State University of New York at Stony Brook
College of Engineering and Applied Sciences

Technical Report No. 750

Location Tracking in Hierarchical Cellular Systems: An Adaptive Algorithm

by

Daqing Gu and Stephen S. Rappaport

Department of Electrical Engineering
State University of New York
Stony Brook, New York 11794-2350

e-mail: dgu@sbee.sunysb.edu , rappaport@sunysb.edu

Date: November 14, 1997
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State University of New York at Stony Brook
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e-mail: dgu@sbcc.sunysb.edu, rappaport@sunysb.edu

Abstract

Evolving personal communication systems (PCS) may have a layered hierarchical structure consisting of small cells overlayed by macrocells which may in turn be overlayed by even larger macrocells, and so on in like manner. For example, if a system uses both terrestrial and satellite segments, satellite spot beams may overlay clusters of macrocells in the terrestrial segment, while each macrocell overlays clusters of microcells. We consider location tracking in such a system. In our location management scheme, at any time a mobile user is registered at a certain level of the hierarchy and lies within a location area (LA) that is defined at that level. The registered level of a mobile user can be adjusted according to the class of mobile users to which it belongs. This class is largely determined by mobility and call rate characteristics for that user. The size of the LA for a mobile user is the number of cells at the registered level. Both the size and shape of the LA at registered level for each individual mobile user are optimized according to estimates of the mobile user's mobility and incoming call arrival rate as well as the user's movement behavior and the surrounding geographical conditions in its current location, so that the combined average cost of location updating and paging for each mobile user is minimized. This guarantees that the LAs are suitable and optimal for different kinds of mobile users, such as, pedestrians, vehicles, buses, etc. The location tracking procedure also includes a paging algorithm. Same-level paging (through cells of the registered level) and cross-level paging (through cells of other levels instead of the registered level) are discussed. Iterative algorithms for computing optimal location areas for each mobile user are developed.

The research reported in this paper was supported in part by the U.S. National Science Foundation under Grant no. NCR 94-15530 and in part by BMD0/EST under grant no. N00014-9511217 administered by the U.S. Office of Naval Research. General research support from Hughes Network Systems is gratefully acknowledged.
1. Introduction

To establish a call connection for a mobile user in a mobile communication system, the location of the mobile user in terms of network access, must be established. In some of the first generation mobile systems, the system pages a particular mobile user over the entire service area to establish such a connection [8]. This method is appropriate for small and low density systems but is not suitable for large and high density cellular systems since it produces excessive signaling traffic for paging.

A commonly used strategy for tracking mobile users in current cellular systems is to partition the entire service area into contiguous and distinct location areas (LAs), each consisting of a group of cells [7],[8],[9]. The LA of a mobile user is updated when the mobile user enters a new LA. When an incoming call arrives, the system pages the called mobile user in the mobile user’s current LA. This approach is a fixed scheme in the sense that the positions, sizes and shapes of LAs are fixed and predefined. The positions, sizes and shapes of the LAs in this scheme can be chosen to provide optimal performance for average mobility, average incoming call arrival rate and typical movement behavior. This approach considers the average over all mobile users in the system and seeks to minimize the average total signaling cost for location updating and paging. For systems that support a mixture of platforms whose mobility characteristics are very varied, such as pedestrians, highway vehicles, street vehicles, buses, etc., this approach may lead to a solution which is not good for many users. We propose and analyze an adaptive scheme which overcomes this shortcoming of fixed schemes.

Generally larger LAs allow less frequent location updating for mobile users and thus less location updating signaling traffic. However, the accompanying larger paging areas introduce more paging signaling traffic. Therefore, a tradeoff exists between the location updating cost and the paging cost [5], [7], [8], [9], [10], [13]. A general observation for a desirable LA size is that a larger LA is preferred for a mobile user with high Mobility-to-Call-Arrival-Rate Ratio (MCARR), while a smaller LA is preferred for a mobile user with low MCARR. Moreover, the shape of LAs for a mobile user should match the user’s movement behavior and local
geographical conditions. Thus, optimal location areas for a mobile user should be user dependent.

Various location tracking strategies and location management schemes have been proposed [5], [6], [7], [8], [9], [10]. In [5], the optimal size of square-shaped LAs for a mobile user is dynamically determined according to the mobile user's current mobility and incoming call arrival rate. Three dynamic location update strategies, time-based, movement-based, and distance-based, are investigated in [6]. In these, a mobile user performs location updating, respectively, according to the time elapsed, the number of cell changes encountered, and the distance traveled, since the last location updating. In [7], a combined distance-based location updating and paging scheme, in which the optimal update threshold distance is determined using an iterative algorithm, is discussed. A scheme which exploits the fact that the mobility behavior of many mobile users can be predicted over a certain time period by storing each mobile user's mobility statistics is discussed in [8]. Multilevel LAs for location updating in hierarchical cellular structures are discussed in [9] where the registered level is dynamically changed according to the mobile user's past as well as present mobility. In [10], a dynamic location tracking strategy is proposed. The size of each LA for a mobile user is dynamically determined on the basis of the mobile user's previous and present mobility estimates and current incoming call arrival rate. The shape of each LA for a mobile user is individually and dynamically tailored to match the user's movement and the surrounding geographical conditions by grouping those cells that the mobile user are most likely to visit into a LA. A brief survey of location tracking strategies appears in [13].

For future personal communication systems (PCS), the issue of location tracking strategies should be reconsidered. This is because PCS will serve a very large number of mobile users with a broad range of mobility within an integrated worldwide system. Hierarchical cellular structures [1], [2], [3], [4] to accommodate different kinds of mobile users may include both satellite and terrestrial segments. The terrestrial segment may contain large macrocells and small microcells that have evolved from existing cellular systems [9]. Such a system
configuration is schematically illustrated in Fig. 1. Specifically, a spot beam is used to overlay a cluster of macrocells, while each macrocell in turn overlays a cluster of microcells. With this hierarchical cellular structure, at any time, the LA of a mobile user can be considered at any of the system levels (microcell, macrocell and spot beam level). Since cells at these levels cover the same geographical location area, the mobile user can be paged through any level according to a paging algorithm. The use of resources for location tracking in such a system introduces new aspects for consideration.

In this paper, we consider a hierarchical overlaid cellular structure. A mobile user can have an LA at one level at a given time. This level is defined as the registered level of the mobile user. The registered level of a mobile user is not necessarily the same as the service level through which call attempts are served. The registered level of a mobile user can be adjusted according to the mobile user’s class. The LA size (in term of number of cells) of the mobile user at the registered level is dynamically determined on the basis of the mobile user’s previous and present mobility estimates and the current incoming call arrival rate, and the shape of each LA for a mobile user is individually and dynamically tailored to match the user’s movement behavior and the surrounding geographical conditions, in this way the combined average cost of location updating and paging for each mobile user is minimized. The design of optimal location areas at each registered level is formulated as an optimization problem. An iterative algorithm is developed to find optimal LAs for each mobile user that minimizes the combined average cost of location updating and paging. This guarantees that the LAs are suitable and optimal for different kinds of mobile users such as pedestrians, vehicles, buses, etc. Same-level paging (through cells of the registered level) and cross-level paging (through cells of levels other than the registered level) are discussed.

Recently, a location update scheme was reported [18]. The objective was to find the optimal LA shape that minimizes the location update cost subject to a constraint on the LA size. To reduce the analysis and computation complexity, a square cellular structure and shortest distance mobility model were used. In this mobility model, a mobile user always follows the shortest path (measured by the number of cells traversed) to its destination within a new LA. A
mobile user just can move straight, left and right in each cell. The set of unconditional probabilities of a mobile user going straight, left and right was assumed to be same in all cells. The impact of each individual user's movement behavior and of geographical conditions on the shape of the LA were not considered.

In the scheme proposed here, we model a mobile user's movement at microcell level by a continuous-time Markov process whose behaviors at state transition instants are governed by the cell-to-cell transition probabilities at microcell level given by the cell transition probability matrix (TPM) [11]. This takes into account the user's movement behavior and geographical conditions. The concept of cell dwell time is used [12] in the mobility model. An iterative algorithm is used to find the optimal location area at each registered level. No special prior assumptions about the size and shape of cells are made. Thus, the proposed scheme is general, broadly applicable and robust. Moreover, the proposed scheme is especially suitable for systems having a hierarchical cellular structure.

2. System Description

The system under discussion is considered to have a hierarchical cellular structure that includes a satellite segment and a terrestrial segment as shown in Fig. 1. The satellite segment contains spot beams, and a terrestrial segment contains large macrocells and small microcells. Mathematically, the proposed scheme can be applied to a hierarchical cellular structure that has any number of levels. To simplify the present discussion, we consider a system three levels: microcell level, macrocell level and spot beam level. Spot beams are used to overlay a cluster of macrocells, while each macrocell in turn overlays a cluster of microcells. These three levels cover the same geographical region.

The entire service area, which may include multiple spot beams, is traversed randomly by a large number of mobile users. A call attempt to a mobile user can be served by one of these three levels, this level is called the service level of the mobile user. A mobile user can have a LA at one of these three levels at a given time. This means that the LA of the mobile user consists of cells of this level. We define this level as the registered level of the mobile user. The registered
level of a mobile user can be adjusted according to the user's MCARR. For example, mobile users with low MCARR tend to have small LAs. Mobile users in this class are assigned to register at the microcell level. Mobile users with medium and high MCARR tend to have medium and large LAs. They are therefore assigned to register at the macrocell level and spot beam level, respectively. In this way we can balance the number of cells in LAs for different classes of mobile users and reduce computation when determining LAs for a mobile user. When a mobile user's MCARR changes to a different category, the registered level must be changed accordingly.

Every cell at each level in the service area periodically broadcasts its own cell ID and the level it belongs to through its broadcast channel. We assume that each mobile user "knows" its current registered level and maintains a list of cells identifiers (IDs) for those cells in its current LA. By monitoring the strongest downlink broadcast channel at its registered level and comparing any newly received cell ID with stored cell IDs, a mobile user can easily determine whether a location update is required. If the newly received cell ID is not on the user's list, a location update is initiated and the list of stored cell IDs will be replaced by its new LA. At this time of location updating the mobile user's registered level can also be changed according to its current MCARR.

3. Mobility Model

The mobility model given here makes use of dwell time concepts[10], [12] but makes appropriate adaptations for hierarchical cellular structures. Mobile users are free to move and change cells and registered levels in the service area. We model the mobility of each mobile user at any level by cell changes of that level and refer to such changes as movements from cell to cell at that level.

We assume that a mobile user can change cells at the microcell level at any time instant and that mobile users' movements at the microcell level are probabilistic and independent from one mobile user to another. Specifically the movement of a mobile user at the microcell level is modeled by a continuous-time Markov process whose behavior at state transition instants are
governed by microcell-to-microcell transition probabilities given by the transition probability matrix (TPM). Conceptually we treat microcells in the service area as states. Cell changes of a at the microcell level correspond to state transitions of the continuous-time Markov process describing the mobility of the user.

For a system with \( m \) microcells, the TPM for a mobile user is an \( m \times m \) matrix whose elements, \( p_{ij} \) (i.e., \( i = 1, 2, 3, \ldots, m \)), give the probability with which a mobile user will next move to microcell \( j \) upon leaving its current microcell \( i \). Thus,

\[
0 \leq p_{ij} \leq 1 \quad i, j = 1, 2, 3, \ldots, m \\
p_{ii} = 0 \quad i = 1, 2, 3, \ldots, m \\
\sum_{j=1}^{m} p_{ij} = 1 \quad i = 1, 2, 3, \ldots, m
\]

Generally, \( p_{ij} \) depends on the current microcell \( i \) and the microcell \( j \) as well as the mobile user. A TPM for each individual mobile user can be estimated from collected data which takes into account the user's movement behavior and geographical condition. In this formulation any two microcells in the service area can be considered to be neighbors regardless of their physical positions. The TPM of a mobile user will be used to group a cluster of cells into a LA such that the optimal size and shape of the LA for the mobile user can be obtained. This mobility model at microcell level is summarized below:

- A cell change of a mobile user at microcell level can occur at any time instant.
- The time interval between successive cell changes at microcell level is called dwell time of the mobile user in a microcell.
- The dwell time in a microcell for a mobile user is exponentially distributed with the parameter that depends on the user and the microcell.
- At some time instant, a mobile user moves into microcell \( i \). After it spends a dwell time in microcell \( i \), it will move to one of its neighboring cells, say microcell \( j \), with probability \( p_{ij} \).

Consider the movement of a mobile user at other levels (macrocell level, spot beam level). Since a spot beam overlays a cluster of macrocells, and a macrocell further overlays a
cluster of microcells, a spot beam or a macrocell can be seen as a combination of states of the Markov movement process at the microcell level. In general, if we combine some states of a Markov process into a large state the process is no longer Markovian [17]. Thus, the movements of a mobile user at both macrocell level and spot beam level can not be treated as Markov processes under the assumption that the movement of the mobile user at the microcell level is modeled by a continuous-time Markov process. However, relevant quantities related to the movement of a mobile user at the higher (macrocell and spot beam) levels can be obtained from the description of the user’s movement at the microcell level where it is assumed to be Markovian.

4. Determination of Registered Level

A mobile user can register at one of the three levels at a given time. The registered level is determined by the mobile user’s MCARR. A mobile user’s mobility is directly related to its dwell time in a cell. The relation between dwell time in a cell for a mobile user and its velocity and cell size has been suggested and analyzed in [12]. Generally, a mobile user with high mobility has a small dwell time in a cell, while a low mobility user has relatively large cell dwell time for a given cell size. It is reasonable to assume that the user’s mobility is inversely proportional to its cell dwell time, since the cells’ sizes are constant for a given cellular system.

Let \( \overline{T}_d \) be the average dwell time of a mobile user in a cell, and let \( \lambda \) be the incoming call arrival rate to this user. Then the mobile user’s MCARR represented by \( R \) can be defined by

\[
R = \frac{1}{\overline{T}_d} = \frac{1}{\lambda \cdot \lambda}.
\]  (4)

Theoretically, the average cell dwell time \( \overline{T}_d \) can be the average dwell time of a mobile user in a microcell, macrocell or spot beam. In order to be consistent with the assumed mobility model, the average dwell time \( \overline{T}_d \) in equation (4) represents average dwell time in a microcell in the system. It can be estimated by recording individual cell dwell times of microcells that the user has visited in the area covered by its current LA. Even though this mobile user may be registered at the macrocell level or spot beam level, its LA also covers a cluster of microcells since the three levels are overlaid on the same region.
A mobile user travels in the service area with changing mobility and changing incoming call arrival rate, the mobile user’s MCARR varies with time. For the proposed scheme, the range of a user’s MCARR is classified into three distinct classes, each class corresponds a particular registered level. Specifically, these three classes and their corresponding range and corresponding registered levels can be depicted as follows:

<table>
<thead>
<tr>
<th>Classes</th>
<th>Ranges of MCARR</th>
<th>Registered Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low MCARR</td>
<td>[0, $R_1$)</td>
<td>Microcell level</td>
</tr>
<tr>
<td>Medium MCARR</td>
<td>($R_1$, $R_2$)</td>
<td>Macrancell level</td>
</tr>
<tr>
<td>High MCARR</td>
<td>($R_2$, $\infty$)</td>
<td>Spot beam level</td>
</tr>
</tbody>
</table>

Where $R_1$ and $R_2$ are two constants which can be set by the system designer. A user’s class at any time is determined by the value of its MCARR. At each location update, a mobile user checks its current MCARR, if its current MCARR falls in a different range, the mobile users changes its registered level appropriately.

5. Location Area Configurations and Paging Algorithms

The overall issue of LA design is to select a cluster of cells to be included in an LA in order to minimize the signaling cost incurred by location updating and paging in the system. Since optimal location areas for a mobile user depends on the user’s mobility, incoming call arrival rate as well as its movement behavior and geographical conditions, optimal LAs of a mobile user should be dynamically determined. In the present scheme, each mobile user always maintains in its memory; 1) a list of IDs of those cells in its current LA; and, 2) its current registered level. When a mobile moves in the service area and enters a new cell of its registered level whose ID is not on the list, a location update is performed and the list is replaced by a new list which defines its new LA. This new cell is defined as initial cell of the user’s new LA.

The physical position where the mobile user last performed location update is simultaneously covered by a microcell, a macrocell, and a spot beam due to a hierarchical cellular structure. We define the microcell, the macrocell and the spot beam as the user’s initial
cell at corresponding levels, and call them the user’s initial microcell, initial macrocell, and initial spot beam, respectively. We see that initial spot beam covers initial macrocell, while initial macrocell in turn covers initial microcell. The motivation here is that for a given number of cells, say \( r \), we want to select \( r \) cells from the user’s registered level such that the LA formed by these \( r \) cells has a lower location update rate than an LA formed by any other \( r \) cells.

5.1. Location Area Configurations at Microcell Level

At each location update, the mobile user determines its class according to the value of its MCARR. If the mobile user currently has a low MCARR, then its current registered level is microcell level, and its new LA is formed by a cluster of microcells. The new LA of the mobile user must include the initial microcell; otherwise, the mobile user always keeps updating its LA. We here suppose that there are \( r \), \( 1 \leq r \leq m \), microcells in the new LA of a mobile user. Without loss of generality, we renumber the microcells from 1 through \( r \) with the initial microcell numbered 1. Let the transition probability matrix of these \( r \) microcells be

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1r} \\
    a_{21} & a_{22} & \cdots & a_{2r} \\
    \cdots & \cdots & \cdots & \cdots \\
    a_{r1} & a_{r2} & \cdots & a_{rr}
\end{bmatrix}
\]  

(5)

in which, \( a_{ij}, i, j = 1, 2, 3, \ldots, r \), is the probability that the mobile user will next move into microcell \( j \) upon leaving the current microcell \( i \). The matrix \( A \) can be obtained from the user’s TPM by simply deleting those rows and columns of the TPM that are not corresponding to these \( r \) microcells.

If we consider microcells that are not in the new LA to be absorbing states of the Markov process, then the mobile user’s movement in the LA can be modeled by an absorbing Markov process with the initial position being initial microcell. In this description the mobile user entering an absorbing state means that it leaves the LA. For an absorbing Markov process given the initial state, the average number of times that a state has been occupied before the process enters an absorbing state can be found from its fundamental matrix [17]. In the fundamental matrix of an absorbing Markov process, the element with indices, \((i, j)\) is the average number of
times that state $j$ is visited by the process before the process enters an absorbing state, given that the process was initially in state $i$. Let $B$ be the fundamental matrix of the absorbing Markov process. According to [17], we have

$$B = (I - A)^{-1}$$  \hspace{1cm} (6)

where $I$ denotes a unity $r 	imes r$ matrix. The matrix, $B$ has the form

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & \cdots & b_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rr} \end{bmatrix}$$ \hspace{1cm} (7)

Since the initial microcell was numbered 1 in the matrix $A$, the element $b_{ij}$, $1 \leq j \leq r$, is the average number of times that microcell $j$ has been visited by the mobile user before the mobile user leaves the LA. So, the average number, $\bar{N}$, of microcell changes needed for the mobile user to leave the LA is given by

$$\bar{N} = \sum_{j=1}^{r} b_{ij}$$ \hspace{1cm} (8)

Let $\bar{T}_j$ denote the average dwell time of the mobile user in microcell $j$, $1 \leq j \leq r$. Then the average time interval that the mobile user resides in the LA is

$$\bar{T}_d = \sum_{j=1}^{r} (b_{ij} \cdot \bar{T}_j)$$ \hspace{1cm} (9)

This is called the dwell time of the mobile user in the LA. So the average location update rate, $R_u$, is

$$R_u = 1 / \bar{T}_d$$ \hspace{1cm} (10)

The objective here is to select these $r$ microcells to form the new LA for a mobile user such that the location update rate, $R_u$, is least. This is an optimization problem and can be formulated as follows:

\begin{align*}
\text{Minimize} & \quad R_u = 1 / \sum_{j=1}^{r} (b_{ij} \cdot \bar{T}_j) \\
\text{Subject to} & \quad \text{given } r \text{ and initial microcell}
\end{align*}

\hspace{1cm} (11)
The exact solution to this optimization problem can be obtained by exhaustive enumeration. However, if all combinations are enumerated, the required computation dramatically increases with \( r \) and the total number of microcells, \( m \), in the service area. For the situation on hand however, we note that a mobile user's new LA is formed from the initial microcell and a cluster of microcells that are close or adjacent to the initial microcell. Therefore, we only need to select \( r \) microcells from this cluster of microcells, rather than from all microcells in the service area. In this way the total number of combinations of \( r \) microcells to be examined can be greatly reduced. Further reduction can be made if we consider only qualified and non-redundant combinations. Qualified combinations are those combinations that include \( r \) distinct microcells and the mobile user can move into any of these \( r \) microcells from its initial microcell without passing through the other microcells that are not in this combination. Redundant combinations are those combinations that are the same as one of those combinations that have been computed regardless of the order of the microcells in the combination. In the following we develop an algorithm to find the optimal LA shape at the microcell level given the number of microcells in the LA, \( r \), and the initial microcell. The idea behind this algorithm is to consider alternative lists each containing \( r \) microcells. For each list, compute the location update rate for a mobile user in the LA defined by the list of \( r \) microcells. Then select (as the mobile user’s new LA) the list having the minimum location update rate. In order to clearly describe the algorithm, we first define the following symbols:

\[ L: \] the list that will be used to contain \( r \) microcells.

\[ Set(i): \] a set of microcell IDs that a mobile user will visit with nonzero probability during its first \( i \) microcell changes (the initial state is being counted as the first microcell change).

Algorithm 1 (optimal algorithm):

1. Initialize the list \( L \).
   
   Add the initial microcell to the list \( L \). Thus \( L \) contains only the initial microcell.
(II) Identify alternative combinations of \( r \) microcells.
    Select one microcell from each \( \text{Set}(i), 2 \leq i \leq r \). Then append these \( r-1 \) microcells to the list \( L \). The selection can be arranged to avoid redundant combinations.

(III) For each of the combinations of \( r \) microcells, determine if it is a qualified combination.

(IV) Delete all unqualified combinations.

(V) For each of the combinations left, compute the average location update rate, \( R_u \), using equations (5) - (10) and compare them.

(VI) Keep the combination with the least location update rate as the mobile user’s new LA.

In this way, the optimal shape of the LA at the microcell level for a mobile user can be found given the user’s initial microcell and \( r \). The selection is optimal in the sense that the average location update rate is minimized. The computational complexity is greatly reduced in the above algorithm since we delete all unqualified and redundant combinations.

In the present scheme, since a mobile user tends to register at an appropriate level according to its \( MCARR \), the number of cells in a LA is relatively small for all classes of mobile users, it is feasible to use this optimal algorithm to compute the optimal LA at least for small LAs.

For large values of \( r \), algorithm 1 still has a high computation time complexity. In order to increase computation efficiency, we developed a heuristic algorithm to iteratively compute a near optimal solution of the equations (11) and (12). The approach is to begin with only the initial microcell in the list and then add one microcell, which is one of the neighboring microcells of the LA formed by the current LA, to the list at each iteration. Neighboring microcells are those which are not on the list but which can be reached (with one microcell change) from at
least one microcell on the list. Specifically, a neighboring microcell is any which has a nonzero, 1-step, transition probability from at least one microcell on \( L(n) \) and which itself is not already on the list. The addition of the microcell to the list is made such that the LA formed by the new list results in a lower location update rate, (i.e., a larger dwell time of the mobile user in the LA), than that by any other neighboring microcell added to the list. This heuristic algorithm is given below:

**Algorithm 2 (heuristic algorithm):**

(I) Let \( n = 1 \). Put the initial microcell on the list. The first list, \( L(n) \), \((n = 1)\) contains only the initial microcell.

(II) Individually identify neighboring microcells of the LA formed by \( L(n) \). If there are no neighboring microcells identified in this step, the procedure terminates with \( L(n) \) as the list defining the location area.

(III) Choose one of the neighboring microcells identified in step II, and append it to the list \( L(n) \). This forms a candidate list \( L^*(n+1) \). For this candidate list, determine the average location update rate, \( R_u \), using equations (5) - (10).

(IV) Remove the last neighboring microcell that was appended to the list. That is, revert to \( L(n) \) and repeat step III for each of the neighboring microcells identified in step II until all neighboring microcells have been considered. Retain the list \( L^*(n+1) \) having the smallest value of \( R_u \).

(V) The new list, \( L(n+1) \) is taken as \( L^*(n+1) \).

(VI) If \( n = r - 1 \), the procedure is terminated. Otherwise replace \( n \) by \( n + 1 \) and go back to step II.

The resultant list from the above procedure is an ordered list. That is, the first \( k \), \( k \leq r \), microcells in the list constitute the near optimal location area with \( k \) microcells.
5.2. Location Area Configurations at Macrocell level and Spot Beam Level

If a mobile user is registered at the macrocell level, its new LA should consist of a cluster of macrocells. That means the list stored in the user's memory contains a number of macrocell IDs. Similar to the case for which the LA is configured at the microcell level, we want to select \( t \) macrocells to form the optimal LA at macrocell level such that the average location update rate for a mobile user is minimized given the user's initial position and \( t \), and the new LA must include the initial macrocell. This is also a optimization problem. However, since the users' movements at the macrocell level are not a Markov process, the users' dwell time in the LA at macrocell level cannot be directly obtained from the users' movements at the macrocell level. Because the macrocell and microcell levels cover the same geographical area, we can use the Markovian property of a mobile user's movement at the microcell level to compute the user's dwell time in the LA at macrocell level.

Suppose these \( t \) macrocells cover \( k \) microcells. (These \( k \) microcells contain the initial microcell). The event that the mobile user leaves the LA is equivalent to its moving out of these \( k \) microcells. The average time interval that the mobile user spends in these \( k \) microcells and the average location update rate, \( R_u \), can be obtained using equations (5) - (10). Thus, we successfully convert an optimization problem at the macrocell level to one at microcell level. The algorithm 1 and algorithm 2 need the following minor modifications to be suitable for computations of the LA at the macrocell level:

1) The cell IDs on the list are macrocell IDs rather than microcell IDs with the initial macrocell replacing initial microcell.

2) Using the microcells covered by the list of macrocells to compute the average location update rate, \( R_u \).

In the same way, the optimal and near optimal LA configurations for a mobile user at the spot beam level can be obtained.
5.3 Paging Algorithms

When an incoming call arrives, the system pages the called mobile user in the user's current LA. The simplest paging algorithm is to page the called mobile user in all cells of the user's current LA simultaneously. This paging strategy is called * simultaneous paging *. But, this algorithm requires excessive paging signaling traffic. In order to reduce average paging signaling traffic, multistep paging algorithms can be used [7], [11], [13], [15]. In multistep paging, the current LA of a mobile user is partitioned into a number of sub-areas called * paging zones * according to some algorithm. The system polls one * paging zone * at a time until the called mobile user is found.

In the hierarchical cellular structure, a new paging strategy has to be provided for this mixed cell environment. For example, the mobile user can register at one of three levels, and the paging can be performed through cells at the user's registered level or through cells at other levels. We call these strategies * same-level paging * and * cross-level paging *, respectively. The level at which the paging is performed is called the paging level of the mobile user.

To determine the paging level, we assume that the paging signaling costs for single page in a microcell, a macrocell and a spot beam are respectively \( C_n, C_m, C_s \) in term of bandwidth consumed. Since the available bandwidth are different at different levels due to users' density, frequency reuse pattern at each level, etc. It is reasonable to assume that the bandwidth cost are different at different levels. We here assume that relative bandwidth costs at macrocell and spot beam levels compared to that at microcell level are respectively \( \beta_m, \beta_s \). For a mobile user registered at one of these three levels, suppose the region covered by the current LA of the mobile user includes \( r \) microcells at microcell level, \( t \) macrocells at macrocell level, and \( s \) spot beams at spot beam level. Considering a * simultaneous paging * algorithm, if the paging is through these \( r \) microcells, the total paging cost for one search is

\[
C_{mp} = C_m \cdot r
\]  

(13)

If the paging is performed at macrocell level, the total paging cost is

\[
C_{mp} = \beta_m \cdot C_m \cdot t
\]  

(14)
If the paging is through these s spot beam, the total paging cost is
\[ C_{p} = \beta_{i} \cdot C_{p} \cdot s \]  
(15)

Comparing the total paging costs at these three levels, we choose the level that results in the smallest total paging cost as paging level.

6. Combined Cost Function and Optimal Location Areas

Since the goal of LA design is to find the optimal LA for a mobile user so as to minimize the combined cost for location updating and paging. In the following, we first give a combined cost function of location updating and paging for a mobile user. Then an algorithm to find the optimal LA for the mobile user is determined.

6.1 Combined Cost Function

A location update is initiated whenever a mobile user enters a new LA. Note that, a mobile user’s expected dwell time in an LA can be calculated at the microcell level regardless of the mobile user’s registered level. Also recall that \( \bar{N} \) is the average number of microcell changes needed for a mobile user to leave an LA and the average dwell time of the mobile user in a microcell averaged over all microcells it has visited in the LA is \( \bar{T}_{d} \). Thus, location updating occurs at an average rate \( (1 / \bar{N} \cdot \bar{T}_{d}) \). Let \( C_{s} \) be the signaling cost for a mobile user to perform single location update in term of bandwidth consumed, then the average location updating cost of a mobile user per unit time \( C_{u} \) is:
\[ C_{u} = \frac{C_{s}}{\bar{N} \cdot \bar{T}_{d}} \]  
(16)

When an incoming call attempts to reach a mobile user, the system pages the mobile user in the user’s current LA. Since \( \lambda \) is the incoming call arrival rate of the mobile user, then the average paging cost for the mobile user per unit time, \( C_{p} \), is
\[ C_{p} = \lambda \cdot C_{p} \]  
(17)
Where \( C_p \) in \( m, M, x \), depends on the paging level and can be obtained from equation (13)-(15). The combined average cost for location updating and paging for the mobile user per unit time, \( C \), is

\[
C = C_a + C_p
\]  
(18)

6.2. Optimal Location Areas

In Sections (5.1) and (5.2), we discussed the optimal LA configurations at different levels given the number of cells in a LA. The optimal LA configurations greatly depends on the user's TPM. Since there is a tradeoff between location updating cost and paging cost, the optimal size of a LA, i.e., the number of cells in the LA, for a mobile user exists such that the combined average cost is minimized. Thus we want to solve the following optimization problem for a given mobile user.

\[
\text{Minimize} \quad C = C_a + C_p
\]  
(19)

The optimal size of LA depends on the user's current mobility and incoming arrival rate. Generally, \( C_a \) in (18) decreases with increasing of the number of cells (microcells, macrocells, or spot beams) in the LA, \( n \), while the average paging cost, \( C_p \) in (18), increases with \( n \) increasing. Thus, the combined average cost function, \( C \), of location updating and paging for the mobile user may have a local minimum for some \( n \). In the following, we present an algorithm to compute optimal size LA for a mobile user.

(I) Let \( n=1 \). Compute the combined average cost, \( C(n) \), (of location updating and paging) for the mobile user - for the LA that includes only the initial cell (initial microcell, initial macrocell, or initial spot beam for different registered level) by using equations (16) - (18).

(II) Let \( n=n+1 \), then select the optimal LA configuration with \( n \) cells according the algorithm presented in Section (5.1) or Section (5.2). Compute the combined average cost of location updating and paging, \( C(n) \), for the LA using (16) - (18).

(III) If the combined cost \( C(n) \geq C(n-1) \), then the procedure terminates. The LA with \( (n-1) \) cells at the \( (n-1) \)th iteration is the optimal LA for the mobile user. Otherwise, go back to (II).
In this way, the optimal size and shape of LAs for a mobile user can be dynamically and individually determined according to its current mobility and incoming call arrival rate as well as its TPM. The combined average cost for a mobile user is minimized.

7. Numerical Results

The performance of the proposed scheme depends on the users’ actual TPMs and the system’s cell layout. In order to evaluate the proposed scheme, we assume the system has two levels: microcell level and macrocell level. At microcell level, the service area is divided into hexagonal cells of the same size. Each cell has six neighbors. At macrocell level, each macrocell overlays seven microcells as illustrated in Fig. 2. A mobile user of a given class can move to one of its six neighboring microcells upon leaving the current microcell with the probabilities \( p_1, p_2, p_3, p_4, p_5, p_6 \). Generally, these six probabilities are not the same and also depend on the current microcell. For numerical purposes, we assume that the cellular system is homogeneous. So the set of six probabilities is the same in all cells (note, however, that generally \( p_1 \neq p_2 \neq p_3 \ldots \neq p_6 \)).

7.1. LA shapes at microcell level

Example 1:

In this example, we assume \( p_1 = 0.1 \), \( p_2 = 0.3 \), \( p_3 = 0.1 \), \( p_4 = 0.1 \), \( p_5 = 0.3 \), \( p_6 = 0.1 \) and given the mobile user’s initial microcell and the number of microcells in the LA. The near optimal LA shape obtained according to the algorithm 2 given in section (5.1) is shown in Fig. 3. The label numbers shown in the figure indicate the size of the LA.

Example 2:

In this example, we assume that the transition probabilities from the current microcell to its neighboring microcells are given by a function \( f(\theta) \) that is independent of the cell, where \( \theta \) is the angle between the user’s moving direction and horizontal line. We assume \( f(\theta) \) has truncated Gaussian distribution in \([0, 2\pi]\), so \( f(\theta) \) has the following form:
\[ f(\theta) = \frac{1}{\Delta_1 \sqrt{2\pi\sigma^2}} e^{-\frac{\theta^2}{2\sigma^2}} \quad \theta \in [0, 2\pi] \]  

(20)

Where \( \Delta_1 \) is a normalized factor, and

\[ \Delta_1 = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^{\infty} e^{-\frac{\theta^2}{2\sigma^2}} d\theta \]  

(21)

Let \( \bar{\theta} = \pi/3 \) and \( \sigma = 2 \), the near optimal LA shape for a given initial microcell and the number of microcells in the LA at microcell level obtained according to the algorithm 2 given in section (5.1) is shown in Fig. 4.

7.2. Comparison between the optimal algorithm and the heuristic algorithm

In order to compare the optimal algorithm (algorithm 1) and heuristic algorithm

(algorithm 2), we assume that a mobile user has the same dwell time in each microcell, \( T_{d} = 0.1 \) hr, for the given system. Since the optimal algorithm is computation intensive, we only compare these two algorithms for the small size LA at microcell level. Fig. 5 and Fig. 6 show the average location update rate as a function of number of microcells in the LA with different TPMs for these two algorithms. In Fig. 5, we assume that the transition probabilities are given by \( p_1 = 0.1 \), \( p_2 = 0.3 \), \( p_3 = 0.1 \), \( p_4 = 0.1 \), \( p_5 = 0.3 \), \( p_6 = 0.1 \). In this case, the two curves are identical to each other for small size of LA. That means these two algorithms get the same solution for this example. In Fig. 6, we assume that the transition probabilities are given by equation (20) with \( \bar{\theta} = \pi/3 \) and \( \sigma = 2 \). In this case, the two curves are identical to each other when size of LA is smaller than eight microcells. When the size of LA are of eight microcells and nine microcells, the optimal algorithm has slight lower average location update rate than the heuristic algorithm. (Because of the resolution of the figure, we can't see the difference in Fig. 6.)

7.3. The performance of proposed scheme

In order to compute the performance of the proposed scheme, we assume that the system has two-level cellular structure as shown in Fig. 2 and \( p_1 = 0.3 \), \( p_2 = 0.1 \), \( p_3 = 0.1 \), \( p_4 = 0.3 \), \( p_5 = 0.1 \), \( p_6 = 0.1 \). For simplicity, the same-level paging and a simultaneous paging algorithm are used, and
let \( C_n = C_M \). \( C_I/C_n = C_I/C_M = 4 \) and \( \beta_M = 7 \). This assumption is reasonable since the location updating cost is more costly than paging cost and each macrocell covers 7 microcells. We assume that the probability density function of incoming call arrival rate \( \lambda \) for all mobile users has a Gaussian distribution with \( \bar{\lambda} \) and variance \( \sigma^2 \). Because the incoming arrival rate for a mobile user can never be negative or infinite, we consider \( \lambda \) as a truncated Gaussian which density is non-zero only in the interval \([0, \lambda_{\text{max}}]\). Normalizing \( \lambda \) such that the area under its probability density function (p.d.f.) equals 1, we obtain the p.d.f. of \( \lambda 
abla \)

\[
f(\lambda) = \frac{1}{\Delta_1 \sqrt{2\pi\sigma^2}} e^{-\frac{(\lambda - \bar{\lambda})^2}{2\sigma^2}} \quad \lambda \in [0, \lambda_{\text{max}}] \tag{20}
\]

in which the normalizing factor, \( \Delta_1 \), is

\[
\Delta_1 = \int_0^{\lambda_{\text{max}}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\lambda - \bar{\lambda})^2}{2\sigma^2}} d\lambda \tag{21}
\]

Since the combined average cost function (18) is the function of \( \lambda, \bar{T}_d, \) and optimal \( r \), while optimal \( r \) is the function of \( \lambda, \bar{T}_d \), the combined cost \( C \) can be written as

\[
C = C(\lambda, \bar{T}_d, r(\lambda, \bar{T}_d)) \tag{22}
\]

The normalized combined cost relative to \( C_n \), the paging cost for a single page in a microcell, is

\[
C_n = \frac{1}{C_n} \int_0^{\lambda_{\text{max}}} f(\lambda) C(\lambda, \bar{T}_d, r(\lambda, \bar{T}_d)) d\lambda \tag{23}
\]

The performance of the proposed schemes of algorithm 2 is plotted in Fig. 7 and Fig. 8 with different \( \bar{T}_d \). Here, the parameters were chosen such that \( \lambda_{\text{max}} = 12 \) arrivals/hr and \( \sigma = 2 \). In each of these two figures, three curves are given for different MCARR ranges. That is when a mobile user’s MCARR is smaller than \( R_t \) showed in the figures, we assign the mobile user to register at microcell level, otherwise, we assign it to macrocell level. The figures show that the higher \( R_t \) is, the lower normalized combined average cost the system has. This is because the more mobile users are assigned to microcell level when \( R_t \) is higher, and microcells have a relative small size and can be more precisely fit into the user’s LA.
8. Conclusions

In this paper, we use an adaptive algorithm to find the optimal location areas for a mobile user at different level in a hierarchical cellular system. In this scheme, the size and shape of LAs for a mobile user as well as its registered level are dynamically determined. Thus the proposed scheme has the following advantages: 1) The combined average cost for location updating and paging is minimized. 2) The use of LAs at different levels allows use of a computational approach which is significantly simplified in comparison with microcell level computation.
9. References


Fig. 1: Hierarchical cellular structure. Microcells, macrocells, and spot beams constitute different levels of the hierarchy.
Fig. 2 Two level cellular structure: each macrocell overlays 7 microcells.
Fig. 3. Near optimal LA shape at microcell level.
Parameters: $p_1 = 0.1$, $p_2 = 0.3$, $p_3 = 0.1$, $p_4 = 0.1$, $p_5 = 0.3$, $p_6 = 0.1$
Fig. 4. Near optimal LA shape at microcell level
Parameters: $\theta = \pi/3$ and $\sigma = 2$. 
Fig. 5. Location update rate for different LA sizes at microcell level
Parameters: \( p_1 = 0.1, p_2 = 0.3, p_3 = 0.1, p_4 = 0.1, p_5 = 0.3, p_6 = 0.1, T_p = 0.1 \) hr.
Fig. 6. Location update rate for different LA sizes at microcell level
Parameters: $\theta = \pi/3$, $\sigma = 2$, and $T_u = 0.1$ hr.
Fig. 7. Normalized combined average cost for location updating and paging.

Parameters: \( p_1 = 0.1 \), \( p_2 = 0.3 \), \( p_3 = 0.1 \), \( p_4 = 0.1 \), \( p_5 = 0.3 \), \( p_6 = 0.1 \).
\( \overline{T}_d = 0.1 \text{ hr.} \), \( \lambda_{\text{arr}} = 12 \text{ arrivals/hr.} \), \( \sigma = 2 \), \( C_i / C_n = 3 \), \( C_i / C_n = 4 \), \( \beta_n = 7 \).
Fig. 8. Normalized combined average cost for location updating and paging.
Parameters: \( p_1 = 0.1 \), \( p_2 = 0.3 \), \( p_3 = 0.1 \), \( p_4 = 0.4 \), \( p_5 = 0.3 \), \( p_6 = 0.1 \).
\( \bar{T}_d = 0.4 \) hr., \( \lambda_{max} = 12 \) arrivals/hr., \( \sigma = 2 \), \( C_L / C_w = C_L / C_w = 4 \), \( \beta_w = 7 \).