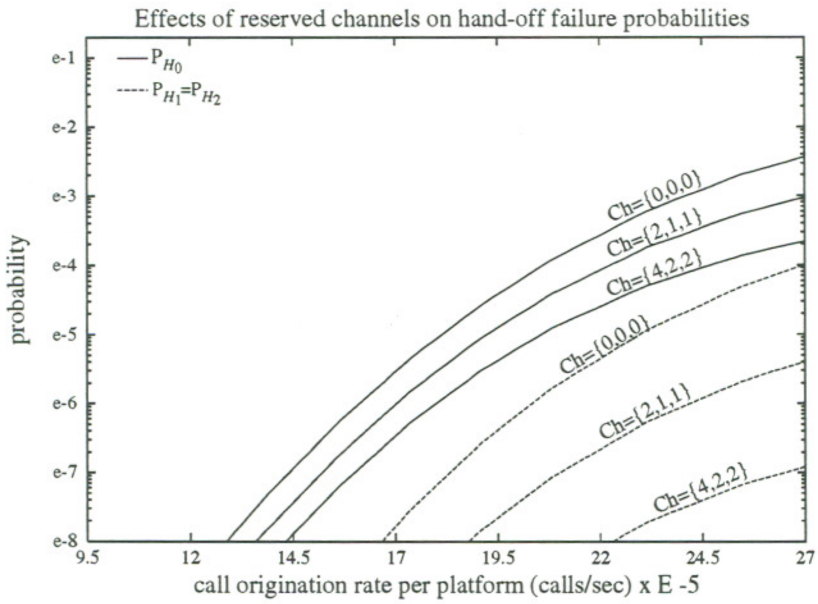

Figure 8: Effects of reserved channels on hand-off failure probabilities. The channel allocation pattern is {32, 14, 14}, and teletraffic demand is uniformly distributed. The parameters used are the same as Fig. 5.

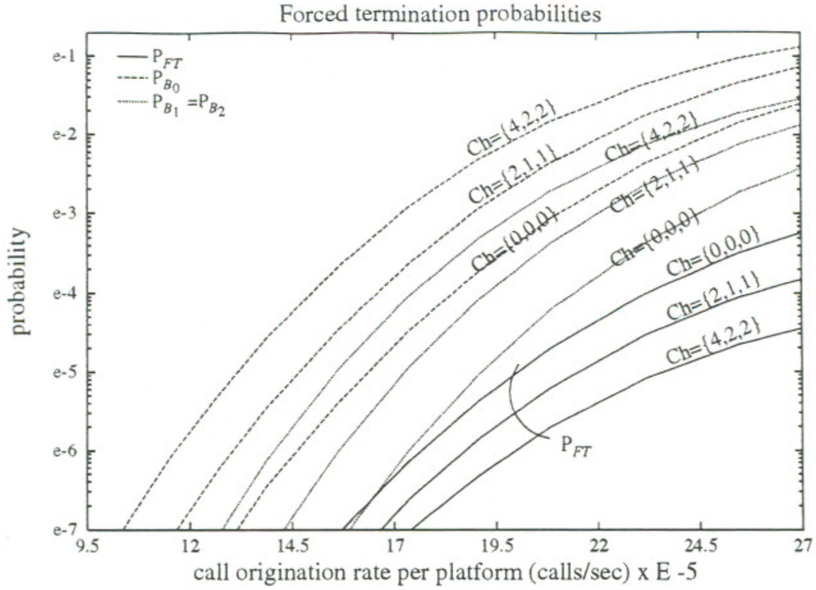

Figure 9: Forced termination probabilities. The channel allocation pattern is {32, 14, 14}, and teletraffic demand is uniformly distributed. The parameters used are the same as Fig. 5.

Fig. 9 shows the forced termination probabilities of a prescribed path for prioritized and nonprioritized hand-off schemes, with the same parameters as in Fig. 5. The channel allocation in a macro-area is again {32, 14, 14}. The path is assumed starting in cell 1, through cell 2, and end up in the macrocell. The prioritized hand-off scheme is seen to provide substantial improvement for forced termination probability. However, this improvement is at the expense of higher blocking probabilities. Fig. 10 shows the traffics carried in the macrocell and a microcell for both prioritized and nonprioritized hand-off schemes. Observed that microcells have lower channel utilization when the number of reserved channels in a microcell is increased. Thus, the total traffic carried in a macro-area is reduced if the number of reserved channels in a cell is increased.

• *Effects of Some Nonuniform Teletraffic Demand Profiles on System Performance*

Sensitivity of the system performance to nonuniform spatial teletraffic distribution is important. A nonuniform teletraffic demand profile was selected to give an indication of the effects on system performance. The total offered load in the *macro-area*

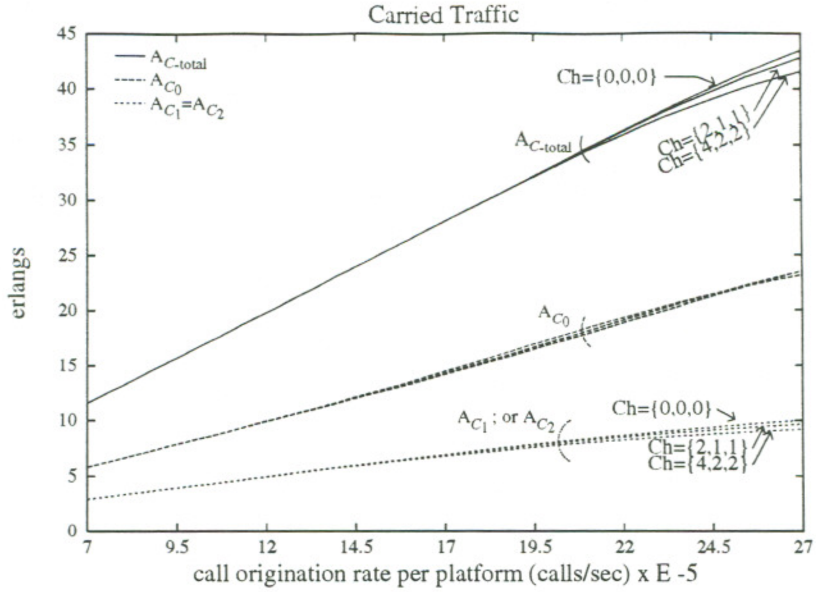

Figure 10: Carried traffic in a macro-area. The channel allocation pattern is {32, 14, 14}, and teletraffic demand is uniformly distributed. The parameters used are the same as Fig. 5.

for nonuniform teletraffic distribution is the same as that of uniform distribution, but each cell may have different loading.

Let $\{L_0, L_1, L_2\}$ denote *relative* offered load in the macrocell, microcell 1, and microcell 2, respectively. For the uniform demand distribution, the relative loading is {0.33, 0.33, 0.33}. The nonuniform demand distribution considered here is {0.2, 0.4, 0.4}. This distribution is selected because the macrocell is to cover low teletraffic area while microcells are to cover high teletraffic areas. It can be seen from Fig. 11 that the blocking and forced termination probabilities for the nonuniform teletraffic distribution is significantly lower than those of the uniform distribution. This is because less teletraffic offered to the macrocell reduces the blocking probability in the macrocell and increases the capability of providing backup channels for subordinate microcells. This shows that even the teletraffic demand in microcells is increased by 20%, under the hierarchically overlaying operation, the system performance is robust to the teletraffic fluctuation.

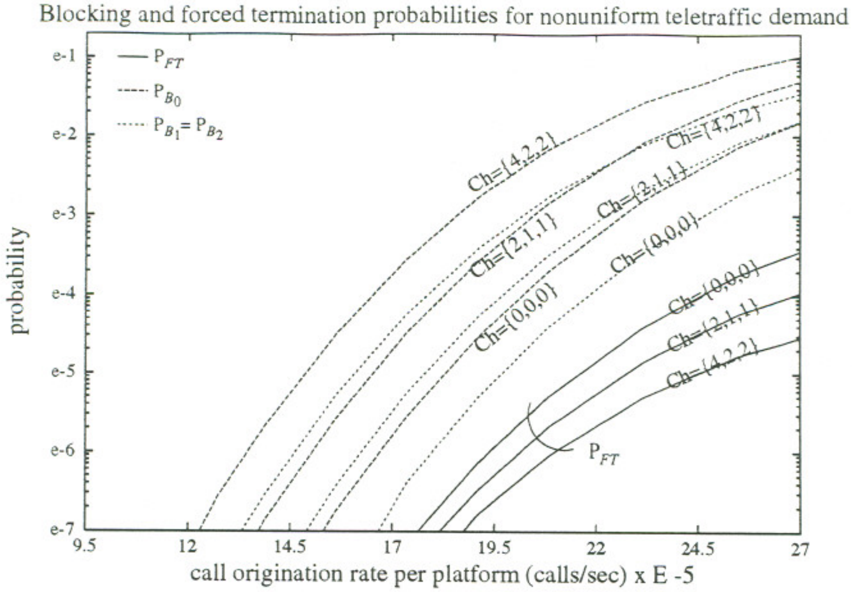

Figure 11: Effects of nonuniform teletraffic on blocking and forced termination probabilities. The relative offered load in a macro-area is {0.2, 0.4, 0.4}. The parameters used are the same as Fig. 5.

- *Multi-Layer Hierarchically Overlaying System*

The concept of overlaying macrocells described here can be extended to form a *multi-layer hierarchically overlaying system.* An example based on hexagonal geometry is shown in Fig. 12. The cellular system consists of overlaying hexagonal cells of different sizes. The smallest is called *level one cell.* Seven microcells are overlaid by a *level 2 cell*, and seven level 2 cells are in turn overlaid by a *level 3 cell.* The overlaying techniques can be further applied to higher levels of the system as needed. The system can be operated in various ways. One is to allow the overlaying cells (level 2 cells and higher) receive the (overflow) new calls from subordinate cells that can not serve them. Priority for user classes can be established by allowing overflow of a call to a certain prescribed hierarchical level depending on user class. Strategies for handling hand-offs can also provide another choice for system operation. Several types of hand-off control strategies between the same levels or different levels can be devised. For example, hand-offs may be accommodated either by a level 1 target cell or by some other higher level cell, based on user class, call priority, and/or resources

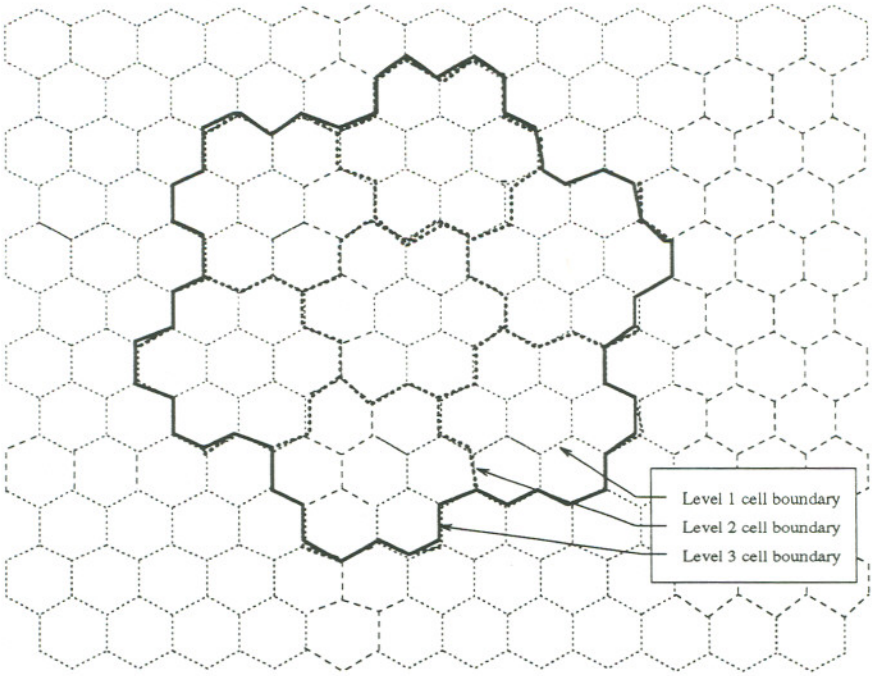

Figure 12: The architecture of multi-layer hierarchically overlaying system.

needed or available.

To achieve higher *spectral efficiency*, the number of channels allocated to higher hierarchical levels of the system must be limited. In fact, system will be less spectrally efficient if a large number of channels are assigned to the overlaying cells. For the convenient operation, each level in the system hierarchy may be allocated different sets of channels and have generally different reuse patterns. Alternately, schemes for coordinated use of the same channels among the hierarchical levels can be devised.

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