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OVERLAPPING COVERAGE AND CHANNEL REARRANGEMENT IN MICROCELLULAR COMMUNICATION SYSTEMS

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Abstract

Overlapping coverage areas of nearby base stations arise naturally in cellular communication systems - especially in small-cell high-capacity microcellular configurations. With overlap, some users may have access to channels at more than one base station. This enhanced access can be used to improve teletraffic performance characteristics. We consider this issue for cellular systems in which hand-off is important. Channel rearrangements are used to benefit users who are in range of only one base station.

An analytical model is developed to determine blocking probability, forced termination probability, hand-off activity and carried traffic for systems with overlapping coverage and channel rearrangement. Example performance characteristics are displayed. These quantify the potential improvement provided by these schemes in comparison with the fixed channel assignment.

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1 Introduction

In cellular communication systems, a user's call is served by a base station that provides best signal quality in some sense. In many cases a mobile unit can establish a communication link of acceptable quality with more than one base. Succinctly, there is overlapping coverage, usually by nearby base stations [1]. This overlap can be used to advantage in special circumstances, such as base station failures and hot spots. Even under uniform and normal operating conditions, with appropriate system control strategies the overlapping coverage can be used to improve teletraffic performance characteristics. Several schemes that do this have been suggested [2, 3, 4, 5]. In [2] a Generalized Fixed Channel Assignment (GFCA) scheme was discussed. GFCA allows a call to be served by any of several nearby base stations. Directed retry and load sharing was considered in [3] and [4]. Directed retry allows a new call that cannot be served at one base to attempt access via a nearby alternate base. Load sharing is an enhancement of directed retry and allows calls in congested cells to be served by neighboring cells. In [5] highway microcells with an overlayed macrocell was analyzed.

Here we consider exploiting overlapping coverage for a microcellular environment in which hand-off is important. When overlapping coverage exists in a system, it is not likely that users in all areas can establish an acceptable quality link with several bases. Users in some areas may have access to only one base while others can have access to more than one. While those calls that can access more than one base have the advantage, calls which can access *only* one base may encounter increased blocking or hand-off failure due to higher channel usage that is possible. In order to let these calls benefit from overlapping coverage, channel rearrangement can be used. Channel rearrangement allows an ongoing call in an overlapped area to continue service through an alternative base. This can free a channel for use by a new or hand-off call that has access to only one base. By using this scheme, it is not always necessary to block a call which originates in a non-overlapping region and finds no channel available at the corresponding base. Channel rearrangement can also be used to accommodate a hand-off call which enters a non-overlapping region and finds all channels occupied at the only target base. Because these schemes tend to accommodate calls that are more distant from the base, the teletraffic improvements may be at the expense of signal quality. However due to the fact that the cluster size is discrete, usually the worst case SIR is higher than the required one. Thus we can use this SIR margin in exchange for the traffic performance benefit. It may also be advantageous to give priority access to hand-offs even at the cost of increased blocking of new calls. This is because hand-offs that fail result in interruptions of service. Communication traffic performance and hand-off issues were considered in [6, 7, 8], but exploration of substantial overlapping coverage was not considered. In [2], [3] and [4] hand-off issues were not considered in detail.

This paper is organized as follows. Section 2 presents the basic model. The performance characteristics for overlapping coverage and overlapping coverage with channel rearrangement are analyzed in section 3 and 4, respectively. The discussion includes state characterization, teletraffic model, transition rates and performance measures. Numerical results and conclusions are presented in section 5.

2 Model Description

We consider a cellular system with omni-directional base stations. These bases are organized in a hexagonal pattern. The *cell* for a base is defined as the area where the received signal power from that base is the strongest. Under uniform propagation and flat terrain condition, this corresponds to a hexagon for each base and every base is located at the center of its own cell. Fig.1 shows a base station A surrounded by six neighbors, base B through base G, and their corresponding cells. The *cell radius*, r, is defined as the distance from a base to a vertex of its own cell. The coverage area of a base is the area that in the communication range of that base. This area is bounded by a circle and the base is also located at the center of this circle. The *coverage radius*, R, is defined as the distance from a base to its coverage boundary. Due to overlapping coverage, a new call may have access to two, three or even more base stations depending on the ratio of the coverage radius to the cell radius (R/r). We consider the overlapping coverage up to a maximum of three base stations. That is, we only consider the case that the ratio of R/r is between 1 and 1.5 [1]. Within this range there are three kinds of regions in which a new call can arise. In these regions a call can access one, two or three base stations. The regions are denoted by A_1, A_2 and A_3 respectively and are shown in Fig.2. Thus region A_1 is the non-overlapping region while both A_2 and A_3 are overlapping regions. The amount of area of these three regions depends on the ratio R/r. In [1] the percentage of a cell area that belongs to region A_1, A_2 or A_3 was calculated. They are denoted by p_1, p_2, and p_3. The cell radius, r, is normalized to unity in this paper. Coverage radius, R, is therefore between 1 and 1.5. Each base has C channels of which C_h channels are reserved for hand-off calls. Only a certain number is reserved, not specific channels. Thus a new call has access to no more than $C - C_h$ channels at a base while a hand-off call can have access to all C channels. In this way hand-off calls are given priority.

With overlapping coverage, a new call will be served unless there are more than or equal to $C - C_h$ calls in progress at each of the base stations that it can access. Similarly a hand-off call will continue its service unless all target bases are fully occupied. While overlapping coverage improves the overall performance characteristics, new calls in region A_1 and those hand-off calls that enter A_1 cannot get the advantage. Actually the situation is worse for them because base stations are busier and these calls only have one choice. In order to let these calls benefit, channel rearrangement can be used. Channel rearrangement allows an ongoing call which resides in an overlapping region to be transferred to another base to continue service. The freed channel is used to accommodate a call in region A_1. If there is more than one ongoing call in the area of overlap when a channel rearrangement is needed, (we assume that) the base will choose the call that has least signal power. The system will attempt to transfer this call to another base. The motivation for this is that the call with least signal power is most likely to be near the coverage boundary and (likely) to need a hand-off. This approach will not only fulfill the channel rearrangement requirement, but also will least degrade signal quality for those chosen calls. If the chosen call cannot gain access to a channel in all target bases that can continue its service, this channel rearrangement attempt fails but the chosen call is continued. The call that would like to benefit from the channel rearrangement is blocked or forced to terminate.

With overlapping coverage a base serves some calls from neighboring cells in addition to the calls that arise in its own cell. For those calls from neighboring cells the signal quality may degrade since they are more distant from the base. We use the worst case SIR to quantify the loss of signal quality. The worst case SIR is taken at the coverage boundary and is calculated using the path loss component of the received power. The path loss is inversely proportional to the distance raised to an exponent γ [9]. Table 1 shows the worst case SIR in dB for various cluster sizes, N, and coverage radii, R, with $\gamma = 4$. In Table 1, i and j are the shift parameters for determining the locations of co-channel bases. The cluster size, N, and these shift parameters are related by $N = i^2 + ij + j^2$.

3 Overlapping Coverage

3.1 State Characterization

For the analysis of overlapping coverage and hand-off priority, the state (of a base), s, is defined as the number of calls served by the base. An overall system state is a string of base station states - one component for each base station in the system. This state representation accounts for all events that occur in the system. It characterizes, for example, a hand-off call that leaves one base and is continuously served by another base; or a new call which is served by a neighboring base due to high channel usage of the base in its own cell. However, the huge number of system states precludes pursuing this approach for most cases of interest. A simplified approach is to decouple a base from others by applying average new call arrival and hand-off arrival rates from neighbors and model the statistical behavior of a given base independently from others. This is similar to the approach used in [8]. The method of calculating the average new call and hand-off arrival is described in section 3.3 for considering overlapping coverage. As a result, the state, s, of the given base is characterized by the number of calls served by this base and s is labeled from 0 to C. Let $j(s)$ denote the number of calls carried by a base in state s. In this case, $j(s) = s$.

3.2 Teletraffic Model

The use of base station resources depends on teletraffic processes such as new call arrivals, call completions, hand-off arrivals and hand-off departures. Mobile users are assumed to be uniformly distributed throughout the service area. We also assume that the new call arrival rate is independent of the number of calls in service and new call arrivals follow a Poisson point process with a rate Λ_n per cell. The unencumbered call duration, T, is defined as the time duration that a call would be served if it were not terminated prematurely. We assume T is a random variable with a negative exponential density function with mean $\overline{T}$ $(=\mu^{-1})$.

The time duration that a mobile unit resides within the coverage area of the base which is providing service is defined as the dwell time, T_d. Dwell time is assumed to be a random variable with a negative exponential distribution of mean $\overline{T_d}$ $(=\mu_d^{-1})$. We assume that mean dwell time is proportional to the coverage radius, R. The hand-off arrivals are related to the hand-off departures because any hand-off demand at a target base corresponds to some hand-off departure in the system. The mean hand-off arrival and departure rates can be related in a manner similar to that used in [10].

3.3 Driving Processes and Transition Rates

In order to calculate the state probabilities, $P(s)$, the state transitions and the corresponding transition rates must be characterized and identified. State transitions are due to four driving processes: 1) new call arrivals 2) call completions 3) hand-off arrivals 4) hand-off departures. The transition rate from a current state s to next state s_n due to these driving processes are denoted by $r_n(s, s_n)$, $r_c(s, s_n)$, $r_h(s, s_n)$ and $r_d(s, s_n)$ respectively. Expressions for all transition rates and the relationship between a state and the successor state due to different driving processes are explained below.

3.3.1 New Call Arrivals

Since mobile users are assumed to be uniformly distributed throughout the service area, the fraction of new calls that arise in region A_1 is p_1. Similarly, the fraction of new calls that arise in region A_2 is p_2 and in region A_3 is p_3. A new call will be served by the base of its own cell if there is a channel available. That is, if there are less than $C - C_h$ calls in progress. If this new call cannot be served by its base, there are three different cases: 1) If this new call arises in region A_1, it is blocked. 2) If this new call arises in region A_2 and the alternate base also has no channel available, it is blocked. Otherwise, it will get the service through the alternate base. 3) If this new call arises in region A_3 and both alternate bases have no channel available, it is blocked. If both alternate bases have a channel available, it will be served by the one with the stronger signal power. Otherwise, it will be served by the one which can accommodate it.

Consider the new call arrival demands on base A when base A is able to accommodate a new call. If base A cannot serve a new call, there is no state transition due to new call arrivals. Base A accommodates the demands from its own cell, which is the new call arrival rate, Λ_n. In addition, due to overlapping coverage, it also takes the demands from neighboring cells within its coverage area under some circumstances. There are two kinds of new calls that may be accommodated by base A from neighboring cells. One is the new calls in region A_2, such as in area ILNW shown in Fig.1. The other is the new calls in region A_3, such as in area IJKL. Those new calls that arise in area ILNW and cannot be served by base C will make demands at base A. The probability that there are more than or equal to $C - C_h$ calls in progress at a base is denoted by η. Thus, the probability that base C is not able to serve a new call is η. For now we proceed as if we know η. Actually this quantity is determined by the state probabilities which depend on the dynamics of the system and the driving processes. The probability, η, is given by

$$\eta = Prob\{s : C - C_h \leq j(s) \leq C\} \quad . \tag{1}$$

This quantity, η, is also the fraction of new calls that arise in area ILNW and make demands on base A. There are altogether six areas like area ILNW within the coverage of base A. The new call arrival demands on base A from these six areas is $P_2 \cdot \Lambda_n \cdot \eta$.

For new calls in area IJKL that cannot be served at base C, potentially can be served by base A or D. Those new calls that find base A with a channel available for a new call and base D with no such channel will be served at base A. On the other hand, if base D can also serve a new call, half of the new calls in area IJKL will be served by base A. Therefore given base C has no channel available for a new call, the fraction of new calls in area IJKL making demands on base A is denoted by κ_n, which can be written as

$$\kappa_n = \eta \cdot 1 + (1 - \eta) \cdot \frac{1}{2} \quad . \tag{2}$$

Moreover, there are twelve such areas within the coverage of base A. The new call arrival rate on base A from these twelve areas is $2 \cdot P_3 \cdot \Lambda_n \cdot \eta \cdot \kappa_n$. The probability, η, in these two cases decouples base A from neighbors for the new call arrivals from neighboring cells. In summary, the new call arrival rate of base A, $r_n(s, s_n)$, is

$$r_n(s, s_n) = \begin{cases} \Lambda_n \cdot (1 + p_2 \cdot \eta + 2p_3 \cdot \eta \cdot \kappa_n)\,, & s < C - C_h\,;\ s_n = s + 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{3}$$

3.3.2 Call Completions

Upon a call completion at a base the base will change state so that the value of s is decreased by one. We assume that the unencumbered session duration has a negative exponential density function with mean $1/\mu$. As a result, μ is the call completion rate per call. The transition rate due to call completion, $r_c(s, s_n)$, is

$$r_c(s, s_n) = \begin{cases} s \cdot \mu\,, & s > 0\,;\ s_n = s - 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{4}$$

3.3.3 Hand-off Departures

Recall from section 3.2 that time during which a mobile user resides in the coverage area of the base that is providing service is defined as the dwell time, T_d. This has mean $1/\mu_d$.

Therefore μ_d is the hand-off departure rate per call. A hand-off departure will decrease the value of s by one. As a result, the rate out of state s due to hand-off departure, $r_d(s, s_n)$, is

$$r_d(s, s_n) = \begin{cases} s \cdot \mu_d\,, & s > 0\,;\ s_n = s - 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{5}$$

3.3.4 Hand-off Arrivals

Since any hand-off arrival corresponds to a hand-off departure in the system, the hand-off arrival rate is related to the rate of hand-off departure. Consider the hand-off arrival demands at base A. They are of two kinds. One is hand-offs entering region A_1 of base A; the other is hand-offs entering region A_2 of base A. If one of the neighbors (say base B) has i calls in progress, the total hand-off departure rate of base B is $i \cdot \mu_d$. All these hand-offs occur uniformly distributed on the coverage boundary. The fraction of hand-offs that enter region A_1 of base A is the ratio that the length of arc XY ($2\theta R$) to the length of coverage boundary ($2\pi R$), which is θ/π. Similarly, the fraction of hand-offs that enter region A_2 of base A is α/π. When hand-off calls leave base B by crossing arc XY, base A is the only target base. Thus these hand-offs will make demands on base A. On the other hand, when hand-off calls leave base B by crossing arc WX (or YZ), base A and C (or base A and G) are the potential target bases. Under this condition, we can calculate the fraction of hand-offs that make demands on base A using a similar method developed in section 3.3.1. This fraction is denoted by κ_h and can be calculated as

$$\kappa_h = \nu \cdot 1 + (1 - \nu) \cdot \frac{1}{2} \quad , \tag{6}$$

where ν is the probability that all channels are occupied at a base. Succinctly,

$$\nu = Prob\{s : j(s) = C\} \quad . \tag{7}$$

Therefore, when there are i calls in progress at base B, the hand-off arrival demands (from base B) on base A are

$$\Lambda_h(i) = i \cdot \mu_d(\frac{\theta}{\pi} \cdot 1 + \frac{\alpha}{\pi} \cdot \kappa_h) \quad . \tag{8}$$

The number of calls, i, at base B can take values from 0 to C. By averaging over i, we can calculate the average hand-off arrival rate from base B. To find the average, the probabilities that there are i calls in service at base B must be determined. These probabilities are just the state probabilities, $P(i)$, such that $P(i) = Prob\{s : j(s) = i\}$. Considering that there are six neighbors, the rate $r_h(s, s_n)$ is

$$r_h(s, s_n) = \begin{cases} 6 \sum_{i=1}^{C} P(i) \cdot \Lambda_h(i) , & s < C \, ; \, s_n = s + 1 \\ 0 , & \text{otherwise} . \end{cases} \tag{9}$$

3.4 Performance Measures

There are four performance measures of interest: 1) blocking probability 2) forced termination probability 3) hand-off activity 4) carried traffic. These can be determined once the state probabilities are found. The state probabilities, $P(s)$, can be calculated by solving probability flow balance equations. Since the state probabilities are used to determine the average new call arrival and hand-off arrival rates from neighboring cells, the transition rates are functions of state probabilities. On the other hand, the state probabilities are functions of transition rates. This results in a set of nonlinear equations. The way to solve these equations are in the framework of [7, 8, 10]. A brief explanation is given in Appendix A. The performance measures are developed below.

3.4.1 Blocking Probability

The blocking probability is the average fraction of new calls that cannot gain access to a channel. The probability that the number of calls in progress is more than or equal to $C - C_h$ is η. The blocking probability for new calls in region A_1 is just η because these new calls can access only one base. The blocking probability for new calls in region A_2 is η^2 because they are blocked only when both bases have no channel available. Similarly, the blocking probability for new calls in region A_3 is η^3. Therefore the overall blocking probability is the average over these three regions. It follows that

$$P_B = p_1 \cdot \eta + p_2 \cdot \eta^2 + p_3 \cdot \eta^3 \quad . \tag{10}$$

3.4.2 Forced Termination Probability

A forced termination occurs if a non-blocked call is interrupted due to a hand-off failure during its lifetime. To determine the forced termination probability, P_{FT}, one must first consider the probability that average fraction of hand-off attempts that fail to gain access to a channel because of the lack of an available channel at the target base(s). This is the hand-off failure probability, P_h. The probability that all channels at a base are occupied is ν. Hand-offs may enter region A_1 or A_2 of the target base. For those hand-offs entering region A_1 the failure probability is ν, because they have only one target base. For those hand-offs entering region A_2 the failure probability is ν^2, because failures occur only when channels at both target bases are fully occupied. Therefore P_h is the average over these two situations. Because of the homogeneity assumption, the fraction of hand-offs into region A_1 is $2\theta/(2\theta+\alpha)$ and, on the other hand, the fraction into region A_2 is $\alpha/(2\theta+\alpha)$, where α is the angle of $\angle$WBX and 2θ is the angle of $\angle$XBY. This is shown in Fig.1. Thus P_h can be expressed as

$$P_h = \frac{2\theta}{2\theta+\alpha}\nu + \frac{\alpha}{2\theta+\alpha}\nu^2 \quad . \tag{11}$$

Let p be the probability that a non-blocked call completes before (the next) hand-off occurs and q be the probability that hand-off occurs first. Because of the negative exponential assumption, we can easily calculate p and q using

$$p = \mu/(\mu+\mu_d) \quad , \qquad q = \mu_d/(\mu+\mu_d) \quad . \tag{12}$$

If there is any hand-off failure during the life of a call, it will be forced to terminate. The probability that a non-blocked call is interrupted at the i^{th} hand-off is $q^i(1-P_h)^{i-1}P_h$. Therefore the forced termination probability is

$$P_{FT} = \sum_{i=1}^{\infty} q^i(1-P_h)^{i-1}P_h \quad . \tag{13}$$

This can be simplified as

$$P_{FT} = q \cdot P_h/[1-q\cdot(1-P_h)] \quad . \tag{14}$$

3.4.3 Hand-off Activity

Hand-off activity is the expected number of hand-offs that a non-blocked call will experience. There will be exactly i hand-offs if 1) the call fails at the i^{th} hand-off or 2) it succeeds at the i^{th} hand-off but successfully completes before the $(i+1)^{th}$ hand-off. The probability of the first case is $q^i(1-P_h)^{i-1}P_h$, and the probability of the second case is $q^i(1-P_h)^i p$. Consequently the hand-off activity is

$$H_A = \sum_{i=1}^{\infty} i \cdot q^i (1-P_h)^{i-1}[P_h + (1-P_h)p] \quad . \tag{15}$$

This can be simplified to

$$H_A = q[P_h + (1-P_h)p]/[1-q(1-P_h)]^2 \quad . \tag{16}$$

3.4.4 Carried Traffic

The carried traffic A_C per base station is the average number of channels that are occupied. By definition it is

$$A_C = \sum_{s=0}^{C} j(s) \cdot P(s) \quad . \tag{17}$$

4 Overlapping Coverage with Channel Rearrangement

4.1 State Characterization

Some modification is needed to include consideration of channel rearrangements in our analysis. This is because there must be at least one ongoing call in an overlapping region for channel rearrangement to be possible, but the single-variable state representation cannot provide this information. Therefore a state, s, of a given base is now identified by two nonnegative integers

$$a(s), b(s) \tag{18}$$

where $a(s)$ is the number of calls in service that can access *only* one base in state s and $b(s)$ is the number of calls in service that can access *more than* one base in state s. If, for a particular state, s, $b(s)$ is greater than zero, there is a possibility for channel rearrangement to accommodate an arrival. In this case, the number of calls in service at a base in state s is

$$j(s) = a(s) + b(s) \quad . \tag{19}$$

A permissible state must satisfy the condition that the number of calls carried by a base is less than or equal to C. That is $j(s) \leq C$. All permissible states are labeled from s=0 to s=s_{max}.

4.2 Teletraffic Model

Due to the modification of the state representation, two more random variables, T_a and T_b, are defined. T_a is the time duration a mobile user resides in the non-overlapping region and T_b is the time duration a mobile user resides in the overlapping area within the coverage limit of a base, which includes regions A_2 and A_3. This is shown in Fig.3, in which the region A_1 is approximated by the inner circle with the same area as A_1. The overlapping area is the outer ring of the coverage area of a base. Both T_a and T_b are assumed to be negative exponentially distributed with means $\overline{T_a}$ (=${\mu_a}^{-1}$) and $\overline{T_b}$ (=${\mu_b}^{-1}$), respectively. It is also assumed that these dwell time means, $\overline{T_a}$ and $\overline{T_b}$, are proportional to the square root of the non-overlapping and overlapping area within the coverage limit of a base respectively. When a communicating mobile user in the non-overlapping area completes its dwell time, T_a, it always enters the overlapping area. This call is still served by the same base and does not need a hand-off. On the other hand, when a communicating mobile user in the overlapping area completes its dwell time, T_b, there are two possible outcomes: 1) the user may enter the non-overlapping area of the current base or 2) the user may leave the coverage area of the current base. In the first outcome, this call is still served by the same base, but in the second outcome, it needs a hand-off. We define a parameter w as the probability for the first outcome. Then $1 - w$ is the probability for the second outcome. This parameter, w, can be obtained from the relationship that the dwell time mean within the coverage of a base calculated from $\overline{T_a}$, $\overline{T_b}$ and w is equal to $\overline{T_d}$. The detailed calculation of w is given in [11].

4.3 Driving Processes and Transition Rates

In addition to the four driving processes discussed in section 3.3, three more driving processes must be considered in the case of channel rearrangement. They are status interchange, rearrangement departures and rearrangement arrivals. Status interchange is the process in which calls move from region A_1 to either region A_2 or A_3 (or vice versa) while served by the same base. Rearrangement departures is the process in which calls are transferred to a neighboring base to accommodate new or hand-off calls in region A_1. Rearrangement arrivals is the process in which calls are transferred from neighboring bases. The transition rate due to status interchange is denoted by $r_i(s, s_n)$. The transition rate due to rearrangement departures and rearrangement arrivals are denoted by $r_{rd}(s, s_n)$ and $r_{ra}(s, s_n)$ respectively. These transition rates are discussed below.

4.3.1 New Call Arrivals

The total new call arrival rate is the same as the rate in equation (3). However, due to the change of the state representation, the new call arrival rate must be characterized separately for state variables $a(s)$ and $b(s)$. The new call arrival rate that increases state variable $a(s)$ by one is $\Lambda_n \cdot p_1$. The rest of the new call arrival rate increases state variable $b(s)$ by one. Thus the rate $r_n(s, s_n)$ is as follows

$$r_n(s,s_n) = \begin{cases} \Lambda_n \cdot p_1, & j(s) < C - C_h; s_n = a(s)+1, b(s) \\ \Lambda_n \cdot (1 - p_1 + p_2 \cdot \eta + 2p_3 \cdot \eta \cdot \kappa_n), & j(s) < C - C_h; s_n = a(s), b(s)+1 \\ 0, & \text{otherwise}. \end{cases} \quad (20)$$

4.3.2 Call Completions

The call completion rate must be described separately for the two state variables because a call completion in the non-overlapping or overlapping area results in different next states.

The call completion rate, $r_c(s, s_n)$ is

$$r_c(s, s_n) = \begin{cases} a(s) \cdot \mu\,, & a(s) > 0\,;\; s_n = a(s) - 1, b(s) \\ b(s) \cdot \mu\,, & b(s) > 0\,;\; s_n = a(s), b(s) - 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{21}$$

4.3.3 Hand-off Departures

A hand-off departure occurs only when a communicating user in an overlapping area completes the dwell time, T_b, and the user leaves the coverage area of the current base. The dwell time, T_b, has mean $1/\mu_b$. The probability that this user leaves the coverage area of the current base is $1 - w$. Therefore the hand-off departure rate, $r_d(s, s_n)$, is

$$r_d(s, s_n) = \begin{cases} b(s) \cdot \mu_b \cdot (1 - w)\,, & b(s) > 0\,;\; s_n = a(s), b(s) - 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{22}$$

4.3.4 Hand-off Arrivals

Following the same reasoning as described in section 3.3.4, we can obtain the hand-off arrival rate for the case considering channel rearrangement. However, in the expressions, there are slight differences. The probability $P(i)$ in equation (9) is replaced by ξ_i, which are the probabilities that at a base there are i calls in the area of overlap. This is because only calls in the overlapping area will probably initiate a hand-off. The probability, ξ_i, can be written as

$$\xi_i = Prob\{s : b(s) = i\} \qquad i = 1, 2, \ldots, C \quad . \tag{23}$$

Also, $\Lambda_h(i)$ in equation (8) is decomposed to two terms, $\Lambda_a(i)$ and $\Lambda_b(i)$, for the consideration of two state variables. These two terms can be written as

$$\Lambda_a(i) = i \cdot \mu_b \cdot (1 - w) \cdot \frac{\theta}{\pi} \quad . \tag{24}$$

$$\Lambda_b(i) = i \cdot \mu_b \cdot (1 - w) \cdot \frac{\alpha}{\pi} \cdot \kappa_h \quad . \tag{25}$$

Note that the hand-off departure rate μ_d in equation (8) is replaced by $\mu_b \cdot (1 - w)$ because a hand-off departure has to be initiated by a call leaving the coverage area of the current base. It follows that the rate $r_h(s, s_n)$ is

$$r_h(s, s_n) = \begin{cases} 6\sum_{i=1}^{C} \xi_i \cdot \Lambda_a(i), & j(s) < C\,;\ s_n = a(s)+1, b(s) \\ 6\sum_{i=1}^{C} \xi_i \cdot \Lambda_b(i), & j(s) < C\,;\ s_n = a(s), b(s)+1 \\ 0, & \text{otherwise}. \end{cases} \tag{26}$$

4.3.5 Status Interchange

Because of two state variables, there are state transitions due to a call moving from a non-overlapping area to an overlapping area or vice versa while it is served by the same base. When a call is in an non-overlapping area, the rate of moving to an overlapping area is μ_a. On the other hand, when a call is in an overlapping area the rate of leaving this area is μ_b, but with only probability w to move to the non-overlapping area of the current base where it is still served by this current base. The rate, $r_i(s, s_n)$, is easily to obtain as follows

$$r_i(s, s_n) = \begin{cases} a(s) \cdot \mu_a\,, & a(s) > 0\,;\ s_n = a(s)-1, b(s)+1 \\ b(s) \cdot \mu_b \cdot w\,, & b(s) > 0\,;\ s_n = a(s)+1, b(s)-1 \\ 0, & \text{otherwise}. \end{cases} \tag{27}$$

4.3.6 Rearrangement Departures

Channel rearrangement is used when a new call arises in or a hand-off call enters a non-overlapping region and there is no channel available to accommodate this call. An ongoing call is transferred to some neighboring base and the freed channel is used to serve this new or hand-off call. This results in a state transition, such that variable $b(s)$ is decreased by one and $a(s)$ is increased by one. These events are denoted by rearrangement departures. There are two kinds of rearrangement departures: one for accommodating new calls that arise in region A_1 and the other for accommodating hand-off arrivals that enter region A_1.

First consider a rearrangement departure which accommodates a new call. When the number of calls served by the base is equal to $C - C_h$, a new call which arises in region A_1 is possible to be accommodated by channel rearrangement. However there must be at least one call served by that base in the overlapping area. Therefore, rearrangement can only be used to accommodate a new call for those states, s, that $j(s) = C - C_h$ and $b(s) > 0$. For a successful channel rearrangement, at least one neighboring base must be able to accommodate the call that is chosen to be transferred. This chosen call is not given priority because this rearrangement is for accommodating a new call. There are two cases for this rearrangement departure: 1) If the call chosen to be transferred is in region A_2, it has only one target base. If that target base has less than $C - C_h$ calls in progress, this rearrangement succeeds. The successful probability is $1 - \eta$. 2) If the chosen call is in region A_3, it has two target bases. The successful probability is $1 - \eta^2$. The fractions for case 1 and 2 are $2\theta/(2\theta + \alpha)$ and $\alpha/(2\theta + \alpha)$ respectively, as described in section 3.4.2. Moreover the new call arrival rate in a non-overlapping region is $\Lambda_n \cdot p_1$. As a result, the rearrangement departure rate for new calls can be written as Δ_{dn}, which is

$$\Delta_{dn} = \Lambda_n \cdot p_1 \cdot [\frac{2\theta}{2\theta + \alpha}(1 - \eta) + \frac{\alpha}{2\theta + \alpha}(1 - \eta^2)] \quad . \tag{28}$$

The rearrangement departures for accommodating a hand-off call are possible only when a base is in a state, s, that $j(s) = C$ and $b(s) > 0$. Following a similar development as for accommodating a new call, the rearrangement departure rate for accommodating a hand-off call can be obtained. The component $\Lambda_n \cdot p_1$ in equation (28) is just the new call arrival rate in region A_1. Similarly, the hand-off arrival rate entering region A_1 can be found in equation (26). For convenience, this is denoted by ψ, which is

$$\psi = 6 \sum_{i=1}^{C} \xi_i \cdot \Lambda_a(i) \quad . \tag{29}$$

Then the rearrangement departure rate for accommodating a hand-off call is

$$\Delta_{dh} = \psi \cdot [\frac{2\theta}{2\theta + \alpha}(1 - \nu) + \frac{\alpha}{2\theta + \alpha}(1 - \nu^2)] \quad . \tag{30}$$

Finally, the rearrangement departure rate, $r_{rd}(s, s_n)$, is

$$r_{rd}(s, s_n) = \begin{cases} \Delta_{dn}\,, & j(s) = C - C_h\,; b(s) > 0\,;\ s_n = a(s) + 1, b(s) - 1 \\ \Delta_{dh}\,, & j(s) = C\,; b(s) > 0\,;\ s_n = a(s) + 1, b(s) - 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{31}$$

In the case of $C_h = 0$, $r_{rd}(s, s_n)$ is the sum of Δ_{dn} and Δ_{dh} for the (s, s_n) pairs stated.

4.3.7 Rearrangement Arrivals

A base station also takes the demands from neighboring bases due to channel rearrangement. For example, base A may take an ongoing call that is transferred from base B because base B uses channel rearrangement to accommodate a call in region A_1. This kind of arrivals is denoted by rearrangement arrivals. The rearrangement arrivals are discussed separately for accommodating new calls or hand-off calls in region A_1 of neighboring bases. Now consider the rate for accommodating new calls. In order to serve a rearrangement call that is used for accommodating a new call, there must be less than $C - C_h$ calls in progress at base A. Consider base A to take an ongoing call that is transferred from base B (totally six neighbors may transfer such a call). Channel rearrangements for new calls may occur when base B is in states, s, that $j(s) = C - C_h$ and $b(s) > 0$. The probability of this kind of states is denoted by δ_1, which can be calculated from the state probabilities as

$$\delta_1 = Prob\{s : j(s) = C - C_h \text{ and } b(s) > 0\} \quad . \tag{32}$$

This is the probability that a new call originating in region A_1 may result in a channel rearrangement. In addition, the call that is chosen to be transferred may be in region A_2 or A_3. If this call is in A_2, it will be served by base A. If this call is in region A_3, another base (other than base A) may also accommodate it. For example, if this chosen call is in region A_3 and near arc WX, base C is also possible to accommodate this call. Given that base A can serve this rearrangement call, if base C can also serve this call, one half of the chance it will be served by base A; if base C can not accommodate this call, it will be served by base A. It follows that the rearrangement arrival rate due to new call arrivals in region A_1 of a neighboring base is

$$\Delta_{an} = 6 \cdot \delta_1 \cdot \Lambda_n \cdot p_1 \cdot (\frac{\theta}{\pi} + \frac{\alpha}{\pi} \cdot \kappa_n) \quad . \tag{33}$$

Similarly, the rearrangement arrival rate for accommodating hand-off calls at neighboring bases can be derived. A base can serve a rearrangement arrival which is used for accommodating a hand-off call when channels at this base are not fully occupied. That is under states, s, that $j(s) < C$. By analogy, $\Lambda_n \cdot p_1$ in equation (33) is replaced by ψ, which is the hand-off arrival rate entering region A_1 at a neighboring base and is calculated in equation (29). In addition, the probability δ_1 is replaced by δ_2, which is the probability that a neighboring base may have a channel rearrangement to accommodate a hand-off call. The probability, δ_2, can be written as

$$\delta_2 = Prob\{s : j(s) = C \text{ and } b(s) > 0\} \quad . \tag{34}$$

Therefore the rearrangement arrival rate due to hand-off calls is

$$\Delta_{ah} = 6 \cdot \delta_2 \cdot \psi \cdot (\frac{\theta}{\pi} + \frac{\alpha}{\pi} \cdot \kappa_h) \quad . \tag{35}$$

A base can take rearrangement arrivals for accommodating a new call when this base is in states, s, that $j(s) < C - C_h$ and take rearrangement arrivals for accommodating a hand-off call when $j(s) < C$. As a result, the rearrangement arrival rate, $r_{ra}(s, s_n)$ can be written as follows

$$r_{ra}(s, s_n) = \begin{cases} \Delta_{an} + \Delta_{ah}\,, & j(s) < C - C_h\,;\ s_n = a(s), b(s) + 1 \\ \Delta_{ah}\,, & C - C_h \le j(s) < C\,;\ s_n = a(s), b(s) + 1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{36}$$

4.4 Performance Measures

The nonlinear equations for the state probabilities are solved by the same method used in section 3. (This is explained in Appendix A.) Blocking probability, forced termination probability, hand-off activity and carried traffic are calculated after the state probabilities are found.

4.4.1 Blocking Probability

With channel rearrangement, a new call that originates in region A_1 is blocked if one of the following events prevails at the time of origination: 1) The number of calls in progress at the base is more than $C - C_h$. 2) The number of calls in progress at the base is equal to $C - C_h$ but none is in the overlapping area. 3) The number of calls in progress at the base is equal to $C - C_h$, at least one is in the overlapping area, and the call that is chosen to be transferred to a neighboring base cannot find an available channel at the target base(s). The probability of the first event is denoted by β. This quantity can be expressed as

$$\beta = Prob\{s : j(s) > C - C_h\} \quad . \tag{37}$$

The probability of the second event is just the state probability, $P(s')$, in which $a(s') = C - C_h$ and $b(s') = 0$. For the third event, the probability that there are $C - C_h$ calls in progress with at least one of them in the overlapping area is just δ_1 in (32). The call chosen to be transferred may be in region A_2 or A_3. If it is in A_2, this chosen call has only one target base (another base is currently serving the call), but if it is in A_3, it has two target bases. There is no priority given to this *chosen* call because this rearrangement is for accommodating a new call. Thus the probability that the chosen call cannot find an available channel is η if it is in A_2 and η^2 if it is in A_3. Since the system picks up the call in the overlapping area with least signal power, the chosen call likely to be near the coverage boundary. The location of this call is uniformly distributed near the coverage boundary. It has the probability $2\theta/(2\theta + \alpha)$ of being in region A_2 and the probability $\alpha/(2\theta + \alpha)$ of being in A_3. Then combining above three events, the blocking probability in region A_1 is denoted by P_{b1}, which is

$$P_{b1} = \beta + P(s') + \delta_1(\frac{2\theta}{2\theta + \alpha}\eta + \frac{\alpha}{2\theta + \alpha}\eta^2) \quad . \tag{38}$$

Moreover the channel rearrangement is not used for new calls originate in region A_2 or A_3, the probability for blocking in these two regions are η^2 and η^3 respectively. Finally the average blocking probability, P_B, for the whole system is

$$P_B = p_1 \cdot P_{b1} + p_2 \cdot \eta^2 + p_3 \cdot \eta^3 \quad . \tag{39}$$

4.4.2 Forced Termination Probability

One must distinguish between hand-off arrivals (departures) and rearrangement arrivals (departures). Hand-off arrivals result from a communicating user's entering the coverage area of the given base and leaving the coverage area of the base that is providing service.

Hand-off departures result from a communicating user's leaving the coverage area of a base. Rearrangement arrivals, on the other hand, result from the need of an *adjacent base* to accommodate a call in its non-overlapping area. Rearrangement departures result from the need of a *given base* to accommodate a call in its non-overlapping area. An unsuccessful rearrangement does not result in a forced termination of the call chosen for rearrangement. With channel rearrangements the hand-off failure probability, P_h, is the average fraction of hand-offs that fail. To determine P_h, one must consider the hand-offs that enter region A_1 or A_2 separately. Hand-offs that enter region A_1 of a target base fail if one of the events, E_1 or E_2, prevails at the target base when the hand-off is needed: $\{E_1\}$ All channels are occupied and none of the calls is in the overlapping area; $\{E_2\}$ Event E_2 is the intersection of events E_{2a} and E_{2b}. E_{2a} is the event that all channels are occupied with at least one of the calls in the overlapping area. E_{2b} is the event that the call which is chosen to be transferred to a neighboring base cannot find an available channel at the neighboring base(s). The probability of E_1 is the state probability $P(s^*)$, in which $a(s^*) = C$ and $b(s^*) = 0$. The probability of E_{2a} is δ_2 in (34). The probability of E_{2b} is denoted by δ_3. The chosen call may be in region A_2 or A_3. If it is in A_2, the probability that this call cannot find an available channel is ν, but if it is in A_3, the probability is ν^2. We use a similar development as that in section 4.4.1 to average over these two situations. Thus,

$$\delta_3 = \frac{2\theta}{2\theta + \alpha}\nu + \frac{\alpha}{2\theta + \alpha}\nu^2 \quad . \tag{40}$$

Channel rearrangement is not used for the hand-offs that enter region A_2 of target bases. The probability of failure for this kind of hand-off is ν^2. As a result, the probability P_h is

$$P_h = \frac{2\theta}{2\theta + \alpha}[P(s^*) + \delta_2 \cdot \delta_3] + \frac{\alpha}{2\theta + \alpha} \cdot \nu^2 \quad . \tag{41}$$

Equation (14) can also be used to determine the forced termination probability for the case with channel rearrangement.

4.4.3 Hand-off Activity

Equation (16) (with P_h in (41)) can also be used to determine the hand-off activity with channel rearrangement.

4.4.4 Carried Traffic

By definition, the carried traffic per base, A_C, is

$$A_C = \sum_{s=0}^{s_{max}} j(s) \cdot P(s) \quad . \tag{42}$$

5 Numerical Results And Conclusion

For the purpose of generating example numerical results, the following parameters were used for Fig.4 through Fig.7: $C = 18$, $C_h = 0$, $\overline{T} = 100$ sec and $\overline{T_d} = 25$ sec for the case that coverage radius, R, is unity. For overlapping coverage and channel rearrangement schemes, $\overline{T_d}$ is adjusted proportional to the coverage radius, R. In these figures, the four performance measures are shown and the comparison is made between no overlapping, overlapping and channel rearrangement. The no overlapping case corresponds to the fixed channel assignment (FCA) scheme. All these figures are plotted with respect to the new call arrival rate.

In Fig.4 and Fig.5, blocking and forced termination probabilities are shown. It is seen that the overlapping schemes provide increased improvement as coverage radius R increases. In addition, channel rearrangement allows substantial further improvement. In Fig.5 as R gets larger, the forced termination probability tends to saturate for the channel rearrangement case. This suggests that it is not necessary to use a large R to improve traffic performance. As a result, the degradation of signal quality for calls near the coverage boundary will not be much.

Fig.6 shows the hand-off activity. With larger R, the hand-off activity decreases since the time that a call resides in the coverage area of a base is larger. With channel rearrangement, the hand-off activity is more than without it. This is because a call is harder to be forcedly terminated than without the channel rearrangement.

In Fig.7 carried traffic is shown. It shows that when offered traffic is light, these schemes do not make much difference. However, when offered traffic is heavy, the carried traffic is larger with increase of coverage radius R. Of course, channel rearrangement can further improve the carried traffic over the overlapping coverage.

From these figures, it can be seen that overall performance is improved by using overlapping coverage. There is even further improvement if channel rearrangements are employed. In order to accomplish this, the signal quality is slightly degraded, especially at the coverage boundary. In Table 1, the worst case SIR is shown. When R changes from 1 to 1.1, the SIR degrades less than 2 dB. However, since the cluster size is discrete, usually the SIR is more than the required amount, even at the coverage boundary. Thus this SIR margin can be used in exchange for the improvement of the traffic performance while the system still satisfies the requirement for the signal quality.

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Appendix A

For overlapping coverage, the state probabilities of equilibrium, $P(s)$, can be calculated by solving probability flow balance equations. From the rates given in section 3.3, the transition rate from state s to any possible next state s_n can be found by

$$q(s, s_n) = r_n(s, s_n) + r_c(s, s_n) + r_d(s, s_n) + r_h(s, s_n) \quad , \tag{A.1}$$

in which $s \neq s_n$. The total transition rate out of state s is denoted by $q(s, s)$ and is given by

$$q(s, s) = - \sum_{k=0, k\neq s}^{C} q(s, k) \quad , \tag{A.2}$$

where the minus sign indicates the direction of the flow. In statistical equilibrium, the net probability flow into any state is zero. This can be written as

$$\sum_{i=0}^{C} P(i) q(i, j) = 0, \qquad j = 0, 1, 2, \ldots, C \quad . \tag{A.3}$$

This provides $C + 1$ equations. Because of (A.2) one of these is redundant. The condition that the sum of all state probabilities is unity provides the additional equation. That is,

$$\sum_{s=0}^{C} P(s) = 1 \quad . \tag{A.4}$$

As a result, we have $C + 1$ simultaneous equations to determine $C + 1$ state probabilities. This is a set of nonlinear equations. The way to solve these equations are in the framework of [7, 8, 10].

For channel rearrangement, the method to calculate $P(s)$ is similar. The only difference is that equation (A.1) needs to be modified as follows

$$q(s, s_n) = r_n(s, s_n) + r_c(s, s_n) + r_d(s, s_n) + r_h(s, s_n) + r_i(s, s_n) + r_{rd}(s, s_n) + r_{ra}(s, s_n) \quad . \tag{A.5}$$

In addition, since the state representation is changed, the total number of states is no longer $C + 1$. All the permissible states are labeled from 0 to s_{max}. The total number of states becomes $s_{max} + 1$. Thus equations (A.2),(A.3),(A.4) can also be applied if C is replaced by s_{max}.

N	i	j	R=1.0	R=1.1	R=1.2	R=1.3	R=1.4	R=1.5
4	2	0	12.29	10.30	8.40	6.58	4.80	3.05
7	2	1	17.82	15.99	14.27	12.66	11.14	9.67
9	3	0	20.20	18.40	16.74	15.18	13.71	12.32
12	2	2	22.86	21.10	19.48	17.96	16.54	15.20
13	3	1	23.59	21.84	20.23	18.72	17.31	15.98
16	4	0	25.48	23.75	22.15	20.67	19.28	17.98
19	3	2	27.03	25.31	23.73	22.26	20.89	19.60
21	4	1	27.93	26.21	24.64	23.18	21.82	20.54

Table 1. The worst case SIR (dB) for various cluster sizes N and coverage radii R

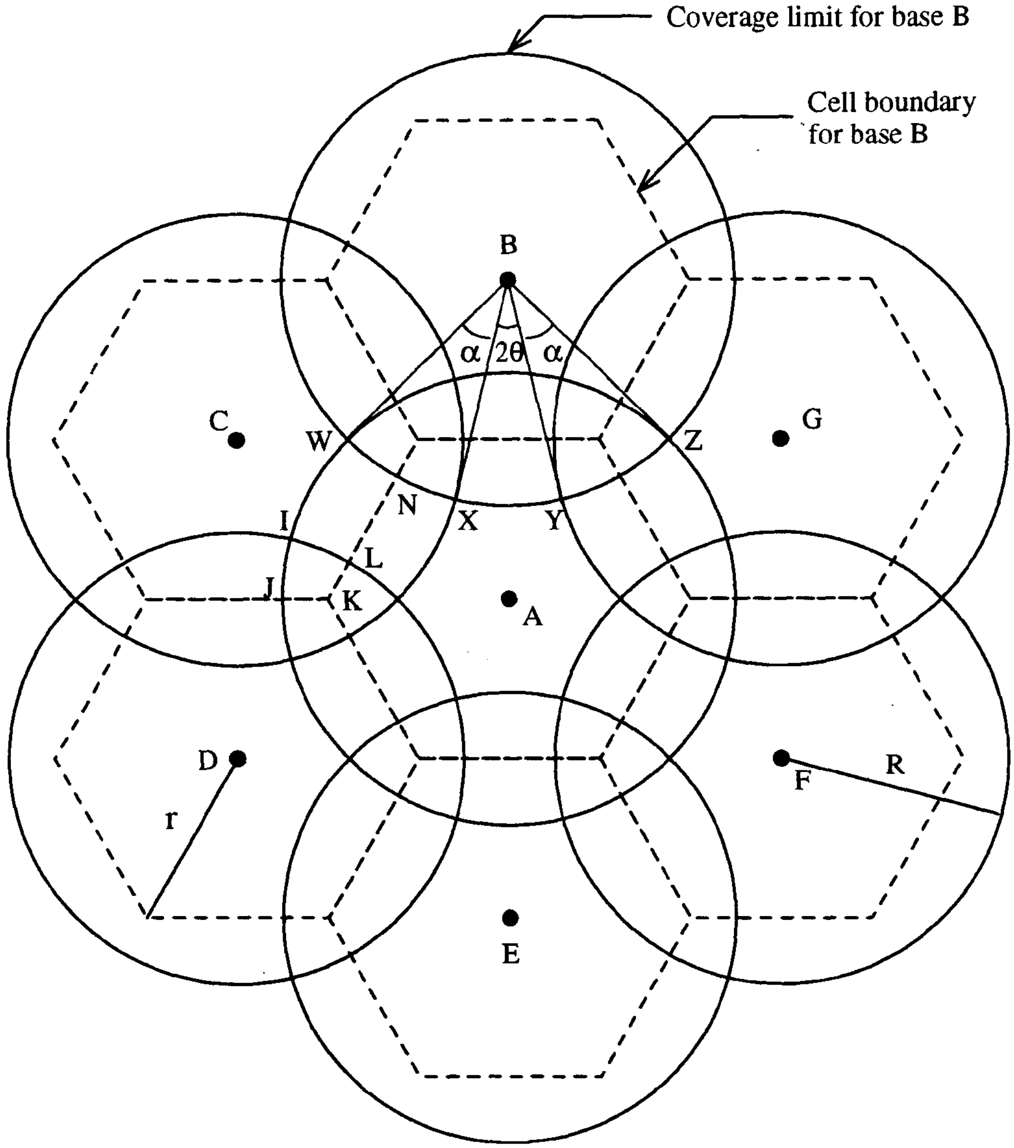

Fig. 1 System layout for overlapping coverage

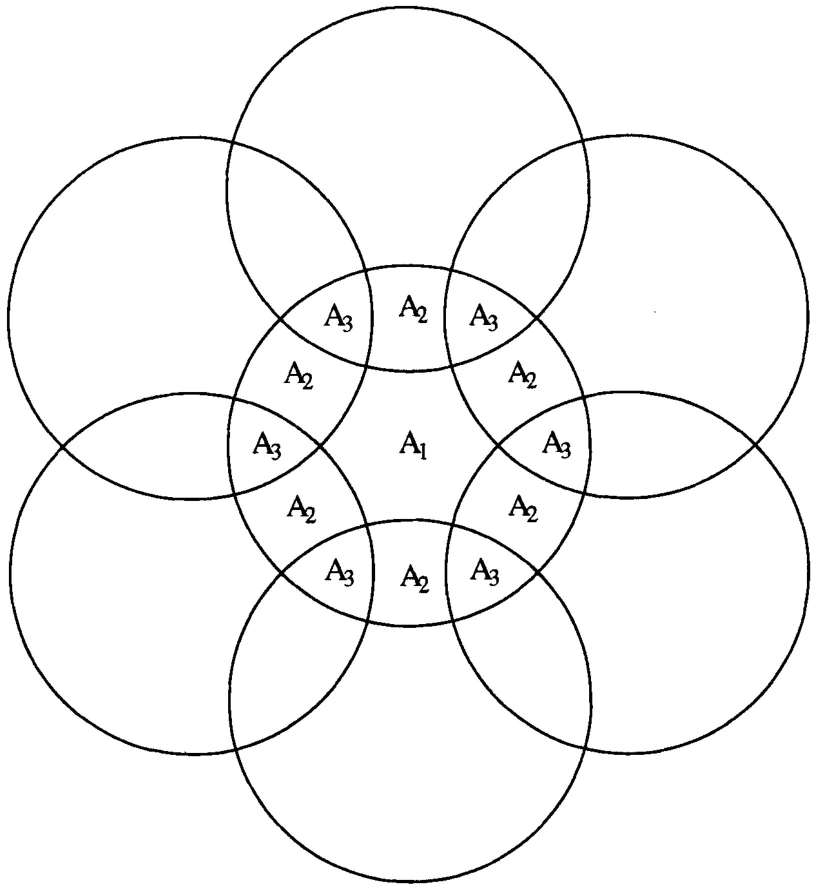

Fig. 2 Three kinds of regions in the coverage area of a base

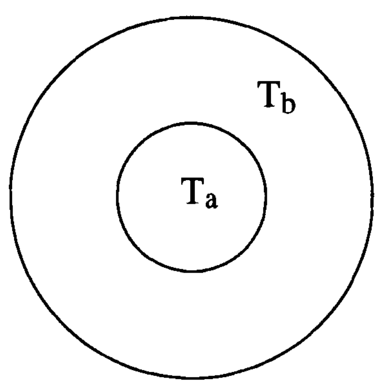

Fig. 3 The dwell time for channel rearrangement

Fig. 4 Blocking Probability

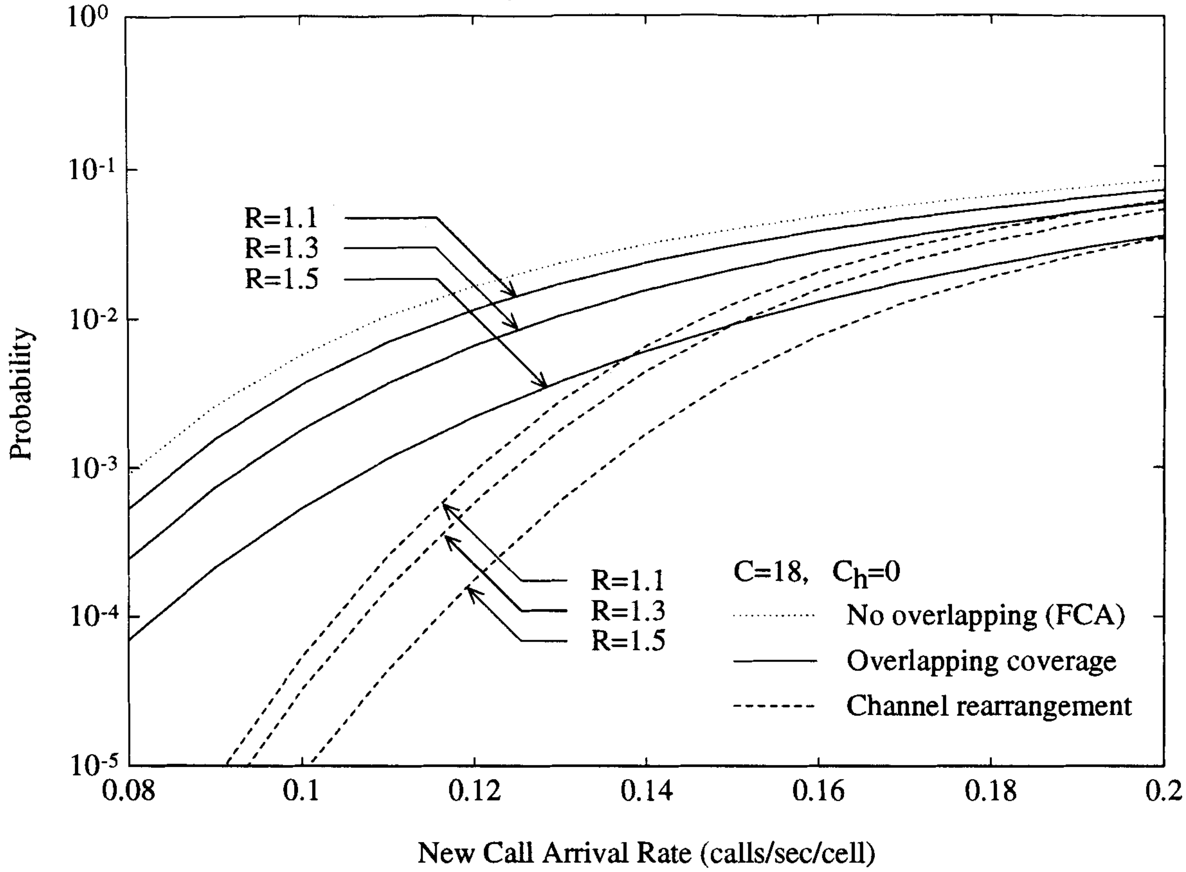

Fig. 5 Forced Termination Probability

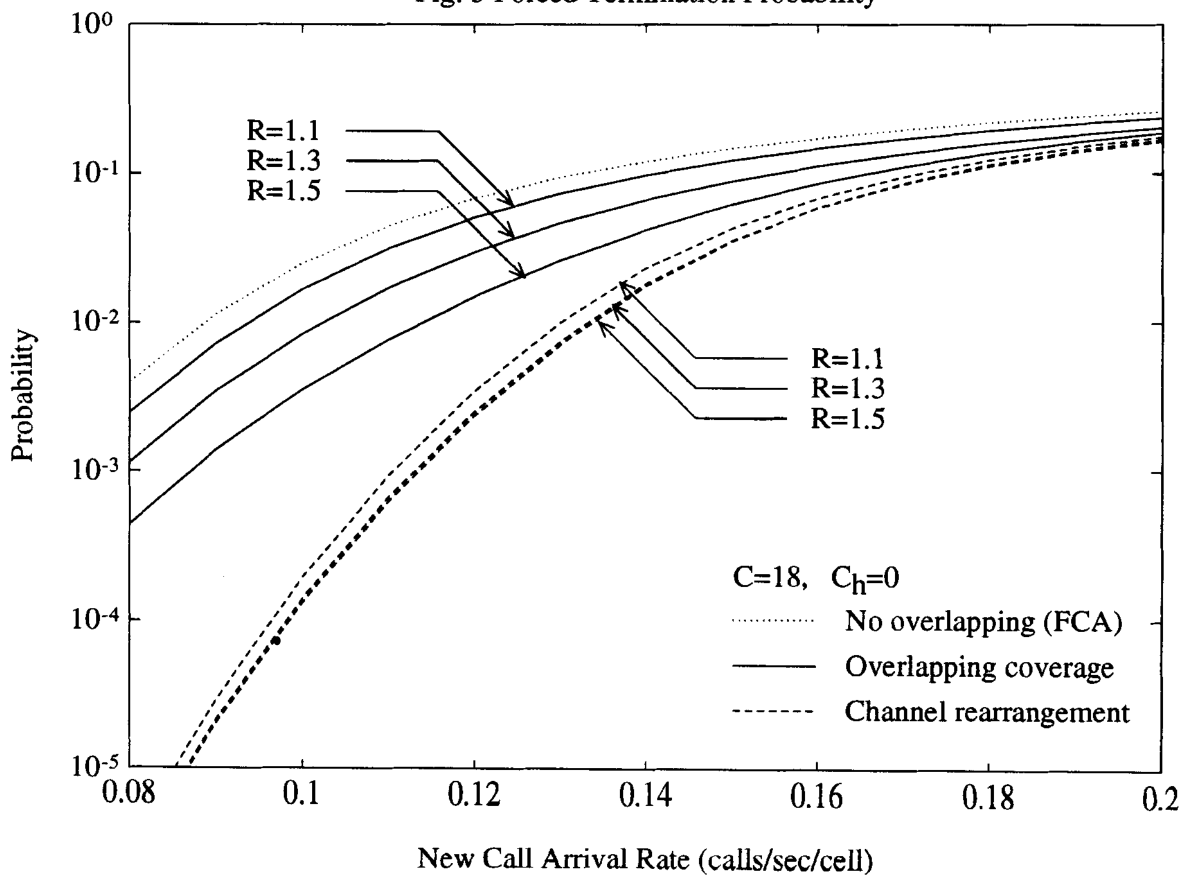

Fig. 6 Hand-off Activity

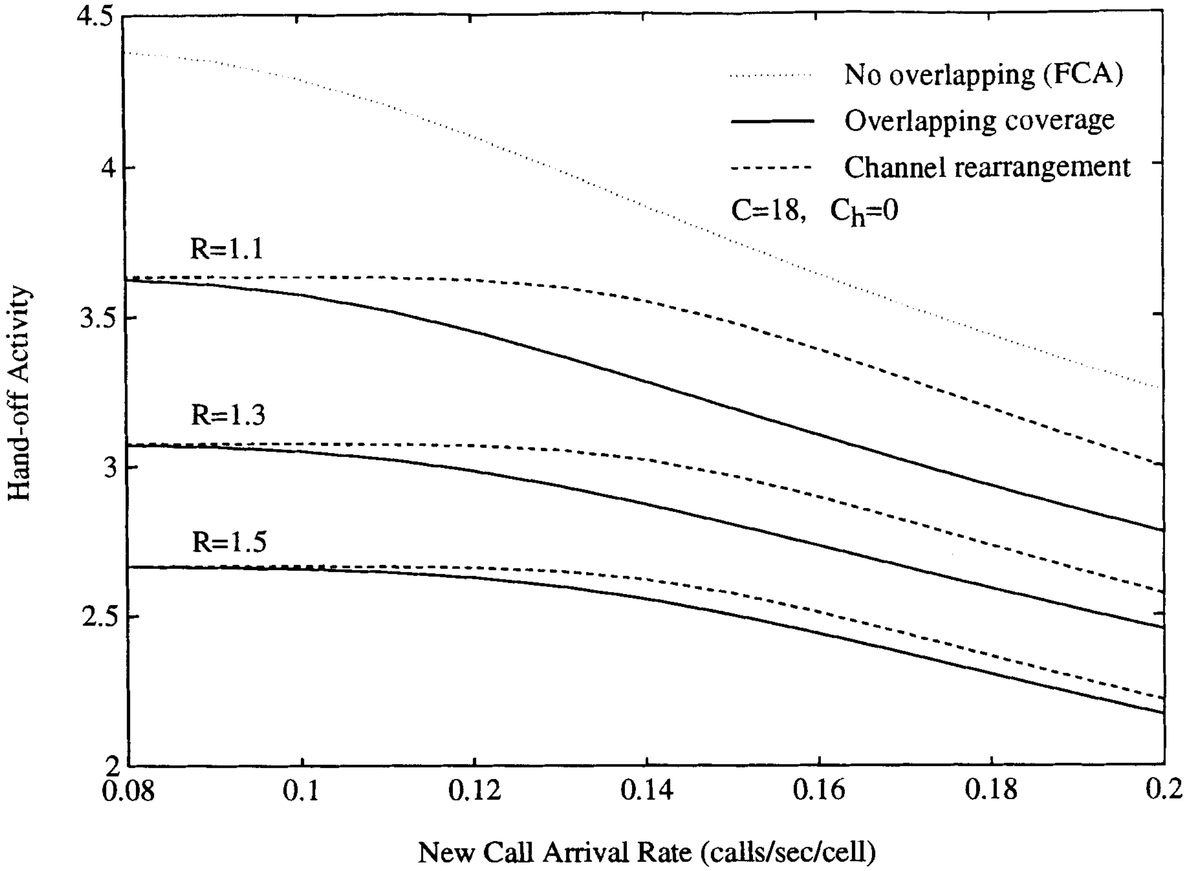

Fig. 7 Carried Traffic