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APPLICATION OF GENERALIZED FUNCTION THEORY TO NETWORK REALIZABILITY THEORY AND TIME DOMAIN SYNTHESIS OF POSITIVE-REAL FUNCTIONS

by

A. H. Zemanian

Contract No.	AF 19(628)-2981
Project No.	5628
Task No.	562806
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FINAL REPORT (Part II)

Period covered: June 1, 1963 to August 31, 1968

August 31, 1968

Contract Monitor: Kurt H. Haase
Data Sciences Laboratory

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ABSTRACT

This is a summary of the work performed under contract AF 19(628)-2981 during the period from June 1, 1963 to August 31, 1968. All significant results have previously been reported in scientific reports and journal publications. The present report summarizes and compares these results.

List of Contributors to the Research Herein

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Ph. D. Theses Supported Under Contract AF 19(628)-2981

Contract AF 19(628)-2981 supported, among other things, the research that culminated in the following Ph. D. theses written in The Department of Applied Analysis, State University of New York at Stony Brook.

1. W. C. Queen, "The n-dimensional Generalized Weierstrass Transformation," May, 1967.
2. J. N. Pandey, "Complex Inversion for the Generalized Convolution Transformation," April, 1967.
3. E. L. Koh, "The Hankel Transformation of Gneralized Functions," June, 1967.

List of Publications Produced Under
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Scientific Reports:

1. A. H. Zemanian, "A Time-Domain Characterization of Rational Positive-Real Matrices." First Scientific Report, AFCRL-63-390, College of Engineering Tech. Rep. 12, State University of New York at Stony Brook; August 5, 1963.

2. A. H. Zemanian, "A Time-Domain Characterization of Positive-Real Matrices." Second Scientific Report, AFCRL-63-391, College of Engineering Tech. Rep. 13, State University of New York at Stony Brook; August 16, 1963.

3. A. H. Zemanian, "The Time-Domain Synthesis of Distributions." Third Scientific Report, AFCRL-64-191, College of Engineering Tech. Rep. 19, State University of New York at Stony Brook; February 1, 1964.

4. A. H. Zemanian, "The Distributional Laplace and Mellin Transformations." Fourth Scientific Report, AFCRL-64-685, College of Engineering Tech. Rep. 26, State University of New York at Stony Brook; August 15, 1964.

5. A. H. Zemanian, "Orthonormal Series Expansion of Certain Distributions and Distributional Transform Calculus." Fifth Scientific Report, AFCRL-64-995, College of Engineering Tech. Rep. 22, State University of New York at Stony Brook; November 15, 1964.

6. A. H. Zemanian, "Applications of Generalized Function Theory to Network Realizability Theory and Time-Domain Synthesis of Positive-Real Functions." Final Report, AFCRL-65-316, College of Engineering Tech. Rep. 41, State University of New York at Stony Brook; May 31, 1965.

7. T. Loughlin, "A Table of Distributional Mellin Transforms," Sixth Scientific Report, AFCRL-65-645, College of Engineering Tech. Rep. 40, State University of New York at Stony Brook; June 15, 1965.

8. A. H. Zemanian, "The Distributional Hankel Transformation," Seventh Scientific Report, AFCRL-65-646, College of Engineering Tech. Rep. 44, State University of New York at Stony Brook; August 9, 1965.

9. A. H. Zemanian, "A Distributional K Transformation with Applications to Time-Varying Electrical Networks," Eighth Scientific Report, AFCRL-65-789, College of Engineering Tech. Rep. 54, State University of New York at Stony Brook; June 1, 1965.

Papers:

1. A. H. Zemanian, "The Time-Domain Synthesis of Distributions." Proceedings of the First Allerton Conference on Circuit Theory, University of Illinois; 1963.

2. A. H. Zemanian, "The Approximation of Distributions by the Impulse Responses of RLC Two-ports." Proceedings of the International Conference of Microwaves, Circuit Theory, and Information Theory; Tokyo; September, 1964.

3. A. H. Zemanian, "The Time-Domain Synthesis of Distributions." IEEE Transactions on Circuit Theory, Vol. CT-11, pp. 487-493; December, 1964.

4. A. H. Zemanian, "A Characterization of the Inverse Laplace Transforms of Rational Positive-Real Functions." J. Soc. Indust. Appl. Math., Vol. 13 (June, 1965), pp. 463-468.

5. A. H. Zemanian, "Some Convergence Properties of Exponential Series Expansions of Distributions." J. Math. Anal. Appl., Vol. 34 (1966), pp. 195-204.

6. H. Konig and A. H. Zemanian, "Necessary and Sufficient Conditions for a Matrix Distribution to Have a Positive-Real Laplace Transform." J. Soc. Indust. Appl. Math., Vol. 13 (1965), pp. 1036-1040.

7. A. H. Zemanian, "The Distributional Laplace and Mellin Transformations." J. Soc. Indust. Appl. Math., Vol. 14 (1966), pp. 41-59.

8. A. H. Zemanian, "Inversion Formulas for the Distributional Laplace Transformation." J. Soc. Indust. Appl. Math., Vol. 14 (1966), pp. 159-166.

9. A. H. Zemanian, "Paper Networks." IEEE Trans. on Circuit Theory, Vol. CT-12 (Sept., 1965) pp. 425-426.

10. A. H. Zemanian, "Paper Networks II." IEEE Trans. on Circuit Theory, Vol. CT-13 (1966), pp. 110-111.

11. A. H. Zemanian, "Orthonormal Series Expansion of Certain Distributions and Distributional Transformations." J. Math. Anal. Appl., Vol. 14 (1966) pp. 263-275.

12. A. H. Zemanian, "The Convolution Transformation of Certain Generalized Functions and its Inversion." Bull. Amer. Math. Soc., Vol. 72 (1966) pp. 725-727.

13. A. H. Zemanian, "A Distributional Hankel Transformation." J. Soc. Indust. Appl. Math., Vol. 14 (1966) pp. 561-576.

14. A. H. Zemanian, "The Hankel Transformation of Certain Distributions of Rapid Growth." J. SIAM Appl. Math., Vol. 14 (1966) pp. 678-690.

15. A. H. Zemanian, "A Distributional K. Transformation." J. SIAM Appl. Math., Vol. 14 (1966) pp. 1350-1365.

16. A. H. Zemanian, "Some Abelian Theorems for the Distributional Hankel and K Transformations." J. SIAM Appl. Math., Vol. 14 (1966) pp. 1255-1265.

17. A. H. Zemanian, "A Generalized Convolution Transformation." J. SIAM Appl. Math., Vol. 15 (1967) pp. 324-346.

18. A. H. Zemanian, "The Weierstrass Transformation of Certain Generalized Functions." Bulletin Amer. Math. Soc., Vol. 73 (1967) pp. 682-684.

19. A. H. Zemanian, "A Generalized Weierstrass Transformation." J. SIAM Appl. Math., Vol. 15 (1967) pp. 1088-1105.

20. A. H. Zemanian, "A Solution of a Division Problem Arising from Bessel-type Differential Equations." J. SIAM Appl. Math., Vol. 15 (1967) pp. 1106-1112.

21. A. H. Zemanian, "Hankel Transforms of Arbitrary Order." Duke Math. Journal, Vol. 34 (1967) pp. 761-770.

22. J. N. Pandey and A. H. Zemanian, "Complex Inversion for the Generalized Convolution Transformation." Pacific Journal of Mathematics (accepted)

23. A. H. Zemanian, "Applications of Generalized Functions to Network Theory." Invited Address, Proceedings of the Summer School on Circuit Theory 1968, Prague, Czechoslovakia (June 28 to July 12, 1968).

24. A. H. Zemanian, "A Frequency-domain Characterization for the Causality of Active Linear Systems." IEEE Transactions on Circuit Theory (accepted).

25. E. L. Koh and A. H. Zemanian, "The Complex Hankel and I Transformations of Generalized Functions." SIAM J. Appl. Math., (accepted).

26. A. H. Zemanian, "The Postulational Foundations of Linear Systems." J. Math. Anal. Appl. (accepted).

27. J. N. Pandey and A. H. Zemanian, "An Extension of Tanno's Form of the Convolution Transformation." Tohoku Math. J. (accepted).

A Summary of Research Results:

This is a report on the research results obtained during the period of June 1, 1963 to August 31, 1968 under the contract AF 19(628)-2981, sponsored by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, United States Air Force, Bedford, Massachusetts. All significant results obtained so far have been reported in the Scientific Reports and research papers listed in the preceding section. Here we shall summarize and correlate these results.

Our research efforts can be classified under the following five categories.

1. The time-domain significance of positive-reality.
2. The time-domain synthesis of distributions.
3. The distributional generalization of integral transformations.
4. The application of generalized integral transformations to time-varying networks.
5. Contributions to the realizability theory of networks.

We shall discuss each of these separately.

1. The Time-Domain Significance of Positive-Reality.

A necessary and sufficient time-domain characterization of a positive-real matrix has been obtained. Report 1 presents the development for the special case where the positive-real function or matrix is rational. It also appears in the Journal of the Society for Industrial and Applied Mathematics as paper 4. Report 2 gives the development in the most general case where there are no restrictions on the positive-real function or matrix.

In the latter case our main conclusions can be stated as follows:

Theorem: Let $w(t)$ be an $n \times n$ matrix distribution and let $W(s)$ be its Laplace transform. The necessary and sufficient conditions for $W(s)$ to be positive-real are the following:

(i) $w(t) = A\delta^{(1)}(t) + w_0(t)$ where A is a real symmetric nonnegative-definite constant matrix, $\delta^{(1)}(t)$ is the first derivative of the Dirac delta functional and $w_0(t) = d^2g/dt^2$ where the elements of the matrix g are continuous functions whose supports are contained in $0 \leq t < \infty$.

(ii) For every $n \times 1$ constant vector y, $y^*[w(t) + w^T(-t)]y$ is a nonnegative-definite distribution, where w^T is the transpose of w, and $y^* =$ complex-conjugate of y^T.

This most general result appears in the Journal of the Society for Industrial and Applied Mathematics as paper 6.

Actually, the nonnegative definiteness of condition (ii) is a reflection of the passivity of a system whose impulse response matrix is $w(t)$. Thus, this gives a concise and yet completely general link between passivity and positive-reality.

2. The Time-Domain Synthesis of Distributions.

In this area we have developed for the first time a method for the time-domain synthesis of a distribution. This method, which is based upon a Fourier series technique, is described in Report 3. Various aspects of it are published in Papers 1 - 3, and 5. An outstanding feature of this technique is that, if the Laplace transform of a given distribution is known, a realizable approximating signal can be written down without any computation; it only requires values of the Laplace transform at various points in its region of convergence. Since ordinary locally integrable functions are special cases of distributions, the technique is significant for the customary time-domain synthesis problem as well.

More recently, our investigations in this area have developed along a somewhat different line. In particular, we have generalized the Lee-Kautz-Huggins time-domain synthesis procedure which uses an orthonormal series expansion technique. Our generalization now allows one to synthesize a distributional signal in the time-domain, whereas the Lee-Kautz-Huggins procedure was previously restricted to ordinary functions. In order to accomplish this, we had to develop a method for expanding distributions into orthonormal series, something that apparently had not as yet been done in mathematics. This mathematically new result is described in Report 5, as well as in a book by A. H. Zemanian entitled, "Generalized Integral Transformations" and published by Interscience Publishers.

3. The Distributional Generalization of Integral Transformations.

The major portion of our research project has been devoted to the problem of extending the classical integral transformations to generalized functions. The subject of integral transformations is a classical one in mathematics and dates back at least 160 years. For example, Laplace introduced his transformation in a paper on probability theory in 1807. Since that time the subject has expanded into a very broad part of mathematics. In fact, there are presently a great number of different integral transformations, and they are used in solving a variety of physical problems.

On the other hand the subject of generalized functions is of recent origin and dates back to about 1947-1948. It has been one of the major innovations in 20th century mathematics and has solved many problems that previously could not be solved by using mathematics. Integral transformations have also been affected by generalized-function theory, but surprisingly only a few of them had been generalized until very recently. In fact, before 1964 the only transformations that had been extended to generalized functions were the following:

Fourier transformation
Laplace transformation
Mellin transformation
Hilbert transformation.

In 1964 we devised a new method of extending the Laplace and Mellin transformation to generalized functions. This work is described in Reports 4 and 7 and in papers 7 and 8. We then turned to other integral transformations that had not up to that time been so extended, and succeeded in doing so for the following transformations:

Hankel transformation
K transformation
Weierstrass transformation
Convolution transformation
Transformations arising from orthonormal series expansions.

The resulting generalized integral transformations have become thereby much more powerful tools and can now be used to solve a greater variety of physical problems. The theory and applications of the generalized Hankel transformation is given in scientific Report 8 and Papers 13, 14, 16, 20, 21 and 25. For the K transformation, see Report 9 and Papers 15 and 16. For the Weierstrass transformation, see Papers 18 and 19. For the convolution transformation, see Papers 12, 17, 22 and 27. For the transformations arising from orthonormal series expansions, see Report 5 and Paper 11.

These results inject modern mathematical concepts into the theory of integral transformations, a subject that has been dominated by classical mathematics. Moreover, the techniques we have used may be applicable to still other integral transformations. Indeed, we feel that we have opened up a new area of applied mathematical research. For example, two outstanding integral transformations that have been very useful in certain boundary-value problems are the Mehler-Fock transformation and the Kontorovich-Lebedev transformation. We plan to attack the problem of their generalization in the near future.

4. The Application of Generalized Integral Transformations to Time-varying Networks.

The generalized Mellin and K transformations allow us to analyze certain time-varying electrical networks whose voltage and current excitations are generalized functions. This is accomplished via the operational calculus generated by these transformations.

In particular, the Mellin transformation is useful when the network under consideration is described by Euler-Cauchy differential equations. These have the form:

$$a_n t^n D^n y(t) + a_{n-1} t^{n-1} D^{n-1} y(t) + \dots + a_o y(t) = f(t) \qquad t > 0$$

where the a's are constants. One class of networks that are of this type are the finite RLC networks wherein the resistors remain fixed with time and the inductors and capacitors vary linearly with time. Particular examples of such networks and their analysis via the gneralized Mellin transformation are given in the book, "Generalized Integral Transformations."

On the other hand, the K transformation can be used to analyze networks that are characterized by differential equations of the form:

$$a_n S_\mu^n y(t) + a_{n-1} S_\mu^{n-1} y(t) + \dots + a_o y(t) = f(t) \qquad t > 0$$

where the a's are constants and S_μ in the differential operator:

$$S_\mu = t^{\mu-\frac{1}{2}} \frac{d}{dt} t^{-2\mu+1} \frac{d}{dt} t^{\mu-\frac{1}{2}} = \frac{d^2}{dt^2} + \frac{1-4\mu^2}{4t^2}$$

Here, μ is any fixed complex number. There are a number of networks of this type. Such, for instance, are the LC networks wherein the inductors vary proportionally to $t^{-2\mu+1}$ and the capacitors vary proportionally to $t^{2\mu-1}$. For a discussion of this, as well as of other networks that can be analyzed by the generalized K transformation, see Report 9.

5. Contributions to the realizability theory of networks.

Our work in this area is concerned with the foundations of network theory and in particular, with the relations between the fundamental physical propertie of networks (such as linearity, causality and passivity) and the properties of the systems functions for networks (such as analyticity and positive-reality).

In Papers 9 and 10 we pointed out that there are infinite networks that are perfectly-well defined in the time domain, but do not possess system functions in the frequency domain in either the classical sense or in L. Schwartz's distributional sense. However, we showed that, if one generalizes the Fourier or Laplace transform by means of the Gelfand-Shilov approach to generalized functions, then the aforementioned infinite networks do possess system functions in the frequency domain. This opens up frequency-domain analysis to a variety of infinite networks that could not previously be so analyzed.

As an illustration of this we showed in Papers 23 and 24 how the causality of (finite or infinite) networks, whether passive or active, could be related to the generalized frequency-domain descriptions of the network. In particular, we achieved a frequency-domain criterion for the causality of every passive or active network that possesses a convolution representation in the time-domain. This is a new result and of very wide applicability.

Finally, in Paper 26 we propose six new sets of axioms for passive and active systems, as well as single-valued and multivalued systems. These axioms are the weakest ones that one can impose and still have a frequency-domain description for the system. They allow for the greatest generality obtained so far in network realizability theory. Moreover, every set of axioms for such systems that have previously appeared in the literature can be related to one of the sets of axioms proposed in Paper 26.

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13. ABSTRACT

This is a summary of the work performed under contract AF 19(628)-2981 during the period from June 1, 1963 to August 31, 1968. All significant results have previously been reported in scientific reports and journal publications. The present report summarizes and compares these results.

14. KEY WORDS	LINK A		LINK B		LINK C	
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