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Abstract - The dynamic behavior of a two-level periodic marketing
network having no market news is investigated. The present analysis
allows the traders to store goods , in contrast to our prior
analysis of such markets. Storage complicates the analysis,
but its overall effect is to ameliorate price swings. It is
shown that the present model has one and only one equilibrium
state for exogeneously given supply and demand functions in its
various markets. Moreover, price disturbances propagate throughout
the network in a step-by-step fashion, progressing no more than
one market between consecutive market days. In this way, the
absence of market news results in a sluggish system, which
nevertheless does convey market information by means of its
trading activities. Under certain circumstances, the marketing
network may generate apparently contradictory price signals;
that is, an oversupply at one market may trigger a shortfall
in another market with the first market sending an initial price
signal for an oversupply through one part of the marketing
network and the second market sending an initial price signal
for a shortfall through another part of the network.

1. Introduction. Periodic marketing networks are ubiquitous throughout the less developed countries. Moreover, they appear in a variety of forms [1], [2], [8]. In this work we continue a study undertaken in [9] of a certain kind of periodic marketing system. In particular, we consider a two-level system of rural periodic markets coupled to urban wholesale markets as indicated by the bipartite graph of Figure 1. This is a system described by Jones [7]. The nodes m_j denote the rural markets, which we refer to as "farmers' markets"; their market days are assumed to occur periodically and simultaneously at the values of the discrete odd-integer-valued time variable $t = \ldots -3, -1, 1, 3, \ldots$. On those days farmers bring to the m_j supplies of a given commodity and sell them to local consumers and also to traders. The traders bulk and transport the commodity to urban wholesale markets, which we denote by the nodes n_k and refer to as the "consumers' markets". The commodity is then resold in the n_k for retailing. The latter event is assumed to take place at $t+1 = \ldots, -4, -2, 0, 2, 4, \ldots$, but in actuality it may take place over a period of days between two consecutive market days for the m_j. The connections set up by the traders between the farmers' and consumers' markets are indicated by the lines in Figure 1.

Jones [7; p. 116] indicates that the traders tend to buy regularly in the same farmers' and consumers' markets and know very little about prices in nearby markets. We idealize this situation by assuming that each trader confines his activities

to one farmers' market and one consumers' market and that he
knows the prices in his own markets but has no information about
prices in the other markets. In particular, we assume that there
is no market news disseminated between markets. Finally, we
assume that no trading goes on between two farmers' markets or
between two consumers' markets.

It should also be noted that our assumption of simultaneity
for the market days of the various farmers' markets conform with
the conflictive arrangement of periodic marketing, in contrast
to the integrative arrangement. As is noted in [2], the
conflictive arrangement is not uncommon; it occurs for example
in Southern South America and Southern Africa.

Our hypothesis that the commodity flows from the rural
areas into the urban centers, as would an agricultural staple,
can be reversed. We can take the commodity to be a good, such
as an item of clothing, that is manufactured in several urban
centers. The commodity is transported by traders from those centers
to the periodic rural markets, where it is retailed. In
this case, we need merely let the m_j denote the urban centers and
the n_k denote the periodic rural markets.

Furthermore, our model may even be applicable to daily
markets where the bulking of the good, perhaps a food staple,
by the traders occurs in the m_j during the early morning hours
and is resold during the day in the n_k. In this case, t would
denote the early morning hours and t+1 the rest of the day.

The analysis we undertook in [9] and continue in the present
work is an economic one, despite the fact that an overall
understanding of periodic markets requires historical, cultural,

and sociological considerations as well [3], [8]. Our narrow approach is justified by the fact that we're asking a narrow question: What are the dynamic variations in the prices and commodity flows in the marketing network of Figure 1 under particular conditions of supply and demand? A question of economics attacked by an economic analysis. Moreover, our approach is founded initially on an atomistic examination of the behavior of each trader, from which the overall behavior of the marketing network is built up through aggregation.

The principal difference between the present analysis and the one given in [9] is that the traders are now allowed to store goods. This complicates the analysis of the consumers' markets n_k; it leads t supply functions in the n_k that are not perfectly inelastic, as they were assumed to be in [9] where no storage was allowed. Nevertheless, most of the results obtained in [9] are once again acheived herein.

The behavior of the traders in the farmers' markets, where they act as buyers, is assumed to be the same as that used in [9]. In the next section we supplement the arguments of [9] justifying that assumed demand behavior with a derivation based upon a treatment of the trader as a firm supplying the services of bulking and transportation. The resulting aggregate demand functions in the farmers' markets are described in Section 3. In Section 4 we develop a supply behavior for each trader in the consumers' markets based upon a cost schedule for his storage of goods, and this is aggregated in Section 5 to get overall supply functions in the consumers' markets. Our total model for the marketing network consists of a set of nonlinear difference

equations obtained by equating supply and demand in every market.
Time series in the various prices and commodity flows can be
obtained from these equations by recursion (Section 6). The
basic existence and uniqueness theorem for the equilibrium state
of our model appears in Section 7. Our model exhibits Jones'
step-by-step transmittal of price disturbances [7]; this is
discussed in Section 8. Finally, we show in Section 9 that a
sudden oversupply at one market can lead to seemingly contradictory
price signals being propagated in different parts of the marketing
network; in particular, in one part the initial price perturbations
may be downward signaling thereby an oversupply whereas in
another part the initial price perturbations may be upward signaling
thereby a shortfall.

2. The demand behavior of the traders in the farmers'
markets. The economic behavior of a trader in a farmers' market
can be derived from the customary analysis of cost functions
[6; pp. 71-75] if we treat the trader as a firm supplying the
services of bulking and transportation. From this point of view,
his output for a particular trip is directly proportional to
the amount q of goods he acquires in the farmers' market.
We shqll take q to be the measure of his output.

As was discussed by Hay and Beavon [5; pp. 30-31], the
costs incurred in providing the output q might include the
following:

1. normal profit

2. interest on stock in trade

3. variable bulking and transport costs (e.g., additional

wear and tear, gas or feed, labor cost, etc. incurred in carrying
the stock between markets)

4. cost of transportation equipment (rent or depreciation
on truck, bicycle, pack animal, etc.)

5. license fees

6. stall rents

Hay and Beavon [5; p. 30] consider normal profit as being
the major cost item. In their words, "... it is the view of
the authors that the major overhead cost is the trader's time.
This can be approached in the usual way as the normal profit,
or irreducible wage cost of the operation. This is consistent
with the definition of normal profits as being that level of
profits sufficient to induce the entrepeneur to stay in business.
Under conditions prevalent in most developing countries this time
can be conceived either as the incentive necessary to induce the
individual to surrender non-working time or (and more likely) as
the opportunity cost, the loss of income due to a reduction of
effort in other directions (e.g., farming)."

We take normal profit to be a variable cost since it can
be avoided simply by not taking the trip between markets and
performing instead an alternative job. The second and third
items are also variable costs. The fourth may be either a variable
or fixed cost depending on whether the trader rents or owns his
transportation equipment. Items 5 and 6 will be either variable
or fixed costs depending on whether they may be paid on a daily
basis or must be paid for an extended period of time.

In any case, the marginal cost (MC), average variable cost (AVC), and average fixed cost (AFC) curves ight resemble those indicated in Figure 2. Those are drawn on the assumption that the variable costs, especially item 1, are considerably larger than the fixed costs and that the marginal cost of the first unit produced is very high because of item 1 but then falls to small values until the trader's transport capacity is approached. The sharp sudden rises in the MC and AVC curves at the largest values of q reflect the fact that item 3 will increase sharply once the trader's transport capacity is exceeded. (We're assuming that the trader cannot acquire an extra truck or pack animal during the middle of his trip.) The standard argument [6; pp. 73-74] asserts that, as q decreases from large values to zero, the trader's demand curve in the farmer's market follows the MC curve downward until the minimum value T of the AVC curve is reached; it then follows a horizontal line $q = T$ toward the ordinate axis, and, when $q = 0$, it coincides with the ordinate axis along the segment $0 \leq p \leq T$. T is thus the critical price for the trader's services above which he will acquire goods in the farmers' market.

Now, to be more specific, we number all the traders by the index $i = 1, 2, 3, \ldots$. Consider the ith trader and assume that he operates between the farmers' market m_j and the consumers' market n_k. Let T^i_{jk} be his cost corresponding to T in Figure 2. (We need not display the subscripts j and k on T^i_{jk} since every i has a unique j and k assigned to it; but we do so for the sake of clarity.) The trader's derived demand curve in m_j at time t can be obtained from his supply curve of Figure 2 by first observing that the quantity of goods he expects to supply in n_k

upon returning from m_j depends upon the price he expects to receive for his services. The latter is the difference between the price $E_k^i(t+1)$ he expects to receive in n_k at $t+1$ and the price $R_j(t)$ he has to pay in m_j at t. That is, upon setting $p = E_k^i(t+1) - R_j(t)$ and referring to the supply curve of Figure 2, we obtain the amount of goods $Q_{jk}^i(t)$ the trader will buy in m_j at t, this being the amount of goods he expects to sell in n_k at $t+1$. Note that, for $E_k^i(t+1) - R_j(t) > T$, $Q_{jk}^i(t)$ is determined by the MC curve. For $E_k^i(t+1) - R_j(t) < T$, $Q_{jk}^i(t) = 0$. Finally, for $E_k^i(t+1) - R_j(t) = T$, $Q_{jk}^i(t)$ may be ambiguous because of the horizontal jump in the supply schedule dictated by the standard theory of the firm.

In view of these remarks, we can redraw the supply schedule of Figure 2 to get the derived demand curve for the trader in m_j at t, as shown in Figure 3. This merely requires the reversal and shifting of the supply curve in Figure 2 with respect to the p axis. Note that, whenever $E_k^i(t+1)$ varies, the derived demand curve of Figure 3 will shift in the vertical direction by the same amount.

Note also that this derivation assumes that the trader knows exactly what his costs are. He doesn't. What we can assume however is that through experience the trader arrives at a demand schedule *much* like that of Figure 3. That experience would give him a fairly good idea of what his per-unit *critical* cost T_{jk}^i of bulking and transporting goods is. For prices above $E_k^i(t+1) - T_{jk}^i$, he would demand nothing in m_j because he would be expecting a loss on any acquisition.

For prices significantly below $E_k^i(t+1) - T_{jk}^i$, he would expect a profit and would fill up most or all of his transport equipment. Finally, it seems unlikely that he would make an "all or nothing" decision at $p = E_k^i(t+1) - T_{jk}^i$. Instead, there should be some transition region where his dubiousness about the expected profit or loss would cause him to follow a more continuous transition from no load to full load, as indicated in Figure 4. This is the demand curve we used in [9] and will continue to use in this paper.

However, there is still another assumption associated with Figure 4 which we should discuss. We are assuming that the trader behaves with perfect consistency. Given any value of $E_k^i(t+1) - R_j(t)$, the amount of goods he acquires in m_j is precisely determined, whatever be the value of t. More realistically, there should be some uncertainty about his actions. This might be modelled by a probability distribution regarding his possible acquisitions. But just which probability distribution should be used requires still another assumption which might be just as difficult to justify. Worse still, such a probabalistic approach might render our model intractable. In any case, something has to be assumed, or this atomistic approach will have to be abandoned. The precise behavior depicted by Figure 4 seems to be a reasonable compromise between realism and tractability.

With regard to the expected price $E_k^i(t+1)$, we could set this equal to some memory function of past prices:

$$E_k^i(t+1) = M_k^i[P_k(t-1), P_k(t-3), P_k(t-5), \ldots]. \tag{2.1}$$

That is, at time t, each trader arrives in his own way at some expected price $E_k^i(t+1)$ in n_k for t+1 from his memory of past prices therein. This should be some weighted average of past prices, where more recent prices are given greater weights. We also require that

$$M_k^i(a, b, c, \ldots) \leq M_k^i(a', b', c', \ldots) \qquad (2.2)$$

if $a \leq a'$, $b \leq b'$, $c \leq c'$, $\ldots$, with strict inequality holding in (2.2) if strict inequality holds for one of the arguments. Moreover, a history of constant prices should lead to the same expected price; that is, M_k^i should satisfy

$$a = M_k^i(a, a, a, \ldots) \qquad (2.3)$$

for every positive value a. The simplest relation satisfying these requirements is

$$E_k^i(t+1) = P_k(t-1). \qquad (2.4)$$

It's the memory used in Ezekial's classical cobweb model [4].

We denote the function of Figure 4 by

$$V_{jk}^i[E_k^i(t+1) - T_{jk}^i - p] \qquad (2.5)$$

where the function $V_{jk}^i(x)$ is continuous for all x, equal to zero for $x \leq 0$, and strictly monotonic increasing for $x \geq 0$. It rises rapidly for small positive values of x and then levels off and approaches a finite value as $x \to \infty$.

3. Supply and demand in a farmers' market. We now proceed exactly as in [9] to set up the supply and demand functions in m_j at time t. These are shown in Figure 5. $S_j^f(p, t)$ denotes the aggregate supply function of all the farmers selling the commodity under consideration in m_j at t. Demand consists on the one hand of local demand from consumers who shop in m_j. Their aggregate demand function is $L_j(p, t)$; a possible form for it is shown in Figure 5. On the other hand, we have the demand functions, shown in Figure 4, of the various traders who operate in m_j. Upon aggregating demand over all the consumers and all the traders who shop in m_j and then equating supply to demand, we get the following equation for clearance in m_j at t.

$$S_j^f(p, t) = D_j^f(p, t) \qquad (3.1)$$

where

$$D_j^f(p, t) = L_j(p, t) + \sum_{k \in K_j} \sum_i V_{jk}^i[E_k^i(t+1) - T_{jk}^i - p].$$

The inner summation on i in the right-hand side is a sum over the indices i for those raders who operate between m_j and n_k. The outer summation is over the index set K_j of the adjacency of m_j. S_j^f and L_j are assumed to be exogeneously given. Equation (3.1) is illustrated in Figure 5.

The solution $p = R_j(t)$ of (3.1) is the clearance price for m_j at t, and the value of either side of (3.1) at that price is the total quantity $Q_j(t)$ exchanged in m_j at t. Moreover,

$$F_{jk}(t) = \sum_i V_{jk}^i \left[E_k^i(t+1) - T_{jk}^i - R_j(t) \right] \qquad (3.2)$$

is the amount of goods transported from m_j to n_k between times t and t+1.

4. The supply and storage behavior of the traders in the consumers' markets.   In [9] we assumed that every trader sold in his n_k at t+1 all the goods he had acquired in his m_j at t and that he accepted whatever price he could get for his goods. In other words, his supply function in n_k was perfectly inelastic. We now wish to relax this restriction by allowing the traders to store goods. If the price in n_k is so low that the trader would incur a loss upon selling goods in n_k, he might instead store at least some of his goods and wait for a better price. In this circumstance, the amount that he stores depends not only on the price in n_k but also on how much it costs him to store goods.

We postulate therefore that the ith trader, who we assume is operating between m_j and n_k, has a per-unit cost $Z_{jk}^i(q)$ for storing goods for one market period (i.e., from t-1 to t+1 for any t); that cost depends upon the amount q being stored. We also postulate that the function Z_{jk}^i has the form depicted in Figure 6. That is, it costs the trader very little to store a few goods, but the per-unit cost increases as the amount stored expands. B_{jk}^i is his storage capacity, and I_{jk}^i is the corresponding per-unit storage cost at full capacity. For $q > B_{jk}^i$, we assume that $Z_{jk}^i(q) = \infty$, which is just another way of saying that the trader won't store any more than B_{jk}^i.

All this leads to a supply schedule in n_k at time $t+1$ for the ith trader as shown in Figure 7. Recall that $R_j(t)$ is the price that the trader has paid in m_j at t. $A^i_{jk}(t-1)$ is the amount of goods in storage between $t-1$ and $t+1$. (We're assuming that storage takes place at n_k.) Also, $C^i_{jk}(t+1)$ is the total amount of goods he has available in n_k at $t+1$; it consists of the amount transported into n_k plus the carry-over stocks from storage. Indeed,

$$C^i_{jk}(t+1) = A^i_{jk}(t-1) + V^i_{jk}[E^i_k(t+1) - T^i_{jk} - R_j(t)]. \qquad (4.1)$$

Now, the question at hand is, "How much will the trader be willing to sell when the price in n_k at $t+1$ is p?"

Remember that T^i_{jk} is the minimum expected price rise from m_j to n_k below which the trader will cease operations in m_j. For an actual price difference larger than T^i_{jk} he makes some profit, perhaps more or less than the profit he was planning on if and when he acquired goods in m_j, but nevertheless a profit. We assume therefore that he is willing to sell the amount

$$V^i_{jk}[E^i_k(t+1) - T^i_{jk} - R_j(t)] \qquad (4.2)$$

he has just transported into n_k so long as he can make some profit on that sale.

With regard to the amount $A^i_{jk}(t-1)$ he has in storage, we assume that he values those goods in the same way as the goods he has just transported into n_k, despite the fact that he may have paid a price for his stored goods different from $R_j(t)$. In other words, his valuation of stored goods is determined by

current market conditions, a practical viewpoint. Consequently, for $p \geq R_j(t) + T_{jk}^i$ we assume that he sells all his goods at hand, namely, $C_{jk}^i(t+1)$.

If however $p < R_j(t) + T_{jk}^i$, the trader will suffer a loss on his sales, and so he must decide whether to incur a loss on sales or a loss resulting from the storage cost between $t+1$ and $t+3$. More precisely, he must decide how much to sell and how much to store. Given a clearance price $p = P_k(t)$ somewhere between $R_j(t) + T_{jk}^i$ and $R_j(t) + T_{jk}^i - I_{jk}^i$, his smallest loss occurs when his per-unit loss on sales equals his per-unit cost of storage for the amount $A_{jk}^i(t+1)$ he decides to store. Any smaller (larger) storage would mean that his per-unit storage loss would be less (respectively, greater) than his per-unit sales loss, and it would therefore pay him to store more (respectively, less) goods. This is the situation depicted in Figure 7, which shows the trader's supply function in n_k at $t+1$. The vertical line at $q = C_{jk}^i(t+1)$ for $p \geq R_j(t) + T_{jk}^i$ results from the reasoning given in the preceding two paragraphs. The curved part of that supply schedule is a reversal and shifting of the storage-cost function of Figure 6. For the clearance price $P_k(t+1)$ indicated, the trader stores $A_{jk}^i(t+1)$ and sells $Q_{jk}^i(t+1)$.

Finally, for $p < R_j(t) + T_{jk}^i - I_{jk}^i$, the trader stores as much as he possibly can, namely, B_{jk}^i and then sells the amount $C_{jk}^i(t+1) - B_{jk}^i$ in excess of that storage capacity at whatever price he can get. This is represented by the lower vertical segment of the supply schedule in Figure 7. This assumes of course that $B_{jk}^i < C_{jk}^i(t+1)$. If $B_{jk}^i \geq C_{jk}^i(t+1)$, then as p decreases, the supply schedule curves in toward the ordinate axis until it either meets

the ordinate axis or meets the abscissa axis. In the former case, it follows the ordinate axis toward the origin. In the latter case, it terminates at the abscissa intercept. An ordinate segment means that the trader exhausts his supply before his storage capacity is filled. An abscissa intercept means that it is cheaper for the trader to give some of his goods away rather than trying to store all of them.

The supply function $S_{jk}^i(p, t+1)$ in n_k at $t+1$ for the ith trader can be written as follows. (As before, we are assuming that the ith trader operates between m_j and n_k.)

$$S_{jk}^i[p, R_j(t), E_k^i(t+1), A_{jk}^i(t-1)]$$

$$= \begin{cases} C_{jk}^i(t+1) & \text{for } p \geq R_j(t) + T_{jk}^i \\[2ex] \max\left\{0, \quad C_{jk}^i(t+1) - W_{jk}^i[R_j(t) + T_{jk}^i - p]\right\} \\ \qquad \text{for } R_j(t) + T_{jk}^i - I_{jk}^i \leq p \leq R_j(t) + T_{jk}^i \\[2ex] \max\left\{0, \quad C_{jk}^i(t+1) - B_{jk}^i\right\} & \text{for } p \leq R_j(t) + T_{jk}^i - I_{jk}^i \end{cases} \qquad (4.3)$$

Here, W_{jk}^i is the function-inverse of Z_{jk}^i. Also, it is understood that p is restricted to positive values.

We mentioned before that the trader values the goods he has at hand in n_k at $t+1$ by the per-unit cost $R_j(t) + T_{jk}^i$; this includes the stored goods as well even though a price different from $R_j(t)$ may have been paid for them. We can relax this assumption - and complicate our model still further - by replacing every $R_j(t)$ in (4.3) by a memory value of prior m_j prices:

$$N_j^i\left[R_j(t),\ R_j(t-2),\ R_j(t-4),\ \ldots\right] \tag{4.4}$$

where N_j^i satisfies conditions similar to those imposed on M_k^i. This would allow greater flexibility in our treatment of trader behavior.

As $R_j(t)$ (or alternatively the value of (4.4)) varies, the supply curve (4.3) shown in Figure 7 shifts vertically, and, as the goods on hand $C_{jk}^i(t+1)$ vary, that curve shifts horizontally. This then is a substantially more complicated model of trader behavior in the n_k than the one used in [9].

5. Supply and demand in a consumers' market. We assume that in each consumers' market n_k at any $t+1$ there is an exogeneously given demand function $D_k^c(p,\ t+1)$ with the conventional slope indicated in Figure 8.

To get the supply function in n_k at $t+1$, we sum the individual supply functions (4.3) for all the traders operating out of n_k. This yields the aggregate supply function

$$S_k^c(p,\ t+1)\ =\ \sum_{j \in J_k} \sum_i S_{jk}^i\left[p,\ R_j(t),\ E_k^i(t+1),\ A_{jk}^i(t-1)\right] \tag{5.1}$$

where J_k is the index set of the adjacency of n_k. $S_k^c(p,\ t+1)$ will have a form like that indicated in Figure 8.

The equation for clearance in n_k at $t+1$ is

$$D_k^c(p,\ t+1)\ =\ S_k^c(p,\ t+1), \tag{5.2}$$

and its solution $p = P_k(t+1)$ is the clearance price. $Q_k(t+1)$ denotes the amount of goods exchanged, namely, the value of either side of (5.2) at clearance.

6. Recursive analysis. In order to perform a recursive analysis on our model and also to establish the theorems presented below, we have to impose precise conditions on the various supply and demand functions, some of which we have already mentioned but now repeat.

Conditions A.

A_1: For each j and t, $L_j(p, t)$ is a continuous nonnegative function on $0 < p < \infty$ and strictly decreasing on $0 < p < R_j^*(t) \leq \infty$. If $R_j^*(t) < \infty$, $L_j(p, t) = 0$ on $R_j^*(t) \leq p < \infty$. If $R_j^*(t) = \infty$, $L_j(p, t) \to 0$ as $p \to \infty$.

A_2: For each j and t, $S_j^f(p, t)$ is a continuous nonnegative function on $0 \leq p < \infty$, equal to zero on $0 \leq p \leq R_j^s(t)$, and strictly increasing on $R_j^s(t) < p < \infty$, where $R_j^s(t) < R_j^*(t)$.

A_3: For each i and its corresponding j and k, $V_{jk}^i(x)$ is a continuous nonnegative function on $-\infty < x < \infty$ such that $V_{jk}^i(x) = 0$ for $x \leq 0$, $V_{jk}^i(x)$ is strictly increasing on $0 \leq x < \infty$, and $V_{jk}^i(x)$ tends to a finite limit as $x \to \infty$. Moreover, it satisfies the Lipschitz condition

$$\left| V_{jk}^i(x) - V_{jk}^i(y) \right| \leq M|x - y|$$

for all x and y. (Since there is only a finite number of traders in the whole marketing system, we can take the constant M to be independent of i, j, and k. It follows that every aggregate function $V_{jk} = \sum_i V_{jk}^i$ also satisfies a Lipschitz condition with a constant that is independent of j and k.)

A_4: For each i and its corresponding j and k, $Z_{jk}^i(x)$ is a continuous nonnegative strictly increasing function on $0 \leq x \leq B_{jk}^i$, and $Z_{jk}^i(0) = 0$.

A_5: For each k and t, $D_k^c(p, t+1)$ is a continuous nonnegative function on $0 < p < \infty$ and strictly decreasing on $0 < p < P_k^*(t+1) \leq \infty$. If $P_k^*(t+1) < \infty$, $D_k^c(p, t+1) = 0$ on $P_k^*(t+1) \leq p < \infty$. If $P_k^*(t+1) = \infty$, $D_k^c(p, t+1) \to 0$ as $p \to \infty$. Also, as $p \to 0+$, $D_k^c(p, t+1) \to \infty$.

We assume that the functions in these five conditions and the T_{jk}^i as well are exogeneously given. Our model then consists of (3.1) and (5.2) for every j and k and for t = 1, 3, 5, ... and in addition some initial conditions. The latter are the initial stored quantities $A_{jk}^i(0)$ and also the consumers' market prices $P_k(0)$, $P_k(-2)$, $P_k(-4)$, ... for all the prices in the arguments of the M_k^i that appear in (2.1) for t = 1. If the memory function (4.4) is used in place of the $R_j(t)$ in (4.3), we will also have to specify those $R_j(-1)$, $R_j(-3)$, $R_j(-5)$, ... (but excluding $R_j(1)$) that appear in (4.4) for t = 1.

Given these initial conditions for all i, j, and k, our Conditions A insure that unique intersections exist between the supply and demand functions in Figures 5 and 8. Indeed, we can compute time series in every price, quantity exchanged, and commodity flow as follows. The initial conditions $P_k(0)$, $P_k(-2)$, $P_k(-4)$, ... determine through (3.1), coupled with (2.1), the farmers' market prices $R_j(1)$ for all j, as indicated in Figure 5. Also determined are the quantities $Q_j(1)$ exchanged at t = 1 in all the farmers' markets, the amounts (4.2) the traders transport just after t = 1, and the $C_{jk}^i(2)$, the last by virtue of the given $A_{jk}^i(0)$ and (4.1). The $R_j(1)$ (and, if (4.4) is used, the $R_j(-1)$, $R_j(-3)$, ...) determine through (5.2), coupled with (5.1) and (4.3), all the consumers' market prices $P_k(2)$. As is indicated in Figure 7, they also determine the amount $A_{jk}^i(2)$ each trader stores and

the amount $Q_{jk}^i(2)$ each trader sells at $t+1 = 2$. Thus, we have
in effect a new set of initial conditions displaced two units
of time later on. Consequently, our computations can be
continued for another period. Continuing in this way, we get
the aforementioned time series.

7. Equilibrium. Our present model deals with three kinds
of variables: prices, commodity flows, and - in contrast to
the prior work [9] - stored amounts as well. The last variable
complicates our discussion of equilibrium states. Such a state
is said to exist if the indicated variables do not vary with
time. (We now denote the constant values for the variables simply
by dropping the arguments in t. Thus, $P_k(t+1)$ is a varying value,
but P_k is a fixed value - not a function.) An equilibrium state
state becomes a possibility when the exogeneously given supply
and demand functions $S_j^f(p)$, $L_j(p)$, and $D_k^c(p)$ are independent of
time. Whether an equilibrium state does exist under these
conditions and whether it is unique is the subject of this section.

Consider the ith trader and assume again that he operates
between m_j and n_k. Since under an equilibrium state all variables
are constant with respect to time and $E_k^i(t+1) = P_k$ for all t
according to (2.3), the amount of goods that trader transports
into n_k on each trip is

$$V_{jk}^i(P_k - T_{jk}^i - R_j).$$ (7.1)

When $P_k > T_{jk}^i + R_j$, (7.1) assumes a positive value, and we have
the situation depicted in Figure 9 where no goods remain in
in storage; C_{jk}^i is equal to (7.1) in this case. On the other

hand, if $P_k \leq T^i_{jk} + R_j$, (7.1) equals zero according to our definition of V^i_{jk}. Thus, in no equilibrium state does the trader perpetually incur a loss on the goods he continually transports into his n_k.

However, equilibrium storage under the condition $P_k < T^i_{jk} + R_j$ is a more complicated affair. According to our model, $S^i_{jk}(p)$ can be any of an infinity of curves, as is illustrated by cases a through e in Figure 10; more specifically, it can be any of the curves obtained by shifting the curve for case e to the left and making it coincide with the ordinate axis below the point where it first meets that axis. Thus, the perpetually stored amount A^i_{jk} can be any value from Q down to zero, the latter value occurring when $S^i_{jk}(p)$ coincides with the ordinate axis for all p (case a). Cases f and g are impossible however. Either one would imply that the amount Q_1 or Q_2 is perpetually transported into n_k, in contradiction to the fact that (7.1) equals zero when its argument $P_k - T^i_{jk} - R_j$ is negative.

Just which possible case will occur depends upon how the equilibrium state arises. For example, assume we have a dynamic variation in prices and flows. If the price $R_j(t)$ in m_j remains at the fixed price R_j but the price $P_k(t+1)$ in n_k jumps suddenly down to the fixed price P_k from prior values well above $T^i_{jk} + R_j$, the curve of case e may ensue after possibly one more sale of goods. The last step may occur as follows. The trader comes to n_k with more goods than Q, finds a low P_k (fixed henceforth), sells all his goods in excess of $Q = A^i_{jk}$, and then remains with the supply curve of case e while storing Q perpetually.

As another example, assume that the price $P_k(t+1)$ gradually approaches a fixed $R_j + T^i_{jk}$ so that the amount brought into n_k approaches zero, but, before these limits are acheived, the $P_k(t+1)$ falls to and stays at the considerably lower value shown in Figure 10. Then a supply curve quite close to case a will ensue. Intermediate cases between a and e can be generated by assuming other paths of convergence for $R_j(t)$ and $P_k(t+1)$.

The equilibrium behavior can be summarized as follows. In an equilibrium state, each trader either sells in his n_k a fixed amount of goods at every value of t+1 or he perpetually keeps in storage a fixed amount of goods, or he neither sells nor stores. However, he cannot both sell and store; f he sells goods, he stores nothing, and, if he stores goods, he sells nothing.

We should comment still further about the fact that our model allows storage to occur perpetually in an equilibrium state. This means that the trader never gives up hoping that the price in n_k will improve sufficiently to allow him to sell at a profit. In this respect our model is not a long-run one. Any trader should eventually recognize that the equilibrium price is not going to improve and would then liquidate his stock to cut his storage costs, perhaps by consuming it himself. He may on the other hand start selling off his stock. In either circumstance, case a should ensue, but the price P_k may be perturbed while the trader sells off his stock. Our model does not encompass this eventual collapse of the permissible supply curves of Figure 10 into the ordinate axis.

Only the stored amounts A_{jk}^i exhibit the ambiguity in the equilibrium state illustrated in Figure 10. The prices and commodity flows are uniquely determined. This can be proven by relating the equilibrium states of our present storage model to the equilibrium states of our no-storage model of [9] as follows.

Let there be given a particular storage model. If discard the cost schedules Z_{jk}^i and assume instead that each trader has a perfectly inelastic supply schedule in his n_k, we get a uniquely determined no-storage model, which we shall refer to as the associated no-storage model. Now, if the storage model has an equilibrium state, then the associated no-storage model has one too with the same prices and quantity flows. To find the latter, note first of all that every trader's equilibrium price P_k in the storage model can occur only where his supply schedule is perfectly inelastic (or at an endpoint of such a range), as is indicated in Figures 9 and 10, and his corresponding equilibrium commodity flow is the amount (7.1). By extending the perfectly inelastic segment where P_k occurs, we get the supply schedule for the no-storage model. Thus, with regard to Figure 8, under equlibrium the supply schedule for the associated no-storage model is a vertical line passing through the equilibrium point (Q_k, P_k) of the storage model.

Conversely, still given the storage model, we know from Theorems 2 and 3 of [9] that the associated no-storage model has a unique equilibrium state. In that state the ith trader transports a nonzero quantity (7.1) if and only if $P_k - T_{jk}^i - R_j > 0$; in this case his supply function in the storage model for equilibrium

can be constructed for prices below $R_j + T^i_{jk}$ from his given cost schedule Z^i_{jk} in accordance with Figure 9. On the other hand, that trader transports no goods if and only if $P_k - T^i_{jk} - R_j \leq 0$; Z^i_{jk} can again be used to construct his supply curve under equilibrium as one of the permissible schedules indicated in Figure 10. However, just which supply curve ensues depends upon the amount stored; the latter can be any amount between 0 and $Q = W^i_{jk}(R_j + T^i_{jk} - P_k)$. In any case, from the equilibrium state of the associated no-storage model, this process yields an equilibrium state for the storage model, where the aggregate supply schedule $S^c_k(p)$ in n_k is uniquely determined for prices below P_k but may be ambiguous above P_k. We can therefore conclude this argument with

Theorem 1. Let there be given a periodic marketing network, as specified in the penultimate paragraph of Section 6, which satisfies Conditions A. Assume that the given $L_j(p)$, $S^f_j(p)$, and $D^c_k(p)$ do not vary with time t. Then, that marketing network has an equilibrium state which is unique with respect to the prices R_j and P_k and the commodity flows $V^i_{jk}(P_k - T^i_{jk} - R_j)$. However, the amount A^i_{jk} stored by each trader can be any nonnegative value up to and including $W^i_{jk}(R_j + T^i_{jk} - P_k)$.

The question of the asymptotic stability of an equilibrium state is obscured by the storage behavior. Indeed, consider an equlilibrium point wherein some traders are storing goods. Each such trader has a supply curve like those indicated in Figure 10. We can alter amount the trader stores to some extent so long as his supply curve remains within the range between cases a and e. This does not change the equilibrium point because the aggregate

supply curve $S_k^c(p)$ has been altered only for prices above the equlibrium price P_k, not at P_k. We can conclude that no equlibrium state involving some storage is asymptotically stable because it is a member of a continuum of equilibrium states.

However it may be true that the prices and flows of any equilibrium state demonstrate asymptotic stability, even though the stored amounts do not. We haven't been able to show this. Our conjecture seems especially reasonable for the case where $P_k - T_{jk}^i - R_j > 0$ for every trader. In this case, no trader stores goods, and in the vicinity of the equilibrium point we have once again the same supply and demand curves of our no-storage model of [9]. That model does demonstrate asymptotic stablity if sufficiently strong assumptions are imposed on the slopes of the demand and supply curves [9; p. 524]. To transfer that result to our present model, we would have to show that any equilibrium state satisfying $P_k - T_{jk}^i - R_j > 0$ for every trader is stable in the sense of Lyapunov. We haven't been able to prove this either.

8. The propagation of disturbances. Since we are assuming that there is no market news at all, the only way a distrubance in supply or demand can propagate is through the trading activity. As a result, any disturbance can progress no more than one market from one value of t to the next. This makes the system sluggish. Moreover, a local disturbance is not ameliorated by the resources of the entire system, as it would be in a developed economy having good transportation, warehouses, wholesale and retail facilities, and of course market news. It should therefore be expected that local prices are more volatile for local

disturbances but more insensitive to variations in distant markets. This is the phenomenon discussed by Jones [9; p. 119]. Our objective in this section is to show how our mathematical model exhibits a similar step-by-step propagation of a disturbance.

In this section, we continue analyzing the more general model wherein the memory function (2.1) and (4.4) are used. We need some more notation. Set

$$F_{jk}^i(t) = V_{jk}^i\left[E_k^i(t+1) - T_{jk}^i - R_j(t)\right] \tag{8.1}$$

and

$$F_{jk}(t) = \sum_i F_{jk}^i(t). \tag{8.2}$$

$F_{jk}(t)$ is the flow of goods along the line (j, k) connecting m_j to n_k; this flow occurs at the end of market day t and before t+1.

Let there be two dynamic processes for which the supply and demand functions $S_j^f(p, t)$, $L_j(p, t)$, and $D_k^c(p, t+1)$ are the same everywhere for all t except at m_1 for t = 1 (and possibly at m_1 for t = 3, 5, 7, ... too). We will denote the variables for the two processes with either a superscript 1 or a superscript 2 and will refer to the two processes as the first and second processes. We partition the two sets of market indices as follows: $Y_1 = \{1\}$; i.e., Y_1 is the set consisting only of the index 1. For $\nu = 3, 5, 7, \ldots$, we let Y_ν (or $Z_{\nu-1}$) denote the set of indices for those m_j (respectively, n_k) that are at a distance $\nu - 1$ (respectively, $\nu - 2$) from m_1. This partitioning is illustrated in Figure 11, which is simply a rearrangement of Figure 1. There is no line between any two nodes of a single index set.

Choose a shortest path P between m_1 and m_j (or n_k).
Consider the following sequence of flows for the ωth process
$(\omega = 1, 2)$:

$$\left\{F^{\omega}_{1k_2}(1), F^{\omega}_{j_3 k_2}(3), F^{\omega}_{j_3 k_4}(3), F^{\omega}_{j_5 k_4}(5), F^{\omega}_{j_5 k_6}(5), \ldots\right\} \qquad (8.3)$$

where $(1, k_2)$ is the first line in P, (j_3, k_2) is the second line
in P, (j_3, k_4) is the third line in P, etc. Therefore, $k_2 \in Z_2$,
$j_3 \in Y_3$, $k_4 \in Z_4$, $j_5 \in Y_5$, $\ldots$. This sequence is called an
m_1-to-m_j passage (or an m_1-to-n_k passage) for the ωth process.
¶ We shall say that coincident cutoff occurs for the given passage
if either or both of the following conditions hold.

1. There is at least one term in the given passage (8.3)
that is zero for both processes.

2. There is at least one consumers'-market index k_ν
appearing in the given passage (8.3) such that all the goods
brought into n_k along the $(j_{\nu-1}, k_\nu)$ line go into storage at
n_{k_ν} at time $t+1 = \nu$ for both processes.

We call the first event a coincident flow citoff and the
second event a coincident storage cutoff.

Theorem 2. Assume that there are two dynamic processes
for a given periodic marketing network of the form of Figure 1,
which satisfy Conditions A and the following three hypotheses.

α. The initial conditions for both processes are the same;
that is, for $t = 1, -1, -3, \ldots$, $P^1_k(t-1) = P^2_k(t-1)$ for all k,
$R^1_j(t-2) = R^2_j(t-2)$ for all j, and $A^{i,1}_{jk}(0) = A^{i,2}_{jk}(0)$ for all
i, j, and k.

β. $S_1^{f,1}(p, 1) < S_1^{f,2}(p, 1)$ for all p such that $S_1^{f,2}(p, 1) > 0$.

γ. Other than the supply function of Hypothesis β, all the exogeneously given supply and demand functions (i.e., those indicated in Conditions A_1, A_2, and A_5) are the same for both processes.

Then, for every t = 1, 3, 5, ..., we have the following results.

a. For every $j \in Y_t$, $R_j^1(t) \geq R_j^2(t)$. Strict inequality holds if and only if there is at least one m_1-to-m_j passage having no coincident cutoff.

b. For every $j \in Y_{t+2} \cup Y_{t+4} \cup \ldots$, $R_j^1(t) = R_j^2(t)$.

c. for every $k \in Z_{t+1}$, $P_k^1(t+1) \geq P_k^2(t+1)$. Strict inequality holds if and only if there is at least one m_1-to-n_k passage having no coincident cutoff.

d. For every $k \in Z_{t+3} \cup Z_{t+5} \cup \ldots$, $P_k^1(t+1) = P_k^2(t+1)$ and $A_{jk}^{i,1}(t+1) = A_{jk}^{i,2}(t+1)$ whatever be i and j.

Proof. Except for the conclusion about the stored amounts, this theorem is the same as Theorem 1 in [9], but the proof is more complicated now because of the possibility of storage. Moreover, in contrast to [9], we now allow the traders to have different memory functions.

It follows from Hypothesis ⊀ that the aggregate demand functions in m_1 are the same for both processes. So, by Hypothesis β, by the strict monotonicities of the supply and demand functions in m_1, and by the conditions that $R_j^{s,1}(1)$ and $R_j^{s,2}(1)$ are both less than $R_j^*(1)$, we have that $R_j^1(1) > R_1^2(1)$. Now, let $k \in Z_2$. By the strict monotonicity of the V_{1k}^i, we also have that $F_{1k}^1(1) < F_{1k}^2(1)$ except when $R_1^2(1) \geq E_k^{i,2}(1) - T_{1k}^i$ for every

trader i operating between m_1 and n_k. In the latter case, $F_{1k}^1(1) = F_{1k}^2(1) = 0$; that is, a coincident flow cutoff occurs on the m_1-to-n_k passage at $t = 1$. Moreover, Hypotheses α and γ imply that $R_j^1(1) = R_j^2(1)$ for $j \in Y_3 \cup Y_5 \cup \ldots$.

Continue to let $k \in Z_2$. Just after market-day 1, the amounts stored in n_k and amounts brought into n_k by the traders from the farmers' markets other than m_1 are the same for both processes. In view of our conclusion about the prices in the various farmers' markets, we also have

$$N_{jk}^i \left[R_j^1(1), R_j^1(-1), R_j^1(-3), \ldots \right]$$

$$\geq N_{jk}^i \left[R_2^2(1), R_j^2(-1), R_j^2(-3), \ldots \right]$$

with strict inequality holding for $j = 1$ and equality holding for $j \in Y_3 \cup Y_5 \cup \ldots$. Consequently, $S_k^{c,1}(p,2) \leq S_k^{c,2}(p,2)$ for all p. By the monotonicities of the supply and demand functions in n_k, we have $P_k^1(2) \geq P_k^2(2)$. Now, not only can coincident flow cutoff occur along the m_1-to-n_k line but also coincident storage cutoff can occur in n_k. The latter occurs when the supply curves for all the m_1-to-n_k traders are of the form of cases a to e in Figure 10 for both processes at time $t+1 = 2$. This means that the aggregate supply functions for both processes in n_k at $t+1 = 2$ differ from each other only above the point where they intersect the $D_k^c(p, 2)$ curve. We can therefore conclude from the strict monotonicity of $D_k^c(p, 2)$ that $P_k^1(2) > P_k^2(2)$ if and only if coincident cutoff does not occur on the m_1-to-n_k passage. On the other hand, for $k \in Z_4 \cup Z_6 \cup \ldots$, we have $P_k^1(2) = P_k^2(2)$ and $A_{jk}^{i,1}(2) = A_{jk}^{i,2}(2)$ for every m_j adjacent to n_k, whatever be i,

because conditions are the same for both processes in m_j at $t = 1$ and in n_k at $t+1 = 2$.

We now construct an inductive argument. Assume that Conclusions c and d both hold for all values of t up to and including a given $t \geq 1$. We have shown that they hold for $t = 1$. By virtue of (2.2), we have for $j \in Y_{t+2}$ that $D_j^{f,1}(p, t+2) \geq D_j^{f,2}(p, t+2)$ with strict inequality holding for at least some $p > 0$ when coincident cutoff does not occur on the m_1-to-m_j passage. In view of Hypothesis γ and the monotonicities of the supply and demand functions, $R_j^1(t+2) \gtrsim R_j^2(t+2)$ for $j \in Y_{t+2}$. Strict inequality occurs if and only if the $S_j^f(p, t+2)$ curve intersects the $D_j^{f,1}(p, t+2)$ and $D_j^{f,2}(p, t+2)$ curves at different points. But, this occurs if and only if there is a $k \in Z_{t+1}$ in at least one m_1-to-m_j passage for which the condition in Conclusion c for strict inequality holds (i.e., no coincident cutoff occurs on the m_1-to-n_k passage) and in addition $F_{jk}^1(t+2) > 0$ (i.e., the difference in $P_k^1(t+1)$ and $P_k^2(t+1)$ is effective in m_j). In other words, $R_j^1(t+2) > R_j^2(t+2)$ if and only if there is some m_1-to-m_j passage having no coincident cutoff. This establishes Conclusion a with t replaced by t+2.

Conclusion b with t replaced by t+2 follows directly from Hypothesis γ and our assumption that Conclusion d holds for the given t. (The two processes are the same at a given market until the perturbation at m_1 reaches that market.)

Next, for $j \in Y_{t+2}$ and $k \in Z_{t+3}$, we have from Conclusion d that, for every i, $E_k^{i,1}(t+3) = E_k^{i,2}(t+3)$, and therefore

$$V_{jk}^i \left[E_k^{i,1}(t+3) - T_{jk}^i - p \right] = V_{jk}^i \left[E_k^{i,2}(t+3) - T_{jk}^i - p \right]$$

for all p. Hence, if $R_j^1(t+2) > R_j^2(t+2)$ and if $R_j^2(t+2) <$ $E_k^{i,2}(t+3) - T_{jk}^i$ for at least one i, we have from the strict monotonicity of the $V_{jk}^i(x)$ for $x > 0$ that $F_{jk}^{i,1}(t+2) < F_{jk}^{i,2}(t+2)$ for some i. On the other hand, $F_{jk}^{i,1}(t+2) = F_{jk}^{i,2}(t+2)$ for all i if either $R_j^1(t+2) = R_j^2(t+2)$ or $R_j^2(t+2) \geq E_k^{i,2}(t+3) - T_{jk}^i$ for all i. The former condition holds if and only if coincident cutoff occurs in every m_1-to-m_j passage, and the latter condition holds if and only if $F_{jk}^1(t+2) = F_{jk}^2(t+2) = 0$ (i.e., coincident flow cutoff occurs in the (j, k) line at t+2). Thus, we have shown that $F_{jk}^1(t+2) \leq F_{jk}^2(t+2)$ and that strict inequality holds if and only if either one of the two stated coincident cutoffs occur. However, the carry-over storage in n_k from t+1 to t+3 is the same under both processes.

On the other hand, for $j \in Y_{t+4}$ and $k \in Z_{t+3}$, it follows from Hypotheses α and γ that $F_{jk}^1(t+2) = F_{jk}^2(t+2)$. Upon combining this with our prior conclusions concerning flows, prices, and carry-over storage, we can see that $S_k^{c,1}(p, t+3) \leq S_k^{c,2}(p, t+3)$. Since $D_k^{c,1}(p, t+3) = D_k^{c,2}(p, t+3)$, we have that $P_k^1(t+3) \geq P_k^2(t+3)$. Strict inequality occurs if and only if the $S_k^{c,1}(p, t+3)$ curve and the $S_k^{c,2}(p, t+3)$ curve intersect the demand curve at different points. But, in view of our preceding paragraph, this occurs if and only if there is some trader, say the ith trader, operating between m_j and n_k, where now $j \in Y_{t+2}$, for whom $0 \leq F_{jk}^{i,1}(t+2)$ $< F_{jk}^{i,2}(t+2)$ and who sells some of his supply under the second process, that is, not all of $F_{jk}^{i,2}(t+2) + A_{jk}^i(t+1)$ goes into storage ($A_{jk}^i(t+1)$ is the same for both processes). All this is

the same as saying that coincident cutoff does not occur on an m_1-to-n_k passage that includes the line on which that trader operates. This proves Conclusion c with t replaced by t+2.

Finally, Conclusion d with t replaced by t+2 follows exactly as did Conclusion b with t replaced by t+2. This completes the proof.

Theorem 2 asserts that, if a sudden fall or a sudden rise in supply occurs in one market, then the disturbance in the price structure can propagate throughout the marketing system no faster than one line (in Figure 1) for each increment in t. Thus, if a second market is at a distance of d lines away from the initially disturbed market, then the disturbance reaches the second market d time units later at the earliest. But, the minimum propagation time d can be realized only if cutoff, of either the flow or storage variety, does not interfere with the propagation of the price signal. If cutoff does interfere, then a price signal may still reach the second market at times later than the minimum possible time, but under certain circumstances it may be of the wrong kind. That is, if a sudden oversupply occurs at the first market and if cutoff prevents the resulting price disturbance from reaching the second market within d units of time, then the first price signal received at the second market might be a rise in price signalling a shortfall - rather than the fall in price that would first be felt had cutoff not occurred. We discuss this phenomenon in the next section. Moreover, even when cutoff does not occur, the subsequent prices at the second market may oscillate up and down after the initial price signal has passed through. This can be shown by example.

In any case, what we have established in this section is that
our model exhibits the sluggish step-by-step propagation of
price signals that was described by Jones [9; p. 119].
Our discussion so far has been based on a disturbance in supply
in a farmers' market. But, other kinds of disturbance are also
possible. There could be a sudden shift in local demand in a
farmers' market or in a consumers' market. These would simply
lead to minor variations of Theorem 2 and its proof.

Before leaving this section, we should also note that
storage has its customary effect of moderating price swings.
Indeed, storage bends the supply curve in n_k to the left as
p decreases, the result being that at low prices some goods
may go into storage rather than being thrown upon the market.
This prevents the price from falling as low as it would were
there no storage. Similarly, a short supply and its consequent
high price may trigger the sale of goods from storage. This
prevents the price from rising as high as it would were there
no storage.

9. The confusion in price signals caused by cutoff. Let us
see how a sudden oversupply at a farmers' market may be initially
felt at a distant market as a rise in price, rather than a fall
in price. Consider the marketing network of Figure 12. Assume
that the network is in an equilibrium state such that the (i, k)
line is cutoff with regard to flows whereas the (l, k) lines
are carrying flows but are not far from cutoff. Also, assume
that the flow along the (l, k) line is much heavier than the
flow along the (j, k) line. Then, a sudden oversupply at m_j at
t = 1 will result in a fall in price in n_k at t+1 = 2. Since

the (i, k) line is already cutoff, it remains so. However, since the (l, k) line is close to cutoff, the fall in price at n_k may induce cutoff in the (l, k) flow at t = 3. If the equilibrium flow in (l, k) was much larger than the flow in (j, k), the effect of cutoff in (l, k) may be to raise the price in n_k at t+1 = 4 above its initial equilibrium value, even though the flow in (j, k) may not have cut off. But now, this subsequent rise in price in n_k may trigger a flow in (i, k) at t = 5. Thus, the first price variation in m_i caused by the oversupply at m_j is a rise - not a fall - in price. So long as the rest of the marketing network does not cutoff, the initial price variation propagating upward beyond m_i will continue to be an increase - not a decrease.

On the other hand, if no cutoff occurs in the (j, s) line or in any line beyond n_s, the first received price variation that propagates downward from m_j in Figure 12 will be a decrease, indicating an oversupply. In this way, we can have a price signal propagating upward from m_i indicating a shortfall and another price signal propagating downward from m_j indicating an oversupply. Although they seem contradictory, neither of them are false signals. The first one is communicating the shortfall in n_k caused by the cutoff phenomenon, and the second one is communicating the initial oversupply in m_j.

Upon continuing the above reasoning, we can also conclude that the subsequent price variations in any market may be oscillatory.

It should be noted that a shortfall in m_j leads to a different phenomenon. That shortfall at t = 1 will induce a

price rise in n_k at t+1 = 2. A price signal may or may not
reach m_i at t = 3 depending on whether (i, k) starts flowing or
remains cut off at t = 3. But, in the former case, m_i will
receive a proper signal for a shortfall, namely, a price rise.
To put all this another way, since (i, k) is cutoff in the
equilibrium state, the only way a price disturbance can reach
m_j is that that disturbance be a price rise in n_k sufficiently
large to induce a flow in (i, k) and thereby a price rise in m_i.

The question remains as to how commonly such seemingly
contradictory initial price signals occur in actual periodic
markets. That depends for one thing on how often lines cut off.
Is it a common phenomenon for a trader to refrain from trading
on a particular market day if he dislikes the prices facing him,
and, if so, are there any days when all the traders operating
between two markets refrain from trading because of unfavorable
prices? The latter will be unlikely if there are many traders
with different T_{jk}^i. If the latter is a rare phenomenon, then
contradic tory initial price signals may also be rare.

Cutoff is more likely when the prices are low in the
consumers' markets and high in the farmers' markets. The former
might be caused by a loss in demand in the consumers' markets
due perhaps to an increase in the supply of a substitute commodity.
The latter may result from a loss in supply in the farmers' markets
due perhaps to a bad harvest. The overall result can be that
the marketing network breaks up into a number of isolated networks.

In any case, our model indicates that periodic markets can
genrate disagreeing signals when cutoff occurs, that cutoff is

more likely if there are just a few traders rather than many traders with many different values of T^i_{jk}, and that accurate and fast market news, which would counter the effects of cutoff as well as the sluggish step-by-step propagation of signals, would be a desirable development.

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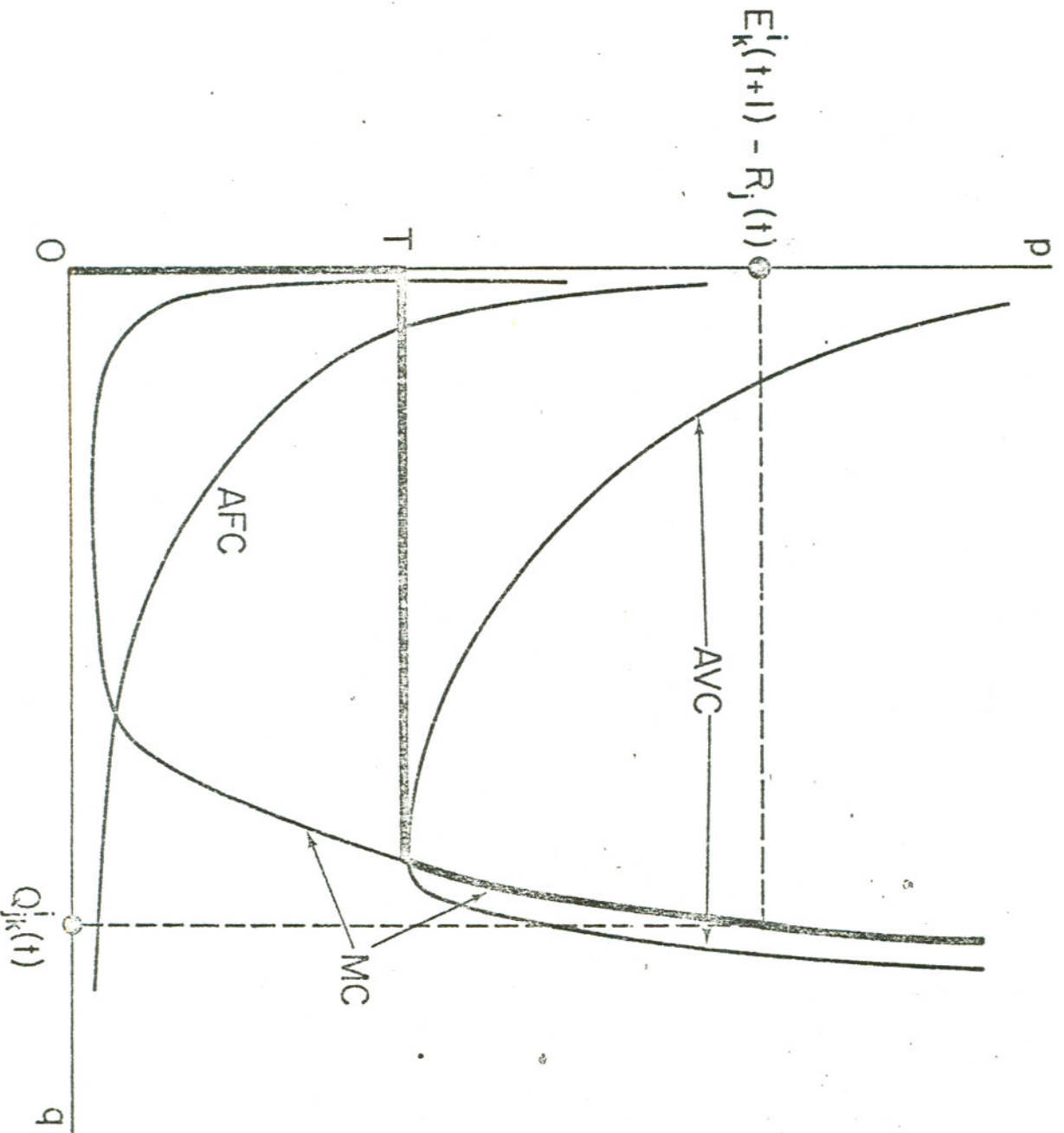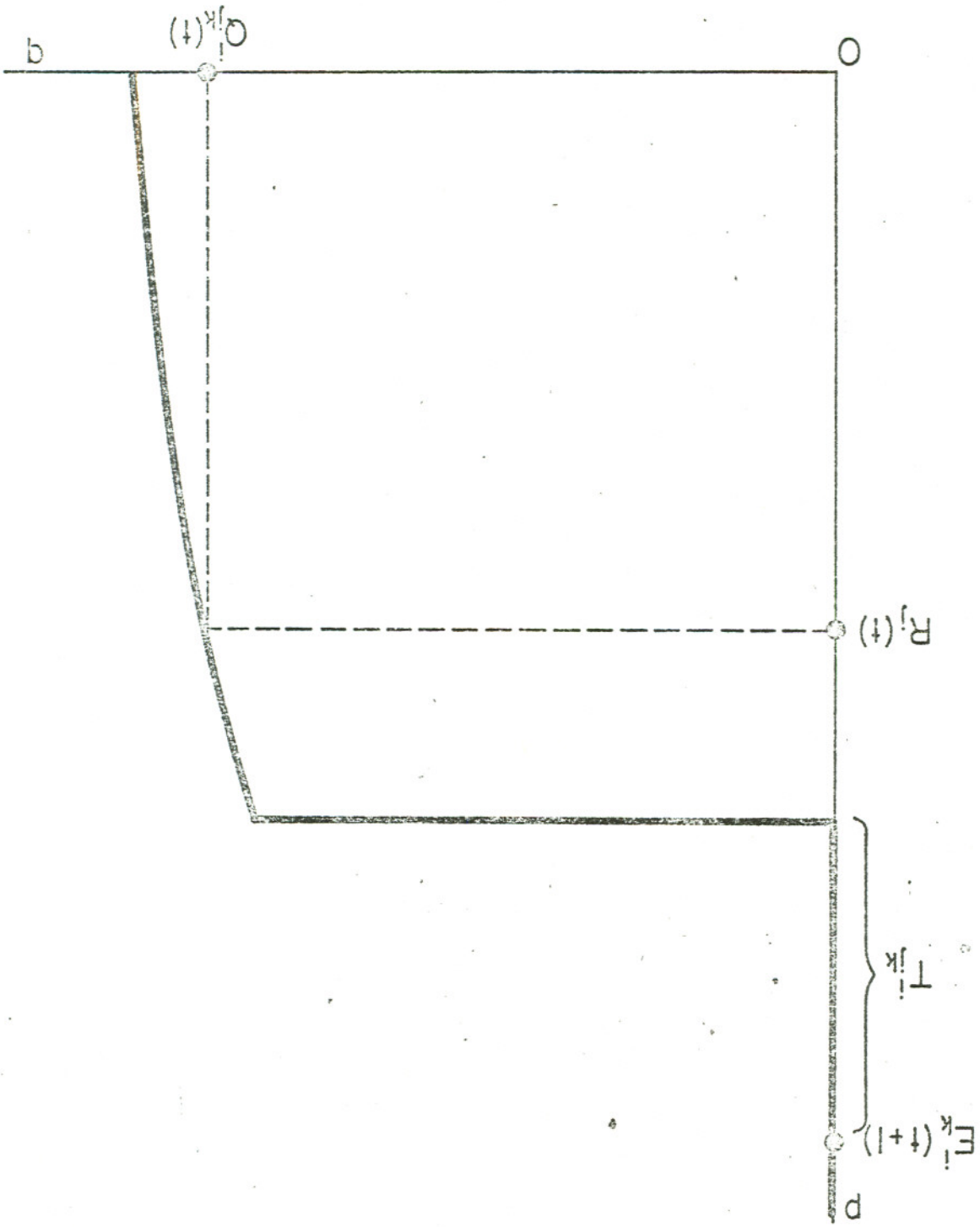   vol. 68 (1979), pp. 509-525.

Figure 1.

Figure 2.
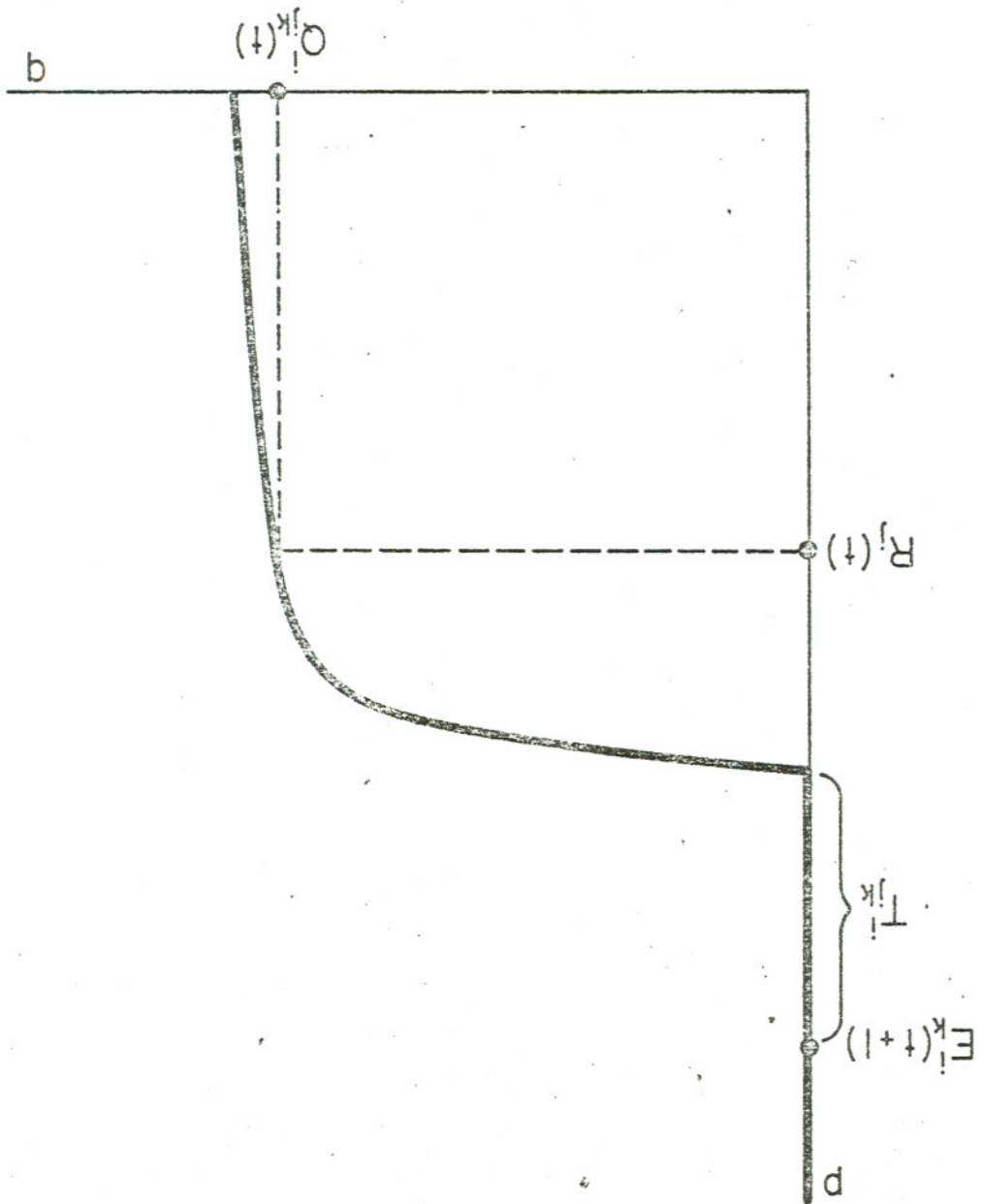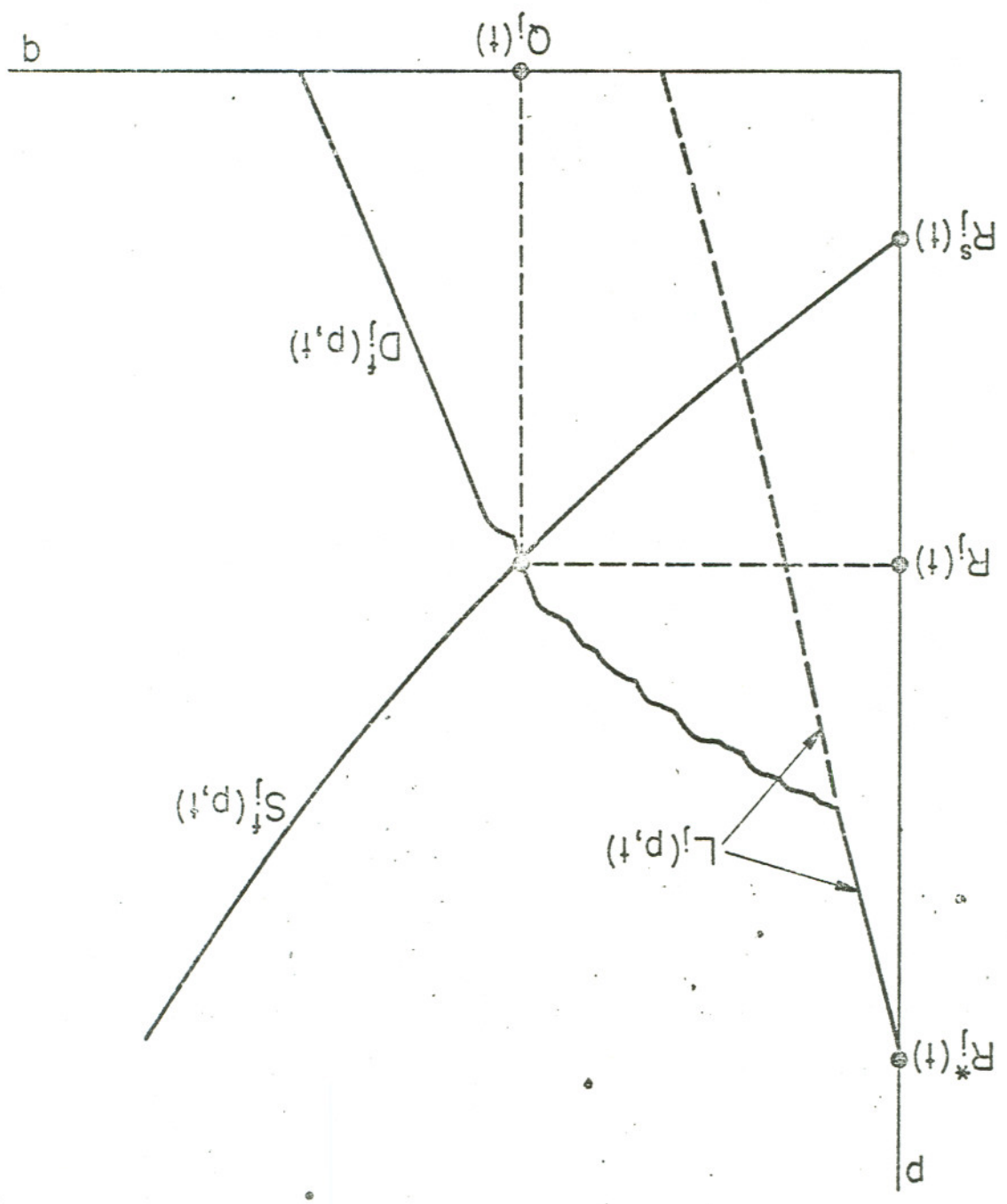
$E_k^i(t+1) - R_j(t)$

$Q_{jk}^i(t)$

AFC

AVC
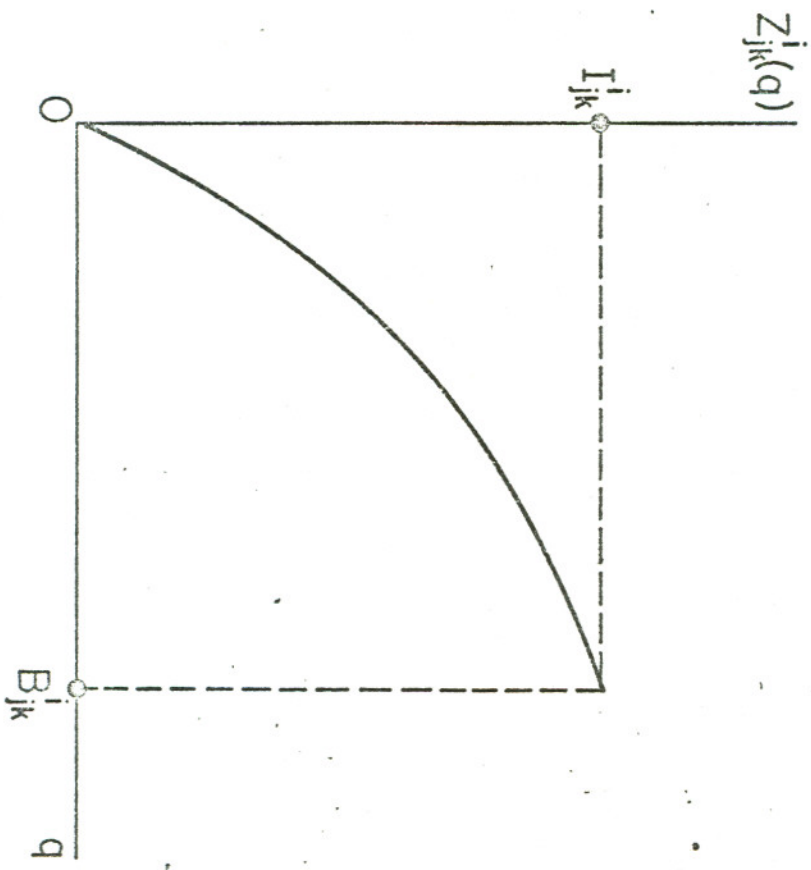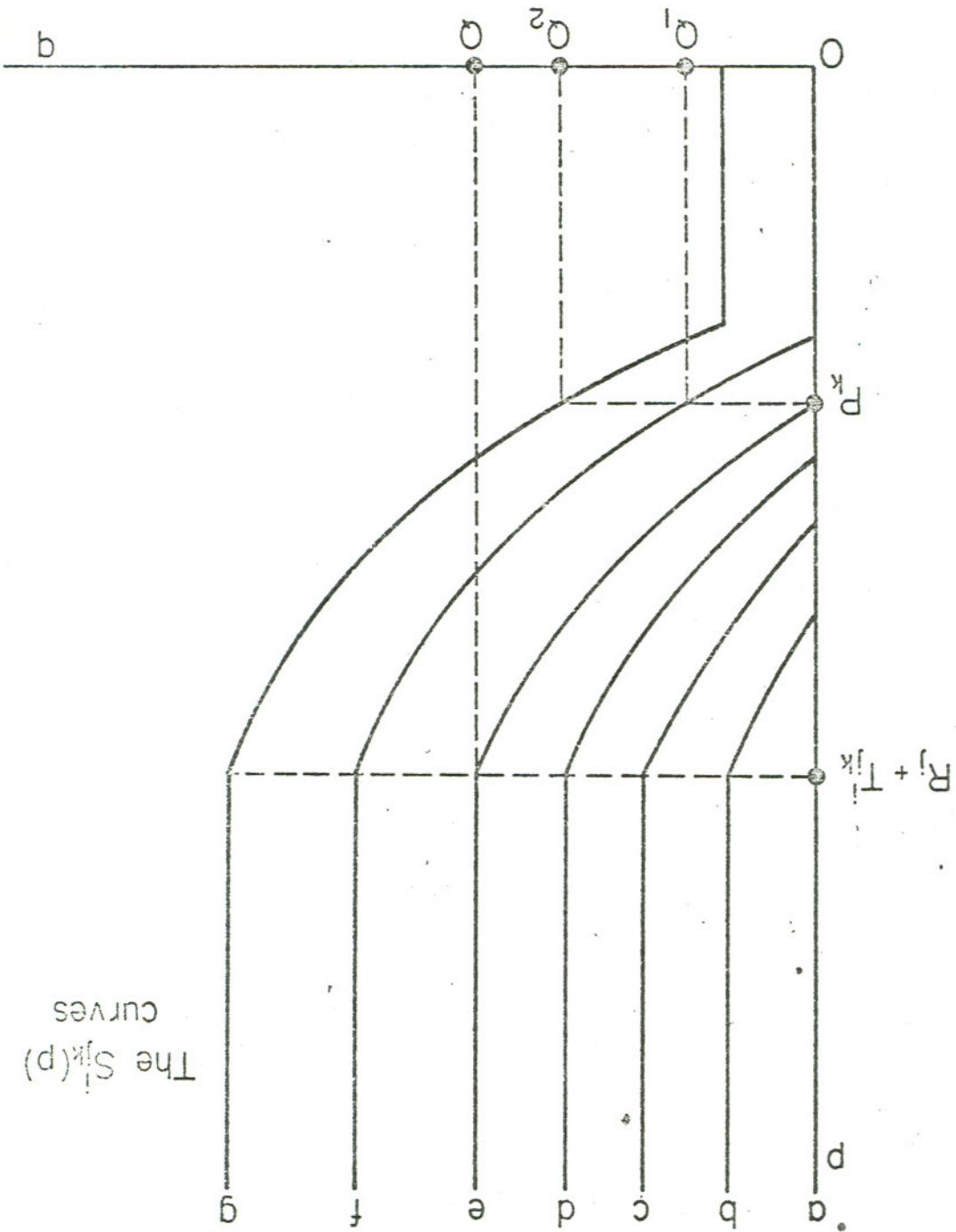
MC

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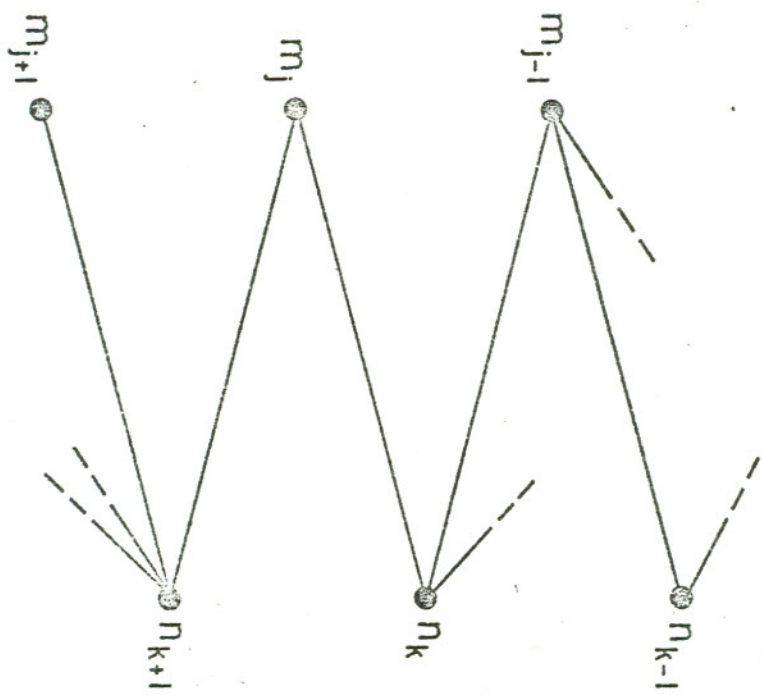
Figure 3.

Figure 4.

Figure 5.

Figure 6.

Figure 7.

Figure 8.

Figure 11.

Figure 12.