Essays on Durable Good Market with Quality Choice

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Abstract of the Dissertation

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My dissertation tries to construct a model of durable-goods oligopoly and monopoly in which quality choice is allowed and analyze the equilibrium behavior of the firms and consumers.

The second chapter considers the case of multiple firms and continuum of consumers whose types are uniformly distributed in infinite horizon setting and shows that there exists an equilibrium where higher-quality good is not offered before certain period even though firms are capable of producing them. The first section of the third chapter analyzes the market which consists of a monopolist who faces a single consumer that has two possible types and shows that the firm charges higher first period price when the consumer's valuation is more likely to be high. The second section of the third chapter examines the market which consists of a monopolist who faces continuum of consumers whose types are again uniformly distributed and shows that lower-quality good is offered only if the monopolist is patient enough.

Contents

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Chapter 1

Introduction

1.1 Introduction

Unlike the case with non-durable goods, the demand for a durable good in the future will be lower when more units are sold today. To mitigate this, a firm can introduce the same kind of good with higher quality. Indeed, this is what we observe in the real world. Most of the durable goods are offered in variety of qualities. In spite of this fact, the equilibrium behavior of consumers and firms which can choose to improve the quality of the products remain unclear. This dissertation analyzes the problem in durable-good market where the firms can produce the good in two qualities. In chapter 2 I consider the case of multiple firms and continuum of consumers whose types are uniformly distributed in infinite horizon setting and show that there exists an equilibrium where higher quality good is not offered before certain period even though firms are capable of producing them. Chapter 3 examines the market which consists of a monopolist who faces a single consumer that has two possible types (3.1) and continuum of consumers whose types are again uniformly distributed (3.2).

1.2 Review of Literature

There is an extensive literature on the behavior of durable-goods monopolist. Ausubel and Deneckere (1989) prove a folk theorem for seller payoffs for the case in which the minimum valuation among consumers is arbitrary close to the monopolist's marginal cost by constructing reputation equilibria. The interpretation of these equilibria is that once a deviation from a main price path which is specified in the equilibrium occurs, consumers believe that the monopolist will act in the way predicted by Coase. So by deviating from the main price path, the monopolist ruins its reputation. This prospect works as a deterrence from deviation and as a result, the monopolist adheres to the main price path which makes static monopoly profit possible. Von der Fehr and Kühn (1995) investigate under what condition the Pacman and the Coase conjectures can be verified as unique subgame perfect equilibria in an infinite-horizon game of durable-good monopolist.[1] Assuming that the strategies of all the players can be conditioned on the actions of single buyer, they consider the case in which number of buyers is finite and price space is continuous and the case in which there are continuum of buyers and the set of prices from which the monopolist chooses from is finite. In the first case, Pacman outcome is a unique subgame perfect outcome. In the second case, on the other hand, the unique subgame perfect equilibrium is Coasian. Their results imply that to have Pacman outcome, allowing the players strategies to be conditioned on a single buyer is not sufficient; it is crucial that a buyer has a significant impact on payoffs.

Von der Fehr and Kühn (1995) extends the analysis to oligopoly and find that the qualitative features of equilibrium outcomes are same as those in monopoly case. Especially, when there is a finite number of buyers, perfect discrimination is achievable and the game ends after finite number of periods. Their result, which is contrast to the one by Gul (1987) who find that

[1] Pacman conjecture says that with buyers who rationally expect future price decreases but know that these decreases will occur only when they have purchased the good, the firms can extract all the surplus as a result of price discrimination.

the total industry profit per period that can be sustained in equilibrium is no greater than static monopoly profit, makes it clear that allowing one buyer's action to influence the payoff of the seller significantly is crucial in oligopoly model as well.

Most of the literatures restrict their attentions to the two-period model when quality choice is involved. Waldman (1996) examines a durable-good monopolist's incentive to innovate with two-period model by studying the effect of investment in R&D on current and overall profit. He argues that when the monopolist cannot commit to amount of investment in R&D, it chooses the amount of investment which is greater than the amount that maximizes its profit. His analysis is restricted to two consumer types and two quality levels. Fudenberg and Tirole (1998) study the monopoly pricing of successive generations of a durable good with continuous consumer type. Their two-period model shows that the pricing of the new generation of a durable good depends heavily on the information the monopolist has about its past customers. Chi (1999) studies the quality choice made by a durable-good monopolist with two-period model. The author proves that a durable-good monopolist without commitment power makes the quality of its product higher than it would choose if it had such power. Inderst (2003) analyzes the optimal strategy of a durable-good monopolist that can offer goods in different qualities. The author shows that if the monopolist can change its product and price policy sufficiently rapidly, the whole market is served in the first period. It is also implied that the monopolist sells the product to the consumers with lower valuation below marginal cost.

Chapter 2

Oligopoly

2.1 Model

There are M firms which can produce any amount of an infinitely durable good. The firms and the consumers have a common discount factor $\delta \in (0, 1)$. In any period except for the first one, all firms can produce goods with low quality Q_L which are denoted by L at a constant marginal cost of c_L or goods with high quality which are labeled H at a constant marginal cost of c_H. Assume $c_H = c_L = 0$ and that it is impossible for firms to produce both H and L in the same period. So, at any period t, each firm has to decide which of H and L to sell.

On the demand side, assume a continuum of nonatomic consumers indexed by $\theta \in \Theta = [0, 1]$, where θ captures a consumer's preference for quality. The preference of these consumers can be specified by a function $f : \Theta \times Q \to R_+$, which is monotone nonincreasing and continuous in Θ, where $Q = \{Q_L, Q_H, Q_\Delta\}$. If buyer θ purchases the good K in period t at price p_K^t, his utility is $\delta^t[f(\theta, K) - p_K^t]$, where $K = H, L$. Define $f(\theta, Q_H) - f(\theta, Q_L) \equiv f(\theta, Q_\Delta)$.

Each period consists of two stages. Within each period, the timing of this oligopoly game is as follows: first, each firm chooses quality of the goods and the price of its own product simultaneously; second, consumers who have not purchased the good decide whether or not to buy. More precisely, a consumer who has purchased L decides whether or not to purchase H if it is offered in that period. And a consumer who has purchased neither H nor L decides a good with which quality K to consume where $K \in \Psi = \{\emptyset, Q_L, Q_H\}$ depending on the quality choices of the firms.[1] I denote by h^t a t-period history that includes all the price and quality offers and the quantity sold by all the firms up to but not including period t. Let P be the set of prices a firm can choose from where $P = [0, \infty)$. Then $h^t \in \Psi^{Mt} \times P_H^{Mt} \times P_L^{Mt} \times \mathrm{I\!R}^{\mathrm{Mt}}$, where Ψ^{Mt}, P_K^{Mt} and $\mathrm{I\!R}^{\mathrm{Mt}}$ are the Mt-fold Cartesian products of Ψ, P_K and $\mathrm{I\!R}$ and an element of $\mathrm{I\!R}^{\mathrm{t}}$ denotes the quantities that the firm sold in the past.[2] A strategy combination for sellers is a sequence of functions $\{q^t\}_{t=1}^{\infty}$ where $q^t : \Psi^{Mt} \times P_H^{Mt} \times P_L^{Mt} \times \mathrm{I\!R}^{\mathrm{Mt}} \to P_H^M \times P_L^M \times \Psi^M$. The objective of each firm is to maximize the discounted sum of the profit $\pi_j = \sum_K \sum_{t=1}^{\infty} p_{K,j}^t \inf_{\{\Theta_{K,j}^t | q^t\} \subseteq S} \mu(S) \delta^t$, where S is measurable, $\Theta_{K,j}^t | q^t$ is the set of buyers who accept the firm j's offer $p_{K,j}^t$ when other firms' offers are given by q^t and μ is the Lebesgue measure.

Define

$$U_L^t(\theta) = \sup_{s \geq t+1, j, k} f(\theta, Q_L) - p_{L,j}^t + \delta^{s-t}(f(\theta, Q_\Delta) - p_{H,k}^s) \tag{2.1}$$

and for each $s \geq t+1$ define

$$U_L^s(\theta) = \sup_{u \geq s+1, j, k} \delta^{s-t}(f(\theta, Q_L) - p_{L,j}^s + \delta^{u-s}(f(\theta, Q_\Delta) - p_{H,k}^u)) \tag{2.2}$$

and

$$U_H^t(\theta) = \sup_{s \geq t, j} \delta^{s-t}(f(\theta, Q_H) - p_{H,j}^s) \tag{2.3}$$

Then in any period t, a consumer θ who has not purchased either H or L accepts $p_{L,j}^t$ if and only if

$$U_L^t(\theta) \geq \max\{U_L^s(\theta), U_H^t(\theta), 0\} \tag{2.4}$$

---

[1]A consumers who has purchased H does not demand either L or H. A consumer who has purchased L, however, has a unit demand for H.

[2]Therefore the history observed by a firm is identical to the one observed by another firm.

for each $s \geq t+1$ and accepts $p^t_{H,j}$ if and only if

$$\max_j\{f(\theta, Q_H) - p^t_{H,j}\} \geq \max\{U^t_L(\theta), U^s_L(\theta), \max_{s,j} \delta^{s-t}(f(\theta, Q_H) - p^s_{H,j})\} \tag{2.5}$$

for each $s \geq t+1$.

And a consumer θ who has purchased L accepts $p^t_{H,j}$ if and only if

$$\max_j\{f(\theta, Q_\Delta) - p^t_{H,j}\} \geq \max_{s,j}\{f(\theta, Q_L), \delta^{s-t}(f(\theta, Q_\Delta) - p^s_{H,j})\}. \tag{2.6}$$

Thus, the pure strategy combination for buyers is a sequence of functions $\{x_t\}_{t=1}^{\infty}$ where $x_t : P_H^{Mt} \times P_L^{Mt} \times Q^M \times P_H^M \times P_L^M \times \Theta \to \{0,1\}_L^M \times \{0,1\}_H^M$ for those who have not purchased either L or H and $P_H^{Mt} \times P_L^{Mt} \times Q^M \times P_H^M \times \Theta \to \{0,1\}_H^M$ for those who have purchased L.

As most of the literatures in this field, attention is restricted to subgame perfect Nash equilibria in pure strategies.

2.2 Results

2.2.1 Equilibrium

Let $E(f,\delta)$ denote the set of equilibria of the game. If δ is sufficiently close to one, there exists $\sigma \in E(f,\delta)$ such that in any $t \leq T$ only L is offered and in any $t > T$ only H is offered except for $t = T+1$, where $T < \infty$ and such that the cutoff types of consumers who purchase L (H) at t are given by the sequence $\{\theta_t\}_{t=1}^{T+1}$ ($\{\theta_t\}_{t=T+1}^{\infty}$) where the mass of consumers who purchase at t is given by $\mu([\theta_{t-1}, \theta_t])$. Along the equilibrium path, we observe the weakly decreasing sequence of prices for L followed by the weakly decreasing sequence of prices for H. Moreover, the skimming property holds along the equilibrium path.[3]

The outcome with above characteristics can be sustained through following trigger strategy by the firms.

Offer L at $\hat{p}_L^1 = (1-\delta)\sum_{k=1}^{T} f(\theta_k, Q_L)\delta^{k-1}$ in the first period. In any $t \leq T$, offer L at $\hat{p}_L^t = (1-\delta)\sum_{k=t}^{T} f(\theta_k, Q_L)\delta^{k-t}$ if all the previous outcomes have been such that only L were offered at the price of $\hat{p}_L^t$ and the mass of consumers who accepted the offer were $\frac{1}{M}\mu([\theta_{t-1}, \theta_t])$ where M is the number of the firms who announced $\hat{p}_L^t$. Otherwise offer H at 0. In $t = T+1$ offer L at 0 and offer H at $\hat{p}_H^t = (1-\delta)\sum_{k=t}^{\infty} f(\theta_{k-T}, Q_\Delta)\delta^{k-t}$ if all the first T outcomes had been such that only L were offered at the price of $\hat{p}_L^t$ where $t \in \{1, \cdots, T\}$. In $t > T+1$ offer H at $\hat{p}_H^t = (1-\delta)\sum_{k=t}^{\infty} f(\theta_{k-T}, Q_\Delta)\delta^{k-t}$ if all the first T outcomes had been such that only L were offered at the price of $\hat{p}_L^t$ where $t \in \{1, \cdots, T\}$ and next $t-1-T$ outcomes have been such that only H were offered at the price of $\hat{p}_H^t$ where $t \in \{T+2, \cdots, t-1\}$ while at $t = T+1$ L were offered at 0 and H were offered at p_H^{T+1}, otherwise offer H at 0.

On the other hand, the strategy of consumer i to support the above equilibrium is the following.

*Consumer $\theta \leq \theta_t$ who has not bought L: Always reject $p_L^t \in ((1-\delta)f(\theta_t, Q_L),$
$(1-\delta)\sum_{k=t}^{T} f(\theta_k, Q_L)\delta^{k-t})$ for L and $p_H^t \in ((1-\delta)f(\theta_t, Q_H), (1-\delta)\sum_{k=t}^{\infty} f(\theta_t, Q_H)\delta^{k-t})$ for H. Accept one of the offers for sure when all sellers charge $\hat{p}_L^t$ for L or $\hat{p}_H^t$ for H. Always buy L when there is some offer $p_L^t \leq (1-\delta)f(\theta_t, Q_L)$. Always buy H when there is some offer $p \leq (1-\delta)f(\theta_t, Q_H)$.*
*Consumer $\theta \leq \theta_t$ who has bought L: Always reject $p_H^t \in ((1-\delta)f(\theta_t, Q_\Delta),$
$(1-\delta)\sum_{k=t}^{\infty} f(\theta_t, Q_\Delta)\delta^{k-t})$.Accept one of the offers for sure when all sellers charge p_H^t. Reject any higher prices. Always buy when there is some offer $p_H^t \leq (1-\delta)f(\theta_t, Q_\Delta)$*

[3]The skimming property says that if an offer is accepted by a buyer with certain valuation type, it is also accepted by buyers with higher valuation.

2.2.2 Proof

Firm's optimality on the equilibrium path

Suppose that no deviation has occurred up to $t-1$. If a firm follows the strategy specified above, its payoff given that other firms and consumers play according to the strategies given above is

$$\begin{aligned}
&(1-\delta)\sum_{k=t}^{T} f(\theta_k, Q_L)\delta^{k-t} \times \frac{\mu([\theta_{t-1},\theta_t])}{M} \\
&+ \delta(1-\delta)\sum_{k=t+1}^{T} f(\theta_k, Q_L)\delta^{k-(t+1)} \times \frac{\mu([\theta_t,\theta_{t+1}])}{M} \\
&+ \cdots + \delta^{T-t}(1-\delta) f(\theta_T, Q_L)\delta^{T-t} \times \frac{\mu([\theta_{T-1},\theta_T])}{M} \\
&+ \delta^{T+1-t}(1-\delta)\sum_{k=T+1}^{\infty} f(\theta_k, Q_\Delta)\delta^{k-(T+1)} \times \frac{\mu([\theta_T,\theta_{T+1}])}{M} + \cdots \\
&\equiv \sum_{k=t}^{\infty}\delta^{k-t}\sum_{k=t}^{\infty}\bar{Z}_k
\end{aligned} \tag{2.7}$$

where $\bar{Z}_t \equiv (1-\delta)\sum_{k=t}^{T} f(\theta_k, Q_L)\delta^{k-t} \times \frac{\mu([\theta_{t-1},\theta_t])}{M}$ for all $t \leq T$ and $\bar{Z}_t \equiv (1-\delta)\sum_{k=T+1}^{\infty} f(\theta_k, Q_\Delta)\delta^{k-(T+1)} \times \frac{\mu([\theta_{t-1},\theta_t])}{M}$ for all $t \geq T+1$. Define $Z \equiv \sum_{k=t}^{\infty}\bar{Z}_t \times M$. Then a firm which deviates at period t can obtain at most $D \equiv (1-\delta)Z$. Therefore the payoff of a firm from playing according to the equilibrium strategy is

$$\begin{aligned}
\sum_{k=t}^{\infty}\bar{Z}_k\delta^{k-t} &= \frac{Z}{M}\sum_{k=t}^{\infty}\delta^{k-t} \\
&= \frac{D}{(1-\delta)M} \times \sum_{k=t}^{\infty}\delta^{k-t} = \frac{D}{(1-\delta)^2 M}
\end{aligned} \tag{2.8}$$

which is greater than the maximum payoff D from deviation as long as $M < \frac{1}{(1-\delta)^2}$.

Firms' optimality off the equilibrium path

Charging zero for H is optimal for each firm given that all other firms will charge zero for H if a deviation has occurred in any previous period.

Consumers' optimality along the equilibrium path

• A consumer $a \in (\theta_{i-1}, \theta_i)$ cannot be better off by adapting a strategy of a consumer who is designated to buy L and H in period s and $s+T$ respectively where $s < i$.

Note that $a \in (\theta_{i-1}, \theta_i)$ buys in period i in the equilibrium described above. From the firms' strategy,

$$\begin{aligned}
p_L^s + & p_H^{s+T}\delta^T \\
&= (1-\delta)\sum_{k=s}^{T} f(\theta_k, Q_L)\delta^{k-s} + [(1-\delta)\sum_{k=s+T}^{i+T-1} f(\theta_{k-T}, Q_\Delta)\delta^{k-(s+T)} + p_H^{i+T}\delta^{i+T-(s+T)}]\delta^T \\
&\geq (1-\delta)\sum_{k=s}^{i-1} f(\theta_i, Q_L)\delta^{k-s} + p_L^i\delta^{i-s} + [(1-\delta)\sum_{k=s+T}^{i+T-1} f(\theta_i, Q_\Delta)\delta^{k-(s+T)} + p_H^{i+T}\delta^{i-s}]\delta^T \\
&= (1-\delta)(\frac{1}{1-\delta} - \frac{\delta^{i-s}}{1-\delta})f(\theta_i, Q_L) + p_L^i\delta^{i-s} \\
&+ [(1-\delta)(\frac{1}{1-\delta} - \frac{\delta^{i-s}}{1-\delta})f(\theta_i, Q_\Delta) + p_H^{i+T}\delta^{i-s}]\delta^T
\end{aligned} \tag{2.9}$$

where $s < i$. Therefore

$$\delta^i(f(a, Q_L) - p_L^i + \delta^T(f(a, Q_\Delta) - p_H^{i+T})) \geq \delta^s(f(a, Q_L) - p_L^s + \delta^T(f(a, Q_\Delta) - p_H^{s+T})) \tag{2.10}$$

Thus a consumer $a \in (\theta_{i-1}, \theta_i)$ cannot gain by adopting a strategy of a consumer who is designated to buy at $s < i$. Similar argument can show that a consumer $a \in (\theta_{i-1}, \theta_i)$ cannot gain by adopting a strategy of a consumer who is designated to buy at $s > i$.

• A consumer $a \in (\theta_{i-1}, \theta_i)$ who is designated to buy L in period i and H in $i+T$ cannot be better off by not buying L and buying H in $i+T$.

Observe that

$$
\begin{aligned}
\delta^i(f(\theta_i, Q_L) - p_L^i + \delta^T(f(\theta_i, Q_\Delta) - p_H^{i+T}))& \\
&= \delta^i((1-\delta)\sum_{k=i}^{\infty} f(\theta_i, Q_L)\delta^{k-1} - p_L^i + \delta^T(f(\theta_i, Q_\Delta) - p_H^{i+T})) \\
&= \delta^i((1-\delta)\sum_{k=i}^{T} f(\theta_i, Q_L)\delta^{k-i} + \delta^T f(\theta_i, Q_L) - p_L^i + \delta^T(f(\theta_i, Q_\Delta) - p_H^{i+T})) \\
&= \delta^i((1-\delta)\sum_{k=i}^{T} f(\theta_i, Q_L)\delta^{k-i} - p_L^i + \delta^T(f(\theta_i, Q_H) - p_H^{i+T})) \\
&> \delta^{i+T} f((\theta_i, Q_H) - p_H^{i+T})
\end{aligned}
\tag{2.11}
$$

since $p_L^i < (1-\delta)\sum_{k=i}^{T} f(\theta_i, Q_L)\delta^{k-i}$. Hence

$$
\delta^i(f(a, Q_L) - p_L^i + \delta^T(f(a, Q_\Delta) - p_H^{i+T})) > \delta^{i+T} f((a, Q_H) - p_H^{i+T}) \tag{2.12}
$$

and $a \in (\theta_{i-1}, \theta_i)$ cannot be better off by buying H in $i+T$ without buying L.

• A consumer $a \in (\theta_{i-1}, \theta_i)$ who is designated to buy L in period i and H in period $i+T$ cannot be better off by not buying L and buying H in $s+T$ where $s < i$.

From (2.10)

$$
\begin{aligned}
\delta^i(f(\theta_i, Q_L) - p_L^i + \delta^T(f(\theta_i, Q_\Delta) - p_H^{i+T})) &\geq \delta^s(f(\theta_i, Q_L) - p_L^s + \delta^T(f(\theta_i, Q_\Delta) - p_H^{s+T}) \\
&= \delta^s((1-\delta)\sum_{k=s}^{T} f(\theta_i, Q_L) + \delta^T f(\theta_i, Q_L) - p_L^s + \delta^T(f(\theta_i, Q_\Delta) - p_H^{s+T})) \\
&= \delta^s((1-\delta)\sum_{k=s}^{T} f(\theta_i, Q_L)\delta^{k-s} - p_L^s + \delta^T(f(\theta_i, Q_H) - p_H^{s+T})) \\
&> \delta^{s+T}(f(\theta_i, Q_H) - p_H^{s+T})
\end{aligned}
\tag{2.13}
$$

Therefore

$$\delta^i(f(a,Q_L) - p_L^i + \delta^T(f(a,Q_\Delta) - p_H^{i+T})) > \delta^{s+T}(f(a,Q_H) - p_H^{s+T}) \tag{2.14}$$

and $a \in (\theta_{i-1}, \theta_i)$ cannot be better off by buying H in $s+T$ without buying L. Similar argument can show that a consumer $a \in (\theta_{i-1}, \theta_i)$ cannot be better off with the case where $s > i$.

• A consumer $a \in (\theta_{i-1}, \theta_i)$ who is designated to buy L in period i and H in $i+T$ cannot be better off by buying L in u and H in $s+T$ where $s < i < u$.
In case of such deviation, the forgone surplus is

$$\delta^i(1-\delta)\sum_{k=i}^{u-1}(f(a,Q_L) - f(\theta_k,Q_L))\delta^{k-i} > 0 \tag{2.15}$$

since $f(a,Q_L) > f(\theta_i,Q_L)$ while the loss incurred is

$$\delta^{s+T}(1-\delta)\sum_{k=s+T}^{i+T-1}(f(a,Q_\Delta) - f(\theta_{k-T},Q_\Delta))\delta^{k-(s+T)} < 0 \tag{2.16}$$

since $f(a,Q_\Delta) < f(\theta_{i-1},Q_\Delta)$. Thus $a \in (\theta_{i-1}, \theta_i)$ cannot be better off by buying L later and H earlier than supposed to. Similar argument can show that a consumer $a \in (\theta_{i-1}, \theta_i)$ cannot be better off with the case where $u < i < s$.

Consumers' optimality off the equilibrium path

If a deviation by some firm and/or consumer has occurred in period $r \leq t$, then every firm charges zero for H in period $t+1$. Hence a consumer who has bought L has no incentive to deviate from rejecting $p_H^t \in ((1-\delta)f(\theta_t,Q_\Delta), \hat{p}_H^t)$ because $f(\theta_t,Q_\Delta) - p_H^t \leq \delta(f(\theta_t,Q_\Delta)) - 0$. And it is optimal to accept $p_H^t \leq (1-\delta)f(\theta_t,Q_\Delta)$ because $f(\theta_t,Q_\Delta) - p_H^t \geq \delta(f(\theta_t,Q_\Delta)) - 0$. It is obviously optimal to reject $p_H^t > \hat{p}_H^t$. Similar argument applies to the case with a consumer who has not bought L.

2.2.3 Summary of the results

Gul (1987) showed that in a oligopoly market of durable goods the firms can extract the total profit arbitrarily close to the one-shot monopoly profit. This section used his technique to show that when a durable good can be produced in two qualities, there exists an equilibrium where the low quality good is offered (and purchased) before high quality one becomes available in the market regardless of the fact that the firms are capable of producing the high quality good which would allow them to extract higher profit.

Chapter 3

Monopoly

3.1 Two-period game with incomplete information

3.1.1 Model and equilibria

Assume for now a single firm, a single consumer and that there are only two periods. The firm can produce a good in low quality (L) in the first period and in low quality and high quality (H) in the second period. The consumer's valuation is (v_L, v_H) with probability q, where v_L is the consumer's valuation for a unit of L and v_H the consumer's valuation for a unit of H, or $\underline{v}$ with probability $1 - q$, where $c_H > c_L = 0$ and $v_H > v_L > \underline{v}$. Such valuation by a consumer reflects a case where he can be either of type who cares about quality or a type who does not. The true value for the consumer of a product with quality k is privately known by the consumer. Let μ be the probability that the consumer's type is (v_L, v_H) conditional on the rejection of p_L^1 and β be the probability that the consumer's type is (v_L, v_H) conditional on the acceptance of p_L^1. Define $v_{\Delta Q} \equiv v_H - v_L$and assume $v_{\Delta Q} > c_H$. [1] In period one, the firm announces p_L^1 and

[1]This implies that producing and selling H generates a higher surplus than producing and selling L.

produces L if it receives an order. In period two, the firm announces a set of prices (p^2_L, p^2_H) and produces if order occurs.

In period two, the firm has to choose between announcing $(p^2_L, p^2_H) = (\underline{v}, v_{\Delta Q} + \underline{v})$ and $(p^2_L, p^2_H) = (v_L, v_H)$. To see this, note that when $p^2_L = \underline{v}$ is offered, type (v_L, v_H) can guarantee himself with surplus of $v_L - \underline{v}$. So in order for the consumer with type (v_L, v_H) to accept p^2_H, it has to satisfy $v_H - p^2_H \geq v_L - \underline{v}$ or $p^2_H \leq v_{\Delta Q} + \underline{v}$. And it is optimal for the firm to offer $p^2_H = v_{\Delta Q} + \underline{v}$ because $v_{\Delta Q} + \underline{v} - c_H > \underline{v}$. The firm can also take chance of earning v_H with probability μ by announcing (v_L, v_H) rather than earning $\underline{v}$ for sure by announcing $(\underline{v}, v_{\Delta Q} + \underline{v})$. Announcing (v_L, v_H) is optimal if $\mu(v_H - c_H) + (1 - \mu)0 \geq \mu(v_{\Delta Q} + \underline{v} - c_H) + (1 - \mu)\underline{v}$ or $\mu \geq \frac{\underline{v}}{v_L}$. Therefore the firm's optimal action in the second period when the consumer rejects p^1_L is

- $(p^2_L, p^2_H) = (\underline{v}, v_{\Delta Q} + \underline{v})$ if $\mu < \frac{\underline{v}}{v_L}$
- $(p^2_L, p^2_H) = (v_L, v_H)$ if $\mu > \frac{\underline{v}}{v_L}$

On the other hand, the firm's optimal action in the second period when the consumer accepts p^1_L is

- $(p^2_L, p^2_H) = (\infty, v_{\Delta Q})$ regardless of the value for β.

Consider $t = 1$. If the rejection of p^1_L generates $\mu < \frac{\underline{v}}{v_L}$, the consumer of type (v_L, v_H) can obtain a surplus of $\delta(v_L - \underline{v})$ by rejecting p^1_L. Therefore it must be the case that $v_L - p^1_L + \delta 0 \geq \delta(v_L - \underline{v})$ or $p^1_L \leq v_L - \delta(v_L - \underline{v})$ in order for type (v_L, v_H) to accept p^1_L if the rejection of p^1_L were to generate $\mu < \frac{\underline{v}}{v_L}$. If the rejection of p^1_L generates $\mu > \frac{\underline{v}}{v_L}$, the consumer of type (v_L, v_H) obtains no surplus by rejecting p^1_L. Therefore it must be the case that $p^1_L \leq v_L$ in order for type (v_L, v_H) to accept p^1_L if the rejection of p^1_L generates $\mu > \frac{\underline{v}}{v_L}$. Since the firm will not announce a price lower than $\underline{v}$, it must be the case that $p^1_L \leq \underline{v}$ in order for type $\underline{v}$ to accept p^1_L.

PBE 1:

The firm's strategy on equilibrium path:

Charge $p_L^1 = (1-\delta)v_L + \delta\underline{v}$ in the first period. If the consumer accepts p_L^1, update the beliefs so that $\beta = 1$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$. If the consumer rejects p_L^1, update the beliefs so that $\mu = 0$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$.

The firm's strategy off equilibrium path:

• If any $\underline{v} < p_L^1 < (1-\delta)v_L + \delta\underline{v}$ is rejected, update the beliefs so that $\mu = 0$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$.

• If any $\underline{v} < p_L^1 < (1-\delta)v_L + \delta\underline{v}$ is accepted, update the beliefs so that $\beta = 1$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$.

• If $p_L^1 \leq \underline{v}$ is rejected, update the beliefs so that $\mu \in [0,1]$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$ or (v_L, v_H).

• If $p_L^1 \leq \underline{v}$ is accepted, update the beliefs so that $\beta = q$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$.

• If any $p_L^1 > (1-\delta)v_L + \delta\underline{v}$ is rejected, update the beliefs so that $\mu = q$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$ or (v_L, v_H).

• If any $p_L^1 > (1-\delta)v_L + \delta\underline{v}$ is accepted, update the beliefs so that $\beta \in [0,1]$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$.

The consumer's strategy:

If he is of type $\underline{v}$, accept $p_k^t \leq \underline{v}$. If he is of type (v_L, v_H), accept $p_L^1 \leq (1-\delta)v_L + \delta\underline{v}$ in period one. In period two, accept $p_H^2 \leq v_{\Delta Q}$ if he bought L in period one and whichever gives the higher payoff as long as $p_L^2 \leq v_L$ and $p_H^2 \leq v_H$ if he did not buy L in period one.

The firm's expected profit is $\pi = q[(1-\delta)v_L + \delta\underline{v} + \delta(v_{\Delta Q} - c_H)] + \delta\underline{v}(1-q)$.

PBE 2:

The firm's strategy on equilibrium path:

Charge $p_L^1 = \underline{v}$ in the first period. If the consumer accepts p_L^1, update the beliefs so that $\beta = q$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$. If the consumer rejects p_L^1, update the beliefs so that $\mu \in [0,1]$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$ or (v_L, v_H).

The firm's strategy off equilibrium path:

• If any $p_L^1 \in (\underline{v}, (1-\delta)v_L + \delta\underline{v}]$ is rejected, update the beliefs so that $\mu = 0$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$.

• If any $p_L^1 \in (\underline{v}, (1-\delta)v_L + \delta\underline{v}]$ is accepted, update the beliefs so that $\beta = 1$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$.

• If any $p_L^1 < \underline{v}$ is rejected, update the beliefs so that $\mu \in [0,1]$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$ or (v_L, v_H).

• If any $p_L^1 < \underline{v}$ is accepted, update the beliefs so that $\beta = q$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$.

• If any $p_L^1 > (1-\delta)v_L + \delta\underline{v}$ is rejected, update the beliefs so that $\mu = q$ and announce $(p_L^2, p_H^2) = (\underline{v}, v_{\Delta Q} + \underline{v})$ or (v_L, v_H).

• If any $p_L^1 > (1-\delta)v_L + \delta\underline{v}$ is accepted, update the beliefs so that $\beta \in (0,1]$ and announce $(p_L^2, p_H^2) = (\infty, v_{\Delta Q})$.

The consumer's strategy:

If he is of type $\underline{v}$, accept $p_k^t \leq \underline{v}$. If he is of type (v_L, v_H), accept $p_L^1 \leq (1-\delta)v_L + \delta\underline{v}$ in period one. In period two, accept $p_H^2 \leq v_{\Delta Q}$ if he bought L in period one and whichever gives the higher payoff as long as $p_H^2 \leq v_H$ and $p_L^2 \leq v_L$ if he did not buy L in period one.

The firm's expected payoff is $\pi = q(\underline{v} + \delta(v_{\Delta Q} - c_H)) + (1-q)\underline{v}$

Note that the consumer of type (v_L, v_H) accepts $p_L^1 \leq (1-\delta)v_L + \underline{v}$ if the rejection generates $\mu \leq \frac{\underline{v}}{v_L}$ and accepts $p_L^1 = v_L$ if the rejection generates $\mu \geq \frac{\underline{v}}{v_L}$. Therefore the only candidates for p_L^1 that are part of equilibria are $\underline{v}, (1-\delta)v_L - \delta\underline{v}$ and v_L. However v_L cannot be a part of an equilibrium because in order for the type (v_L, v_H) to accept $p_L^1 = v_L$, it has to be the case that the rejection generates $\mu \geq \frac{\underline{v}}{v_L}$ so that $(p_L^2, p_H^2) = (v_L, v_H)$. But if accepting $p_L^1 = v_L$ is part of consumer's equilibrium strategy, the rejection must yield $\mu = 0$ with which the firm announces $(p_L^2, p_H^2) = (\underline{v}, \underline{v} + v_{\Delta Q})$.

3.1.2 Summary of the results

In PBE 1, $q > \frac{\underline{v}}{v_L}$ must hold in order for the firm not to deviate. In PBE 2, $q < \frac{\underline{v}}{v_L}$ must hold in order for the firm not to deviate. As described above, $\underline{v}$ and $(1-\delta)v_L - \delta\underline{v}$ are the only first period prices that can constitute an equilibrium. And each of them are rational given the beliefs by the firm which yield unique pair of second period prices. Therefore the equilibria described in this section are the only ones of the game. The results are intuitive. The firm charges the higher price for the low-quality good when the consumer is more likely to be of high type.

3.2 Two period game with continuum of consumers

3.2.1 Model and analysis of each subgame

Assume next a single firm that can produce a good in high quality (H) and in low quality (L) and identify which consumers have purchased in the first period and can fully discriminate on the basis of past consumption [2], continuum of consumers whose types are uniformly distributed in [0,1] and there are only two periods. The preference of the consumers is specified by a function $f(\theta, v_K) = \theta v_K$. So if buyer θ purchases the good with quality K in the first period at price p_K^1, his discounted sum of utility is $(1+\delta)\theta v_K - p_K^1$, where $K = L, H$.If he purchases the good with quality K in the second period, his utility discounted to the first period is $\delta(\theta v_K - p_K^2)$. If he purchases a unit of L in the first period and upgrade it to H in the second period, his utility is $\theta v_L - p_L^1 + \delta(\theta v_H - p_\Delta^2)$ or $(1+\delta)\theta v_L - p_L^1 + \delta(\theta v_\Delta - p_\Delta^2)$ where $v_\Delta = v_H - v_L$ and p_Δ^2 is the price those who bought L in the first period are charged for good H in the second period. His utility is

[2]"identified consumers" case in Fudenberg and Tirole (1998)

0 if he does not purchase. Assume θv_Δ is increasing in θ, $v_H > v_L > v_\Delta$ and $c_H = c_L = 0$. Let $\hat{p}^2_K$ be the expected second period price of the good with quality K where $K = H, L, \Delta$.

Proposition 3.2.1 *For each price pair (p^1_L, p^1_H) announced in the first period such that $p^1_L \leq (1+\delta)v_L - \frac{1}{2}\delta v_H$ and $p^1_H \leq v_H + \frac{1}{2}\delta v_H$ the continuation equilibrium in any perfect Bayesian equilibrium is such that there exists a pair $\bar{\theta}_L, \bar{\theta}_H$ with $\bar{\theta}_L \leq \bar{\theta}_H$ such that consumers with type $\theta \in (0, \bar{\theta}_L)$ do not buy in period 1, consumers with type $\theta \in (\bar{\theta}_L, \bar{\theta}_H)$ buy the good of quality L and consumers with type $\theta \in (\bar{\theta}_H, 1)$ buy the good of quality H.*

Proof First of all, observe that Θ and $Q = \{v_H, v_L, v_\Delta\}$ are completely ordered sets and the function has increasing differences in $(\theta; v_K)$. In order for θ to buy H in the first period, it must be the case that

$$(1+\delta)\theta v_H - p^1_H \geq (1+\delta)\theta v_L - p^1_L \tag{3.1}$$

$$(1+\delta)\theta v_H - p^1_H \geq \delta(\theta v_H - \hat{p}^2_H) \tag{3.2}$$

and

$$(1+\delta)\theta v_H - p^1_H \geq \theta v_L - p^1_L + \delta(\theta v_H - \hat{p}^2_\Delta). \tag{3.3}$$

Because of the increasing differences if (3.1) and (3.3) are satisfied for θ they should be satisfied for $\theta' \geq \theta$ also.[3] Rearranging (3.2) gives

$$\theta v_H \geq p^1_H - \delta \hat{p}^2_H \tag{3.4}$$

[3]With (3.1), for example, let $g(\theta, v_H) \equiv (1+\delta)\theta v_H - p^1_H$ and $g(\theta, v_L) \equiv (1+\delta)\theta v_L - p^1_L$. Then by increasing differences $g(\theta', v_H) - g(\theta', v_L) > g(\theta, v_H) - g(\theta, v_L)$ for all $\theta' > \theta$.

and its LHS is increasing in θ. These facts imply that for each of the three inequalities there exists $\bar{\theta}_H$ such that the inequalities are satisfied only for $\theta \geq \bar{\theta}_H$ so that only $\theta \in (\bar{\theta}_H, 1)$ buy H in period one.

On the other hand if a consumer buys L in the first period, she can take no further action in the second period or she can upgrade L to H at an expected price of $\hat{p}^2_\Delta$. The utility from buying L in the first period is

$$U_L(\theta) = max\{(1+\delta)\theta v_L - p^1_L, \theta v_L - p^1_L + \delta(\theta v_H - \hat{p}^2_\Delta)\}. \tag{3.5}$$

The alternatives to buying L are to buy H either in the first or the second period or to buy nothing. Define

$$U_H(\theta) = max\{(1+\delta)\theta v_H - p^1_H, \delta(\theta v_H - \hat{p}^2_H)\}. \tag{3.6}$$

It is optimal to buy L in the first period if and only if

$$U_L(\theta) \geq max\{U_H(\theta), 0\} \tag{3.7}$$

Assuming $U_L(\theta) = (1+\delta)\theta v_L - p^1_L$, in order for θ to buy L in the first period it must be the case that

$$(1+\delta)\theta v_L - p^1_L \geq (1+\delta)\theta v_H - p^1_H \text{ [4]} \tag{3.8}$$

and

$$(1+\delta)\theta v_L - p^1_L \geq \delta(\theta v_H - \hat{p}^2_H). \tag{3.9}$$

The increasing differences together with (3.1) implies that there exist $\bar{\theta}_H$ and $\bar{\theta}_L$ such that the first inequality is satisfied only for $\theta \leq \bar{\theta}_H$ and the second is satisfied only for $\theta \geq \bar{\theta}_L$ so that only $\theta \in (\bar{\theta}_L, \bar{\theta}_H)$ buy L in period one. Similarly assuming $U_L(\theta) = \theta v_L - p^1_L + \delta(\theta v_H - \hat{p}^2_\Delta)$, in order for θ to buy L in the first period it must be the case that

[4] Note that $\bar{\theta}_H$ obtained through this is equal to the one obtained through (3.1).

$$\theta v_L - p_L^1 + \delta(\theta v_H - \hat{p}_\Delta^2) \geq (1+\delta)\theta v_H - p_H^1 {}^5 \tag{3.10}$$

and

$$\theta v_L - p_L^1 + \delta(\theta v_H - \hat{p}_\Delta^2) \geq \delta(\theta v_H - \hat{p}_H^2). \tag{3.11}$$

Again the increasing differences together with (3.3) implies that only $\theta \in (\bar{\theta}_L, \bar{\theta}_H)$ buy L in the first period. ∎

Corollary 3.2.2 *For the continuation equilibrium described in the proposition, the cutoff type $\bar{\theta}_H$ is given by the indifference of type $\bar{\theta}_H$ between buying H in the first period and buying L in the first period and upgrading it in the second period and the cutoff type $\bar{\theta}_L$ is given by the indifference of type $\bar{\theta}_L$ between buying L in the first period and waiting for the second period to buy H.*

First observe that, by increasing differences, given that $(1+\delta)\theta v_L - p_L^1 \leq \theta v_L - p_L^1 + \delta(\theta v_H - \hat{p}_\Delta^2)$ (or $\hat{p}_\Delta^2 \leq \theta v_\Delta$) for θ, it is also the case for all $\theta' > \theta$. $\hat{p}_\Delta^2 = max\{\bar{\theta}_L v_\Delta, \frac{1}{2}\bar{\theta}_H v_\Delta\}$ because in the second period the monopolist maximizes $(\bar{\theta}_H - \frac{p_\Delta^2}{v_\Delta})p_\Delta^2$ when $\bar{\theta}_L \leq \frac{1}{2}\bar{\theta}_H$ $(((\bar{\theta}_H - \bar{\theta}_L)p_\Delta^2)$ when $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H)$. So when $\hat{p}_\Delta^2 = \frac{1}{2}\bar{\theta}_H v_\Delta (\hat{p}_\Delta^2 = \bar{\theta}_L v_\Delta)$, all $\theta \geq \frac{1}{2}\bar{\theta}_H (\theta \geq \bar{\theta}_L)$ prefer upgrading and thus $\bar{\theta}_H$ is given by

$$(1+\delta)\bar{\theta}_H v_H - p_H^1 = \bar{\theta}_H v_L - p_L^1 + \delta(\bar{\theta}_H v_H - \hat{p}_\Delta^2). \tag{3.12}$$

Similarly if $(1+\delta)\theta v_L - p_L^1 \geq \theta v_L - p_L^1 + \delta(\theta v_H - \hat{p}_\Delta^2)$ (or $\hat{p}_\Delta^2 \geq \theta v_\Delta$) for θ, it is also the case for all $\theta' < \theta$. Therefore $\bar{\theta}_L$ is given by

$$(1+\delta)\bar{\theta}_L v_L - p_L^1 = \delta(\bar{\theta}_L v_H - \hat{p}_H^2).^6 \tag{3.13}$$

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[5] $\bar{\theta}_H$ obtained by this is equal to the one obtained through (3.3).

[6] Note that when $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H$, $(1+\delta)\bar{\theta}_L v_L - p_L^1 = \bar{\theta}_L v_L - p_L^1 + \delta(\bar{\theta}_L v_H - \hat{p}_\Delta^2)$.

Observation *If the first period outcome is such that only consumers above certain type purchase so that the demand that the monopolist faces in the second period is truncated version of the original one, the firm does not offer L in the second period.*[7]

Suppose in the first period, the set of consumers who purchased L or H is $[\bar{\theta}, 1]$. Let $x_L(\theta)$ and $x_H(\theta)$ denote the probability that type $\theta \in [0, \bar{\theta})$ buys L and H at the second period respectively.[8] Also let $U(\theta)$ be the utility of the consumer θ. Then $U(\theta) \equiv \theta(x_L(\theta)v_L + x_H(\theta)v_H) - p(x_L(\theta) + x_H(\theta))$.In the second period, the monopolist's profit function is

$$\begin{aligned}\pi &= \int_0^{\bar{\theta}} \{\theta(x_L(\theta)v_L + x_H(\theta)v_H) - U(\theta) - (c_L x_L(\theta) + c_H x_H(\theta))\} f(\theta) d\theta \\ &= \int_0^{\bar{\theta}} \{\theta(x_L(\theta)v_L + x_H(\theta)v_H) - \int_0^{\theta} (x_L(u)v_L + x_H(u)v_H) du \qquad {}^{9} \\ &\qquad - (c_L x_L(\theta) + c_H x_H(\theta))\} f(\theta) d\theta.\end{aligned} \tag{3.14}$$

Integrating by parts yields

$$\begin{aligned}\pi = &\int_0^{\bar{\theta}} \{\theta(x_L(\theta)v_L + x_H(\theta)v_H) - (c_L x_L(\theta) + c_H x_H(\theta))\} f(\theta) d\theta \\ &- [\int_0^{\theta} (x_L(u)v_L + x_H(u)v_H) du (F(\bar{\theta}) - F(\theta))]_0^1 - \int_0^{\bar{\theta}} (x_L(\theta)v_L + x_H(\theta)v_H)(F(\bar{\theta}) - F(\theta)) d\theta \\ = &\int_0^{\bar{\theta}} \{[\theta(x_L(\theta)v_L + x_H(\theta)v_H) - (c_L x_L(\theta) + c_H x_H(\theta))] f(\theta) \\ &\qquad - (x_L(\theta)v_L + x_H(\theta)v_H)(F(\bar{\theta}) - F(\theta))\} d\theta.\end{aligned} \tag{3.15}$$

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[7]It is obtained by slight modification to "the monopolist's optimal rental policy" in Fudenberg and Tirole (1998).

[8]It is required that $0 \leq x_L(\theta) + x_H(\theta) \leq 1$ for all θ.

[9]By envelope theorem, $U' = x_L(\theta)v_L + x_H(\theta)v_H$. So the utility of the consumer θ can be expressed as $U(\theta) = \int_0^{\theta} (x_L(u)v_L + x_H(u)v_H) du$.

The maximization of the profit function with respect to $x_L(\cdot)$ and $x_H(\cdot)$ gives[10]

$$\theta_L^* v_L - c_L = \frac{F(\bar{\theta}) - F(\theta_L^*)}{f(\theta_L^*)} v_L \tag{3.16}$$

$$\theta_H^* v_H - c_H = \frac{F(\bar{\theta}) - F(\theta_H^*)}{f(\theta_H^*)} v_H \tag{3.17}$$

With the assumptions of the uniform distribution on consumers' types and $c_L = c_H = 0$, one obtains $\theta_L^* = \theta_H^* = \frac{1}{2}\bar{\theta}$ and the firm does not offer L in the second period.

Now the firm's behavior in each subgame has to be analyzed.

• Case i: When $p_H^1 > p_L^1 + (1+\delta)v_\Delta$ or $p_H^1 > v_H + \frac{1}{2}\delta\bar{\theta}v_H$[11] and $p_L^1 < (1+\delta)v_L - \frac{1}{2}\delta v_H \equiv \bar{p}_L^1$, I conjecture that the relationship between the announced pair of (p_L^1, p_H^1) and set of consumers who make purchase is $\bar{\theta} = \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}$[12] where $(\bar{\theta}, 1)$ buy L. The announced pair of prices generates the expectation by the consumers $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}v_H$ and $\hat{p}_\Delta^2 = \frac{1}{2}v_\Delta$ when $\bar{\theta} \leq \frac{1}{2}$ or $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}v_H$ and $\hat{p}_\Delta^2 = \bar{\theta}v_\Delta$ when $\bar{\theta} > \frac{1}{2}$ because in the second period the monopolist maximizes $(\bar{\theta} - \frac{p_H^2}{v_H})p_H^2$ and $(1 - max\{\bar{\theta}, \frac{p_\Delta^2}{v_\Delta}\})p_\Delta^2$ with respect to p_H^2 and p_Δ^2. The firm's profit function can be written as follows

$$\begin{aligned}
\pi &= (1-\bar{\theta})p_L^1 + \delta[(\bar{\theta} - \frac{p_H^2}{v_H})p_H^2 + (1 - \frac{p_\Delta^2}{v_\Delta})p_\Delta^2] \\
&= (1-\bar{\theta})p_L^1 + \delta(\frac{1}{4}\bar{\theta}^2 v_H + \frac{1}{4}v_\Delta) \\
&= (1 - \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})p_L^1 + \delta\frac{1}{4}((\frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})^2 v_H + v_\Delta)
\end{aligned} \tag{3.18}$$

[10] With the uniform distribution, hazard rate $\frac{f(\theta)}{1-F(\theta)}$ increases with θ and the first order condition is sufficient.

[11] It is obtained by $(1+\delta)\theta v_H - p_H^1 < \delta(\theta v_H - \hat{p}_H^2)$ when $\theta = 1$ and $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}v_H$.

[12] The condition $(1+\delta)\theta v_L - p_L^1 \geq (1+\delta)\theta v_H - p_H^1$ is satisfied for any $\theta \leq 1$ given the conditions for p_H^1. So the cutoff type must be given by indifference of type $\bar{\theta}$ in $(1+\delta)\bar{\theta}v_L - p_L^1 = \delta(\bar{\theta}v_H - \hat{p}_H^2)$ where $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}v_H$.

if it chooses p_L^1 such that $\bar{\theta} \leq \frac{1}{2}$ and

$$\begin{aligned}
\pi &= (1-\bar{\theta})p_L^1 + \delta[(\bar{\theta} - \frac{p_H^2}{v_H})p_H^2 + (1-\bar{\theta})p_\Delta^2] \\
&= (1-\bar{\theta})p_L^1 + \delta[\frac{1}{4}\bar{\theta}^2 v_H + (1-\bar{\theta})\bar{\theta}v_\Delta] \\
&= (1 - \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})p_L^1 \\
&+ \delta[\frac{1}{4}(\frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})^2 v_H + (1 - \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})\frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}v_\Delta]
\end{aligned} \tag{3.19}$$

if it choses p_L^1 which makes $\bar{\theta} > \frac{1}{2}$. Maximizing the profit function with respect to p_L^1 gives $\bar{\theta} = \frac{(1+\delta)v_L - \frac{1}{2}\delta v_H}{2(1+\delta)v_L - \frac{3}{2}\delta v_H}$ when $max\{\bar{\theta}, \frac{p_\Delta^2}{v_\Delta}\} = \frac{p_\Delta^2}{v_\Delta}$ and $\bar{\theta} = \frac{v_L + \frac{1}{2}\delta v_H}{2v_L + \frac{1}{2}\delta v_H}$ when $max\{\bar{\theta}, \frac{p_\Delta^2}{v_\Delta}\} = \bar{\theta}$. The first $\bar{\theta}$ does not satisfy the requirement $\bar{\theta} \leq \frac{1}{2}$ so that $\hat{p}_\Delta^2 = \frac{1}{2}v_\Delta$ is not rational while the second $\bar{\theta}$ satisfies $\bar{\theta} > \frac{1}{2}$ and the corresponding p_L^1 is positive and smaller than $\bar{p}_L^1$.

•Case ii: When $p_H^1 > v_H + \frac{1}{2}\delta v_H$ and $p_L^1 > (1+\delta)v_L - \frac{1}{2}\delta v_H$, I conjecture that no consumer buys either H or L so that $\bar{\theta}_H > 1$ and $\bar{\theta}_L > 1$ for any pair of (p_L^1, p_H^1) described above. The announced pair of (p_L^1, p_H^1) generates the expectation by the consumers $\hat{p}_H^2 = \frac{1}{2}v_H$ and $\hat{p}_\Delta^2 = \infty$ because in the second period the monopolist maximizes $(1 - \frac{p_H^2}{v_H})p_H^2$ with respect to p_H^2. To ensure that the conjecture that no one buys either H or L is correct, the following equalities must be solved for $\bar{\theta}_H$ and $\bar{\theta}_L$.

$$(1+\delta)\bar{\theta}_H v_H - p_H^1 \geq \delta(\theta_H v_H - \hat{p}_H^2) \tag{3.20}$$

in order for $\bar{\theta}_H$ to buy H in the first period and

$$\begin{aligned}
&max\{(1+\delta)\bar{\theta}_L v_L - p_L^1, \bar{\theta}_L v_L - p_L^1 + \delta(\bar{\theta}_L v_H - \hat{p}_\Delta^2)\} \\
&\geq \delta(\bar{\theta}_L v_H - \hat{p}_H^2)
\end{aligned} \tag{3.21}$$

in order for $\bar{\theta}_L$ to buy L in the first period. With (3.21) RHS is equal to $(1+\delta)\bar{\theta}_L - p_L^1$ given $\hat{p}_\Delta^2$.

Solving (3.20) and (3.21) gives $\theta \geq 1$ and $\theta \geq 1$ when $p_H^1 = v_H + \frac{1}{2}\delta v_H$, $p_L^1 = (1+\delta)v_L - \frac{1}{2}\delta v_H$ and $\hat{p}_H^2 = \frac{1}{2}v_H$. And (3.2) implies if $\bar{\theta}_H \geq 1$ with $p_H^1 = p_H^1 = v_H + \frac{1}{2}\delta v_H$ then $\bar{\theta} > 1$ with $p_H^1 > p_H^1 = v_H + \frac{1}{2}\delta v_H$. And (3.9) implies if $\bar{\theta}_L \geq 1$ with $p_L^1 = (1+\delta)v_L - \frac{1}{2}\delta v_H$ then $\bar{\theta}_L > 1$ with $p_L^1 > (1+\delta)v_L - \frac{1}{2}\delta v_H$. Therefore no one buys either H or L in the first period given (p_L^1, p_H^1) and the expected second period prices described above.

•Case iii: When $p_H^1 \leq v_H + \frac{1}{2}\delta v_H \equiv \bar{p}_H^1$ and $p_L^1 \geq p_H^1$ (or $p_L^1 \geq v_L - \delta v_\Delta + \frac{1}{2}\delta\bar{\theta}v_H$[13]), I conjecture that the relationship between the announced pair of (p_L^1, p_H^1) and the set of consumers who purchase is $\bar{\theta} = \frac{p_H^1}{(1+\frac{\delta}{2})v_H}$[14] where $(\bar{\theta}, 1)$ buy H. The announced pair of prices generates the expectation by the consumers $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}v_H$ since the monopolist maximizes $(\bar{\theta} - \frac{p_H^2}{v_H})p_H^2$ with respect to p_H^2 and $\hat{p}_\Delta^2 = \infty$. The firm's profit function can be written as follows

$$\begin{aligned} \pi &= (1-\bar{\theta})p_H^1 + \delta(\bar{\theta} - \frac{\hat{p}_H^2}{v_H})p_H^2 \\ &= (1 - \frac{p_H^1}{v_H + \delta\frac{1}{2}v_H})p_H^1 + \delta(\frac{p_H^1}{2v_H + \delta v_H})^2 v_H \end{aligned} \tag{3.22}$$

Maximizing the profit function with respect to p_H^1 gives $p_H^1 = \frac{v_H(2+\delta)^2}{2(4+\delta)}$ which is positive and smaller than $\bar{p}_H^1$ for any values of $\delta \in [0,1]$ and v_H.

•Case iv: When $p_H^1 < v_H + \frac{1}{2}\delta v_H$, $p_L^1 \leq (1+\delta)v_L - \frac{1}{2}\delta v_H$ and $p_L^1 < p_H^1$, I conjecture that the relationship between the announced pair of (p_L^1, p_H^1) and the set of consumers who purchase in the first period is $\bar{\theta}_L = \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}$ and $\bar{\theta}_H = \frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta}$ where $(\bar{\theta}_H, 1)$ buy H and $(\bar{\theta}_L, \bar{\theta}_H)$ buy L. The announced pair of prices generates the expectation by the consumers $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}_L v_H$ and $\hat{p}_\Delta^2 = \frac{1}{2}\bar{\theta}_H v_\Delta$ when $\bar{\theta}_L < \frac{1}{2}\bar{\theta}_H$ and $\hat{p}_\Delta^2 = \bar{\theta}_L v_\Delta$ when $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H$ because the monopolist would

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[13] This condition is obtained by solving $(1+\delta)\theta v_L - p_L^1 \leq \delta(\theta v_H - \hat{p}_H^2)$ when $\theta = 1$ and $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}_H v_H$

[14] By increasing differences, if $(1+\delta)\theta v_L - p_L^1 \geq \theta v_L - p_L^1 + \delta(\theta v_H - \hat{p}_\Delta^2)$for $\theta = 1$, it is also the case for all $\theta' < 1$. The inequality is satisfied for $\theta = 1$ when $\hat{p}_\Delta^2 \geq v_\Delta$. Given that the equilibrium is such that no one buys L in the first period, p_Δ^2 is off equilibrium path so that it is safe to assign $\hat{p}_\Delta^2 \geq v_\Delta$. Therefore the cutoff type is given by the indifference of $\bar{\theta}$ in the equality, $(1+\delta)\bar{\theta}v_H - p_H^1 = \delta(\bar{\theta}v_H - \hat{p}_H^2)$ where $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}$.

maximize $(\bar{\theta}_L - \frac{p_H^2}{v_H})p_H^2$ and $(\bar{\theta}_H - max\{\bar{\theta}_L, \frac{p_\Delta^2}{v_\Delta}\})p_\Delta^2$ with respect to p_H^2 and p_Δ^2. If it chooses p_L^1 such that $\bar{\theta}_L < \frac{1}{2}\bar{\theta}_H$ the firm's profit function is written as follows

$$\begin{aligned} \pi &= (1-\bar{\theta}_H)p_H^1 + (\bar{\theta}_H - \bar{\theta}_L)p_L^1 + \delta[(\bar{\theta}_L - \frac{p_H^2}{v_H})p_H^2 + (\bar{\theta}_H - \frac{p_\Delta^2}{v_\Delta})p_\Delta^2] \\ &= (1 - \frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta})p_H^1 + (\frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta} - \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})p_L^1 \\ &+ \delta[\frac{1}{4}(\frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})^2 v_H + \frac{1}{4}(\frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta})^2 v_\Delta]. \end{aligned} \tag{3.23}$$

Maximizing the profit function with respect to p_H^1 and p_L^1 yields $\bar{\theta}_H = \frac{2+\delta}{4+\delta}$ and $\bar{\theta}_L = \frac{\delta v_H - 2(1+\delta)v_L}{3\delta v_H - 4(1+\delta)v_L}$. Since the value of $\hat{p}_\Delta^2$ depends on the assumption $\bar{\theta}_L < \frac{1}{2}\bar{\theta}_H$, $\bar{\theta}_L$ is required to be smaller than $\frac{1}{2}$. However this is not possible[15] and therefore $\hat{p}_\Delta^2 = \frac{1}{2}\bar{\theta}_H v_\Delta$ is not rational.

Next consider the case where p_L^1 is such that $\bar{\theta}_L > \frac{1}{2}\bar{\theta}_H$ so that $p_\Delta^2 = \bar{\theta}_L v_\Delta$. The profit function can be written as

$$\begin{aligned} \pi &= (1 - \frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta})p_H^1 + (\frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta} - \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})p_L^1 \\ &+ \delta[\frac{1}{4}(\frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})^2 v_H \\ &+ (\frac{p_H^1 - p_L^1 - \delta\hat{p}_\Delta^2}{v_\Delta} - \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H})\frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H} v_\Delta] \end{aligned} \tag{3.24}$$

Maximizing the profit function with respect to p_H^1 and p_L^1 yields $\bar{\theta}_H = \frac{1+\delta(\frac{1}{2}+\bar{\theta}_L)}{2+\delta}$ and $\bar{\theta}_L = \frac{\delta v_H - 2(1+\delta)v_L}{3\delta v_H - 4(1+\delta)v_L - 4\delta v_\Delta}$ indeed satisfy $\bar{\theta}_L > \frac{1}{2}\bar{\theta}_H$.[16]

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[15] To see this, remember $v_L > v_\Delta$ and both the numerator and the denominator of $\bar{\theta}_L$ are negative.

[16] In order for $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H$ to hold, it must be the case that $\bar{\theta}_L \geq \frac{1+\frac{1}{2}\delta}{4+\delta}$. This is satisfied for any $\delta \in [0, 1]$ assuming $v_L > v_\Delta$.

3.2.2 Profit comparison

For the ease of reading, let $(\bar{\theta}, 1) = A$ in case iii. Then

$$\begin{aligned}\pi_{iii} &= Ap_H^1 + \delta\frac{1}{2}Ap_H^2 \\ &= A\frac{v_H(2+\delta)^2}{2(4+\delta)} + \delta\frac{1}{2}A\frac{v_H(2+\delta)}{2(4+\delta)}\end{aligned} \tag{3.25}$$

On the other hand in case iv, as a benchmark assume $\bar{\theta}_L$ is the lowest possible value which satisfies $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H$, that is, $\bar{\theta}_L = \frac{1+\frac{\delta}{2}}{4+\delta}$. Then $\bar{\theta}_H = \frac{2+\delta}{4+\delta}$. Note that this value of $\bar{\theta}_H$ is equal to the value of $\bar{\theta}$ in case iii. Then

$$\begin{aligned}\pi_{iv} &= Ap_H^1 + \frac{1}{2}Ap_L^1 + \delta(\frac{1}{2}Ap_\Delta^2 + \frac{1}{4}Ap_H^2) \\ &= A\frac{(2+\delta)(v_\Delta + v_H)}{2(4+\delta)} + \frac{1}{2}A\frac{(2+\delta)(v_L - \delta v_\Delta)}{2(4+\delta)} + \delta(\frac{1}{2}A\frac{(1+\frac{\delta}{2})v_\Delta}{4+\delta} + \frac{1}{4}A\frac{(1+\frac{\delta}{2})v_H}{2(4+\delta)})\end{aligned} \tag{3.26}$$

It is found that $\pi_{iii} < \pi_{iv}$ only when $\frac{4v_L}{13v_L+5v_\Delta} < \delta$. Remember that π_{iv} given above is not the maximized profit. Therefore it is possible that $\pi_{iii} < \pi_{iv}$ holds for some δ such that $\delta < \frac{4v_L}{13v_L+5v_\Delta}$. On the other hand, when $\delta = 0$, $\pi_{iii} > \pi_{iv}$ because $\bar{\theta}_H = \frac{1}{2}$ and $\bar{\theta}_L = 0$ in case iv. Together with the fact that both π_{iii} and π_{iv} are continuous in δ it can be concluded that there exists $\bar{\delta}$ such that $\pi_{iii} < \pi_{iv}$ only if $\delta > \bar{\delta}$.

3.2.3 Equilibrium

Monopolist's strategy: When $\delta > \bar{\delta}$, announce $(p_L^1, p_H^1) = (\bar{\theta}_L[(1+\delta)v_L - \frac{1}{2}\delta v_H], \bar{\theta}_H v_\Delta + p_L^1 + \delta p_\Delta^2)$ where $\bar{\theta}_L = \frac{\delta v_H - 2(1+\delta)v_L}{3\delta v_H - 4(1+\delta)v_L - 4\delta v_\Delta}$, $\bar{\theta}_H = \frac{1+\delta(\frac{1}{2}+\bar{\theta}_L)}{2+\delta}$ and $p_\Delta^2 = \bar{\theta}_L v_\Delta$ in the first period. In the second period, announce (p_Δ^2, p_H^2) such that the profits are maximized in the market for those who have purchased L and in the market for those who have not purchased L respectively. When $\delta \leq \bar{\delta}$, announce $(p_L^1, p_H^1) = (\tilde{p}_L^1, \frac{v_H(2+\delta)^2}{2(4+\delta)})$ where $\tilde{p}_L^1 \geq p_H^1$ or $\tilde{p}_L^1 \geq v_L - \delta v_\Delta + \frac{1}{2}\delta\bar{\theta}v_H$ and $\bar{\theta} = \frac{2+\delta}{4+\delta}$ in the first period. In the second period, announce (p_Δ^2, p_H^2) such that the profits are maximized in the market for those who have purchased L and in the market for those who have not purchased L respectively.

Consumers' strategy: If (p_L^1, p_H^1) is what is described in case i, accept according to $\bar{\theta} = \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}$ if $\theta \in (\bar{\theta}, 1)$ and reject otherwise. If (p_L^1, p_H^1) is as described in case ii, do not accept any offer. If (p_L^1, p_H^1) is as described in case iii accept according to $\bar{\theta} = \frac{p_H^1}{v_H + \delta\frac{1}{2}v_H}$ if $\theta \in (\bar{\theta}, 1)$ and reject otherwise. If (p_L^1, p_H^1) is what is described in case iv accept according to $\bar{\theta}_L = \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}$ if $\theta \in (\bar{\theta}_L, \bar{\theta}_H)$ and $\bar{\theta}_H = \frac{p_H^1 - p_L^1 - \delta \hat{p}_\Delta^2}{v_\Delta}$ where $\hat{p}_\Delta^2 = \bar{\theta}_L v_\Delta$ if $\theta \in (\bar{\theta}_H, 1)$.

In the second period, accept p_Δ^2 if he bought L in the first period and if $\theta v_H - p_\Delta^2 \geq \theta v_L$. Accept p_H^2 if he did not buy L in the first period and $\theta v_H - p_H^2 \geq 0$.

Beliefs: When (p_L^1, p_H^1) is what is described in case i, consumers believe that the set of consumers who purchase L in the first period is expressed by $\bar{\theta} = \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}$ where $(\bar{\theta}, 1)$ purchase L so that $\hat{p}_H^2 = \frac{1}{2}\bar{\theta} v_H$ and $\hat{p}_\Delta^2 = \frac{1}{2}v_\Delta$ if $p_L^1 < \frac{1}{2}(1+\delta)v_L - \frac{1}{4}\delta v_H$ and $\hat{p}_\Delta^2 = \bar{\theta} v_\Delta$ if $p_L^1 \geq \frac{1}{2}(1+\delta)v_L - \frac{1}{4}\delta v_H$. When (p_L^1, p_H^1) is as described in case ii, consumers believe that no one buys L or H in period one so that $\hat{p}_H^2 = \frac{1}{2}v_H$ and $\hat{p}_\Delta^2 = \infty$. When (p_L^1, p_H^1) is what is described in case iii, consumers believe that the set of consumers who buy H in period 1 is expressed by $\bar{\theta} = \frac{p_H^1}{(1+\frac{\delta}{2})v_H}$ where $(\theta, 1)$ buy H so that $\hat{p}_H^2 = \frac{1}{2}\theta v_H$. When (p_L^1, p_H^1) is as described in case iv consumers believe that the set of consumers who purchase L in the first period is expressed by $\bar{\theta}_L = \frac{p_L^1}{(1+\delta)v_L - \frac{1}{2}\delta v_H}$ and the set of consumers who purchase H in the first period expressed by $\bar{\theta}_H = \frac{p_H^1 - p_L^1 - \delta \hat{p}_\Delta^2}{v_\Delta}$ where $\hat{p}_\Delta^2 = \frac{1}{2}\bar{\theta}_H v_\Delta$ ($\hat{p}_\Delta^2 = \bar{\theta}_L v_\Delta$) when $\bar{\theta}_L < \frac{1}{2}\bar{\theta}_H$ (when $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H$) and where $(\bar{\theta}_L, \bar{\theta}_H)$ and $(\bar{\theta}_H, 1)$ buy L and H respectively so that $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}_L v_H$ and $\hat{p}_\Delta^2 = \frac{1}{2}\bar{\theta}_H v_\Delta$ when $\bar{\theta}_L < \frac{1}{2}\bar{\theta}_H$ and $\hat{p}_H^2 = \frac{1}{2}\bar{\theta}_L v_H$ and $\hat{p}_\Delta^2 = \bar{\theta}_L v_\Delta$ when $\bar{\theta}_L \geq \frac{1}{2}\bar{\theta}_H$.

3.2.4 Summary of the results

When the firm sells L in the first period as well as H, it allows the firm to price discriminate among consumers. For such a scheme to be more profitable than offering just H in the first period, it is required that the firm is sufficiently patient. When the firm is not sufficient enough, it sells only H in the first period.

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