Essays on Strategic Outsourcing

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Abstract of the Dissertation

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This dissertation focuses on strategic outsourcing and contains three essays.

Essay I identifies the previously unstudied function of strategic sourcing in deterring entry. It shows that an incumbent may source its key input to a potential entrant with the sole purpose of blocking its future entry. An entry barrier is created in this case because, through sourcing to the entrant, the incumbent can partly commit to a future quantity, therefore imposes a second mover's disadvantage on the entrant in the event that it actually carries out entry. In addition, there is a collusive effect. Both the incumbent and the entrant are better off relative to what would be the case if the latter attempted entry: through the sourcing transaction, they share the surplus generated from a more-concentrated final-product market. Although the market is less competitive when entry is deterred, the social welfare is generally higher. In some circumstances, even the consumer's welfare increases. The reason for the counter-intuitive finding is that, to block future entry, the incumbent often needs to commit to a large quantity of the final product, a quantity which may even be larger than that under duopoly. These findings hold with either the Cournot or Bertrand competition assumption. The major point — that a supplier is less likely to attempt entry — is consistent with the previous empirical evidence. In the near future, I plan to further my research by examining how well the commitment value of sourcing is preserved when the entrant has incomplete information, and by checking if full-collusion exists under a long-run sourcing contract.

Essay II shows that, intermediate goods can be sourced to firms on the "outside" (that is, firms that do not compete in the final product market), even when there are no economies of scale or cost advantages for these firms. What drives the phenomenon is that if "inside" firms were to accept such orders, they would incur the disadvantage of Stackelberg followers in the ensuing competition to sell the final product. Thus they have an incentive to quote high provider prices to ward off future competitors, driving the latter to source outside. Our game involves simultaneous moves

at various junctures: first, at the very start "insider"s and "outsider"s independently quote prices at which they are willing to supply, likewise later in their competition on the final product. Far from having perfect information in our game, we prove that there exists a continuum of pure strategy SPNE (subgame perfect Nash equilibrium), across which the outputs of the firms differ, but the outsourcing pattern is invariant. As long as the "outsider"s' cost disadvantage is not too significant, in any SPNE, the intermediate goods is sourced to the "outsider"s.

Essay III incorporates economies of scale into the work described above in Essay 3. In this scenario, when a final-product producer sources to an "insider" who produces under scale economies, there are two strategic effects intermingling with each other: an Stackelberg leader's advantage, together with a future cost disadvantage due to the economies of scale. The second effect might outweigh any leadership advantage that the final-product producer obtains by going to "insider"s. Foreseeing a competitor that is fierce in spite of being a follower, the final-product producer would prefer to source outside. Moreover, then economies of scale can drive "insider"s to source to "outsider"s as well! As long as the "outsider"s' cost is not significantly higher than the "insider"s', we find that (i) the final-product producer sources to "outsider"s in any SPNE (subgame perfect Nash equilibrium); (ii) when economies of scale are not too small, "insider"s also source to "outsider"s in any SPNE.

To My Husband, Bin Tang
And To My Parents

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Chapter 2 of this dissertation is a reprint of the materials as it appears in "Outsourcing induced by strategic competition", co-authored with Professor Pradeep Dubey and Professor Debapriya Sen.

Chapter 1

Entry Deterrence through Strategic Sourcing

1.1 Introduction

Seeking appropriate suppliers is crucial to any firm requiring production components from the outside. The uprising worldwide trend of outsourcing is a good evidence of this, since it shows that firms optimally design supply chains in order to strengthen their competence in the global productivity race.

The conventional wisdom on outsourcing explains this behavior as being due to firms pursuing a lower cost, their focusing on core competence, or their desire to have access to the latest technology, and so forth (see, for example, Domberger (1998)). It is only in recent years that strategic considerations have been recognized as one set of factors underlying outsourcing. For example, Shy and Stenbacka (2003) show that outsourcing can be driven by economies of scale, which also lead those firms who outsource to congregate on a unique provider. With Bertrand competition, Chen (2001) and Chen et al. (2004) identify the collusive effect of outsourcing between competing firms, in the sense that it yields higher prices of both intermediate and final products.

There are observations in the real business world, however, which can not be so easily explained by employing the conventional wisdom or on the basis of the strategic reasons identified in the former literature. For example, one observation is the outsourcing between the American aircraft manufacturer Boeing and a Japanese consortium composed of the three biggest industrial giants of Japan: Mitsubishi Heavy Industries, Kawasaki Heavy Industries LTD, and Fuji Heavy Industries. These Japanese firms expressed an interest in entering the market for commercial aircrafts. Consequently, agreements were signed between Boeing and the Japanese firms. According to the agreements, Boeing would outsource to them part of its production of the 767-X fuselage during the 1990s (*Chicago Tribune*, April

14, 1990)[1], and then wings, together with related research and development during the 2000s (*Chicago Tribune*, December 21, 2003). The incentives for outsourcing in this example cannot be explained on the basis of cost-saving, since in the airline industry, costs in Japan "are just as high as or higher than at home" (*Newsweek International Edition*, May 15-22, 2006).

Another observation is the outsourcing between Boeing and Lockheed. Although Lockheed exited the commercial aircraft market after 1981, it still possessed the production capability to reenter and compete with Boeing. Boeing signed a contract with Lockheed to outsource certain parts of commercial aircraft production. (*The Wall Street Journal*, May 10, 1989, p. 87)[2]. Subsequently, Lockheed never reentered the commercial aircraft market.

This work provides a theoretical explanation for these observations by studying the strategic role of outsourcing for entry deterrence. We find that, when confronted with a potential entrant who at the same time is also a potential supplier of the intermediate product, the incumbent can utilize the sourcing strategy as a vehicle for successfully deterring what would be an otherwise profitable entry. Moreover, deterring entry can be the incumbent's sole purpose of sourcing to an entrant: the incumbent is in fact often willing to pay higher relative to what would be its cost in resorting to other resources, in order to lure the entrant to become a supplier.

The basic element which erects the entry barrier is the commitment value of the quantity sourced to the entrant. To be explicit, after the incumbent has ordered from the entrant a certain amount of the intermediate product, it successfully turns this part of cost into sunk when making decisions in the final-product market. Therefore, it can now at least partly commit itself to its future strategy. For the entrant, as a provider, it naturally observes the quantity the incumbent has ordered. If the entrant carries out entry, it will have to accommodate to the incumbent's committed strategy. As a consequence, it acquires a second mover's disadvantage; and it may therefore no longer find entry profitable.

On the other hand, before a binding contract exists between the incumbent and the entrant, the entrant might always opt out of supplying the incumbent, thereafter practicing entry in order to reap profit by selling in the downstream market. In order to lure the entrant to become a supplier and thus ultimately achieve entry deterrence, the incumbent must at minimum remedy the entrant's losses by staying out. Unlike entry deterrence achieved through other tactics, for example, via capacity construction and limit pricing — where the entrant receives nothing when it stays out — the entrant gets at least its duopoly profit as an autarkic producer of the final product with entry deterrence achieved through an sourcing strategy. A collusive effect exists here. With a larger profit generated by a more concentrated downstream market, the incumbent and the potential entrant are both happier sharing the profit

[1] Also cited in Spiegel (1993).

[2] same as in footnote 1.

through their transaction on the intermediate product, than with the duopoly profit each would end up obtaining with entry.

This explains why the incumbent is willing to purchase from the entrant even though by doing so, it pays more than if it sourced to other resources. At the same time, the entrant is willing to commit to supplying even though it understands that by taking on the role of a provider, it loses the chance to enter.

Our framework consists of a model with an incumbent and a potential entrant in the market for a final product. The incumbent is choosing among many potential providers, including the entrant, for supply of a key intermediate product. Under fairly general conditions, we find that, in any equilibrium with entry deterrence, the incumbent is ordering a certain amount from the entrant while paying a higher price than what it would pay to other providers. Although our baseline model is in context of Cournot competition, our results are robust with Bertrand competition, and with several other variants of the model.

It is important to note that, although a collusive effect is there with entry deterred, a pure monetary transmission from the incumbent to the entrant does not in and of itself erect a barrier to entry. In order to prohibit entry, a sizable amount of the intermediate product must be ordered from the entrant, one which is usually larger than the monopoly quantity. Moreover, there exists a range of parameters wherein each firm can improve its payoff by having entry deterred, yet where entry occurs in equilibrium.

Given the preservation of a less competitive market, it is natural to suspect that entry-deterring strategic sourcing is detrimental to social welfare. However, under a large range of parameters, we find that the social welfare is actually improved compared to what it would be under entry. Moreover, even the consumer's welfare shows a net increase in some circumstances. The reason has to do with the fact that a large enough quantity must be sourced in order to convince the entrant that entry is unprofitable. Such a large quantity restricts the consumers' loss; moreover, in the event that it exceeds the duopoly quantity, consumers are better off.

Although the real observations we cited above are in context of outsourcing, the basic idea presented in this work applies to a more general situation. Our work identifies a strategic incentive in a firm's choice of suppliers, thus offering insights regarding strategic supply chain configurations.

While a large number of firms are utilizing a sourcing strategy in order to entrench themselves nowadays, one concern is that key suppliers could turn into fierce competitors in the future, thus becoming detrimental to firms who outsource. For example, Caves and Porter (1977) make the argument that: "*important suppliers to an industry ... are often likely entry candidates.*"[3] Intuitively, downstream firms are

[3]Caves and Porter (1977) argue that the suppliers are likely to possess those key elements for a successful entry, including well-established distribution or service networks, and the ability to produce components transformable into other commodities. In addition, other reasons can exist for

inclined to be wary of the entry potential of their major providers. However, empirical findings tell quite a different story. Smiley (1988) summarized an extensive survey across a broad range of industries regarding what source of entry concerns them the most. One finding is that,

"in the opinion of the respondents, the dominant source for potential entrants into existing product lines was existing rivals (who do not have similar products). Surprisingly few firms were concerned about new entrants ...from (among) their suppliers ...". Moreover, *"manufacturing firms ...are less concerned about entry by related firms such as suppliers"*.

While other factors may be at play, our work indicates that, everything else being equal, a provider who forms a real entry threat is less likely to practice entry, relative to potential entrants who are independent of the incumbent.

The role of sourcing in entry deterrence is analogous to the capacity construction carried out by the incumbent (Spence (1977), Dixit (1979, 1980)), in the sense that both grants the incumbent a first mover's advantage. One major difference is that capacity construction represents a single-sided decision made by the incumbent, while strategic sourcing can never occur if the entrant is unwilling to comply. More strategic interaction between the incumbent and the entrant is involved when entry is deterred through strategic sourcing as opposed to capacity construction. This is the focus of our work.

In Chen (2005) and Chen et. al. (2006), we have also analyzed the strategic use of outsourcing. In these papers we find that the buyer's first mover advantage leads to strategic outsourcing decisions: that is, firms will purchase from providers who are not in the final product market ("outsiders") even if these providers have higher costs compared to "insiders" (firms that also produce the final product). The buyer's first mover advantage is also identified in Bakke et. al. (1998), who explain cross-supplies, the phenomenon that two or more firms operating in the same industry supply one another with their final products. Moreover, Spiegel (1993) finds that subcontracting can be used for entry deterrence under the assumption of a strictly convex production cost. Salop (1979) discussed the incumbent's capability in making binding commitments in the pre-entry period. Aghion and Bolton (1987) showed that an incumbent, by signing contracts with consumers, and prevent the entry of some potential entrants. Basu and Singh (1990) depict the properties of entry deterrence in a Stackelberg game, with production cost for the entrant including an entry cost and a start-up cost. Chen and Ross (2000) find the anti-competitive effect of an alliance, where capacity is shared hence a restrictive post-entry quantity is imposed to the entrant.

The rest of the paper is organized as follows. Section 2 describes the benchmark

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key providers to be entrant candidates. For example, it could be relatively easier for a provider to infer information regarding the downstream market demand or the consumers' tastes, or to grasp technology in converting the intermediate goods into the final products.

model with Cournot competition. Section 3 gives our analysis and major finding. Section 4 checks for the robustness of our results using variations of the benchmark model, including Bertrand competition. Section 5 discusses the results and Section 6 concludes.

1.2 The Model

The model consists of a monopoly incumbent, denoted as firm 0, who is producing the final product good F, and a potential entrant, firm 1. The only intermediate product required for producing good F is good I, which firm 0 can not produce. Firm 1 can produce good I at constant marginal cost $c > 0$.[4] Moreover, by investing $K < \infty$, firm 1 can acquire the same technology as firm 0 in converting good I into good F.

Firm 0 can order good I either from firm 1, or from a perfect competitive market, in which good I is also produced at marginal cost c. The only difference between firm 1 and the providers in the competitive market is that firm 1 has the entry potential for good F, whereas other firms do not. The reason can be that only firm 1 has access to some critical technology or resource for producing good F.

The inverse market demand for good F is $P(Q)$, where $P(\cdot)$ is strictly decreasing in the total quantity Q for $Q < \bar{Q}$, with $P''Q + P' < 0$. Assume that one unit of good I can be converted into one unit of good F, and firm 0's constant average cost in converting good I into good F is normalized to zero.

The strategic interaction among firms 0, 1 and the competitive market is modelled as a three-stage game, which is depicted below.

In stage one, firm 0 proposes firm 1 a take-it-or-leave-it offer, $\{p, x^1\}$, specifying that x^1 units of good I will be ordered by firm 0 from firm 1 at price p. Firm 1 either accepts or rejects the offer.[5]

In stage two, with the outcome in stage one observed, firm 1 decides whether to invest K to enter for good F or not.

Firm 0 observes firm 1's entry decision. In stage three, it can order more good I from the competitive market, which firm 1 does not observe since the transaction is confidential between firm 0 and the chosen provider. If firm 1 has entered, firms 0 and 1 simultaneously decide quantities $\{q_0, q_1\}$ to produce for good F, otherwise firm 0 decides q_0 alone.

The price for good I in the competitive market is pinned down at c due to perfect competition. Denote x^2 as the total quantity firm 0 orders from the competitive

[4]The linearity enables us to have a clear view of the central point. It is not critical to our conclusion, as shown in Section 4.3.

[5]The assumption that firm 0 has all the bargaining power is meant to simplify the analysis. Our basic finding will still hold with a different distribution of bargaining powers. That is, firm 0 will source to firm 1 with the sole purpose of deterring its entry.

market. To make our model more general, assume firm 0 has free disposal with $x \equiv x^1 + x^2$, the total quantity of good I it purchases.[6]

Total profit at the terminal notes to firm $i, i = 0, 1$ are $\Pi_i^e(p, x^1, x^2, q_0, q_1)$ if firm 1 enters, and $\Pi_i^{out}(p, x^1, x^2, q_0)$ if firm 1 stays out, as given below:

$$\Pi_0^e(p, x^1, x^2, q_0, q_1) = P(q_0 + q_1)q_0 - px^1 - cx^E$$

$$\Pi_1^e(p, x^1, x^2, q_0, q_1) = P(q_0 + q_1)q_1 - cq_1 + (p - c)x^1$$
$$\Pi_0^{out}(p, x^1, x^2, q_0) = P(q_0)q_0 - px^1 - cx^E$$
$$\Pi_1^{out}(p, x^1, x^2, q_0) = (p - c)x^1$$

The subgame perfect Nash equilibrium (SPNE) for game Γ is characterized in the following section.

1.3 Model Analysis and Major Result

1.3.1 Model Analysis

We begin our analysis from the last stage. Given $\{p, x^1\}$ and given firm 1's entry decision, let firm i's profit, $i = 0, 1$, in stage three be $\pi_i^e(x^2, q_0, q_1)$ if firm 1 enters, and $\pi_i^{out}(x^2, q_0)$ if it stays out.

Case I. Firm 1 enters.

Firm 1's problem after entry is

$$\max_{q_1} \pi_1^e(q_0, q_1) = P(q_0 + q_1)q_1 - cq_1.$$

In equilibrium firm 0 will order $x^2 > 0$ only if $q_0 > x^1$. That is

$$x^2 = \begin{cases} q_0 - x^1 & \text{if } \; q_0 > x^1 \\ 0 & \text{otherwise} \end{cases}$$

Since firm 0's expenditure on x^1 is already sunk, its problem is

$$\max_{q_0} \pi_0^e(q_0, q_1) = \begin{cases} P(q_0 + q_1)q_0 - c(q_0 - x^1) & \text{if } \; q_0 > x^1 \\ P(q_0 + q_1)q_0 & \text{otherwise} \end{cases}$$

Denote the equilibrium outcome as $q_0^e(x^1), q_1^e(x^1)$.

Firm 0 faces marginal cost c if $q_0 > x^1$ and 0 otherwise, while firm 1's marginal cost is c. Post-entry reaction functions for the duopolists are of Dixit (1980) type, illustrated by Figure 1.1. Firm 1's reaction curve is RR'. Two reference curves

[6] Without free disposal, our conclusion can only be strengthened since the disposal cost helps to make firm 0's order of good I from firm 1 a commitment to its future quantity of good F.

are given for firm 0's reaction function: OO' represents its reaction curve with zero marginal cost, and MM' represents its reaction curve with marginal cost c. At a given x^1, firm 0's reaction function is kinked at $q_0 = x^1$, which overlaps OO' for $q_0 < x^1$ and MM' for $q_0 > x^1$, shown by the heavy kinked line in Figure 1.1. Denote the intersection of RR' and MM' as point W, the intersection of RR' and OO' as point V, with coordinates for W and V given by $(W_0, W_1), (V_0, V_1)$ respectively. As clear in the figure, there are three subcases for the equilibrium.

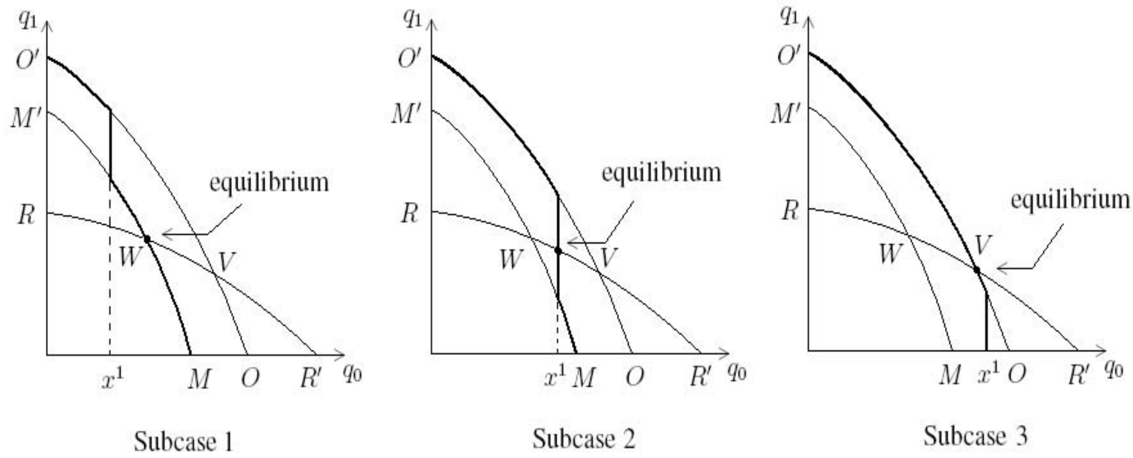

Figure 1.1: Post-entry Reaction Functions

Subcase 1. $x^1 \in [0, W_0]$. Firm 0's reaction function intersects RR' at point W. Define their profits in this subcase as $(\pi_0^c(x^1), \pi_1^c(x^1))$, with

$$\pi_0^c(x^1) \equiv P(W_0 + W_1)W_0 - c(W_0 - x^1), \quad \pi_1^c(x^1) \equiv P(W_0 + W_1)W_1 - cW_1.$$

Particularly, at $x^1 = 0$, standard Cournot-Nash profits are achieved as

$$\pi_0^W \equiv P(W_0 + W_1)W_0 - cW_0, \quad \pi_1^W \equiv P(W_0 + W_1)W_1 - cW_1.$$

Note that $\pi_0^W = \pi_1^W$ since firm 0 and firm 1 are symmetric at $x^1 = 0$. The equilibrium strategies in this subcase are given by Cournot-Nash quantities, $q_0^e(x^1) = W_0, q_1^e(x^1) = W_1$.

Subcase 2. $x^1 \in (W_0, V_0]$. Firm 0's reaction function intersects RR' at $q_0 = x^1$. In stage three, $q_0^e(x^1) = x^1$. Firm 0 will neither source $x^2 > 0$, nor drop any of x^1. Firm 1 knows this and its problem is

$$\max_{q_1} \pi_1^e(x^1, q_1) = P(x^1 + q_1)q_1 - cq_1.$$

Let $q_1^f(x^1)$ be the solution to the first order condition

$$P'(x^1 + q_1)q_1 + P - c = 0.$$

By setting $x^1 \in (W_0, V_0]$, firm 0 can get any point along segment WV as their competition outcome, thus is granted a Stackelberg leader's advantage along WV. Firm 1 acts as a Stackelberg follower by producing $q_1^f(x^1)$ to accommodate the observed x^1. Define firms 0 and 1's profits $\pi_0^l(x^1)$ and $\pi_1^f(x^1)$ as

$$\pi_0^l(x^1) \equiv P(x^1 + q_1^f(x^1))x^1, \quad \pi_1^f(x^1) \equiv P(x^1 + q_1^f(x^1))q_1^f(x^1) - cq_1^f(x^1).$$

At $x^1 = V_0$, their profits are

$$\pi_0^V \equiv P(V_0 + V_1)V_0, \quad \pi_1^V \equiv P(V_0 + V_1)V_1 - cV_1.$$

Subcase 3. $x^1 > V_0$. Firm 0's reaction function intersects RR' at point V. In equilibrium $q_0^e(x^1) = V_0, q_1^e(x^1) = V_1$, and firm 0 has $x^1 - V_0$ of good I left idle. Their profits in stage three are exactly (π_0^V, π_1^V). Since firm 0 is paying $p(x^1 - V_0) > 0$ extra amount to firm 1, this subcase is strictly dominated for firm 0 by $x^1 = V_0$, and should never appear in equilibrium.

To summarize, post-entry equilibrium profits for firms 0 and 1 are

$$(\pi_0^e(x^1), \pi_1^e(x^1)) = \begin{cases} (\pi_0^c(x^1), \pi_1^W) & \text{if} \quad x^1 \leq W_0 \\ (\pi_0^l(x^1), \pi_1^f(x^1)) & \text{if} \quad x^1 \in (W_0, V_0] \\ (\pi_0^V, \pi_1^V) & \text{otherwise} \end{cases}$$

A critical fact to our analysis is that $\pi_1^e(x^1)$ is strictly decreasing for $x^1 \in (W_0, V_0]$, because

$$\frac{d\pi_1^f(x^1)}{dx^1} = \frac{\partial \pi_1^f(x^1)}{\partial x^1} = P' q_1^f(x^1) < 0. \tag{1.1}$$

Case II. Firm 1 stays out.

Firm 0 is a monopolist for good F. It chooses q_0 to maximize profit, with $x^2 = q_0 - x^1$ if x^1 does not meet its demand of good I. It's problem is

$$\max_{q_0} \pi_0^{out}(q_0) = \begin{cases} P(q_0)q_0 - c(q_0 - x^1) & \text{if} \quad q_0 > x^1 \\ P(q_0)q_0 & \text{otherwise} \end{cases}$$

Denote the solution to firm 0's problem as $q_0^{out}(x^1)$. Define

$$M_0 \equiv \arg\max_{q_0} [P(q_0)q_0 - cq_0], \quad O_0 \equiv \arg\max_{q_0} P(q_0)q_0.$$

Thus M_0 is firm 0's monopoly quantity under marginal cost c, and O_0 is its monopoly quantity under marginal cost zero. We have $q_0^{out}(x^1) = M_0$ if $x^1 \leq M_0$, with $x^2 = M_0 - x^1$; $q_0^{out}(x^1) = x^1$ if $x^1 \in (M_0, O_0]$; and $q_0^{out}(x^1) = O_0$ otherwise, with $x^1 - O_0$ amount of good I disposed. In equilibrium, firm 0's post-entry profit is

$$\pi_0^{out}(x^1) = \begin{cases} P(M_0)M_0 - c(M_0 - x^1) & \text{if} \quad x^1 \leq M_0 \\ P(x^1)x^1 & \text{if} \quad x^1 \in (M_0, O_0] \\ P(O_0)O_0 & \text{otherwise} \end{cases}$$

We finish the analysis for stage three. In stage two, firm 1 chooses whether or not to enter the market of good F. W.l.o.g., assume that firm 1 stays out if it is indifferent between entering or not. Before we move back to stage two, we want to rule out the range of K which is uninteresting to our analysis.

If $K \geq \pi_1^W$, firm 1 will never enter since its highest profit upon entry is $\pi_1^W - K \leq 0$. By Bain's terminology, *entry is blockaded*. Firm 0 can act as if there exists no entry threat. Instead, if $K < \pi_1^V$, firm 1 will always enter since its lowest profit upon entry is $\pi_1^V - K > 0$. Firm 0 lacks effective vehicle to deter entry. We call this scenario as *entry can not be deterred*.

From now on, assume $K \in [\pi_1^V, \pi_1^W)$, the range of interests. We are ready to move back to stage two.

If firm 1 enters, its profit is $\pi_1^e(x^1) - K$, where $\pi_1^e(x^1)$ decreases in x^1. If $x^1 \leq W_0$, firm 1 should always enter to get $\pi_1^W - K > 0$; if $x^1 > V_0$, firm 1 should stay out since its post-entry profit is $\pi_1^V - K \leq 0$. For $x^1 \in (W_0, V_0]$, firm 1's post-entry profit is $\pi_1^e(x^1) = \pi_1^f(x^1)$. By the strict monotonicity of $\pi_1^f(x^1)$ (see (1.1)) and the fact that $\pi_1^f(W_0) = \pi_1^W > K$ and $\pi_1^f(V_0) = \pi_1^V \leq K$, there exists a unique intersection of $\pi_1^f(x^1)$ and K. Firm 0's reaction function jumps down at some point on WV and coincides the horizontal axis thereafter, see the heavy curve in Figure 1.2.

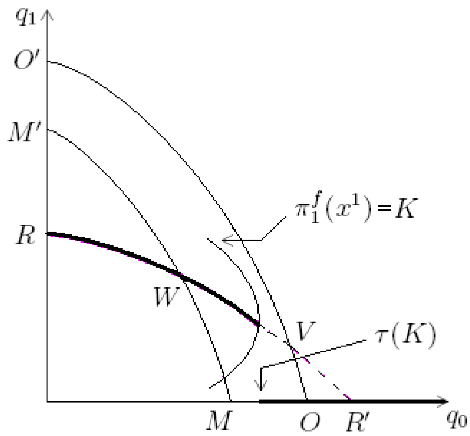

Figure 1.2: Firm 1's Reaction Function with $K \in [\pi_1^V, \pi_1^W)$

Define

$$\tau(K) \equiv \{x^1 | \pi_1^f(x^1) = K\}$$

be the value of x^1 below which firm 1 prefers to enter than to stay out.

Lemma 1 *In any SPNE, firm 1 enters if and only if $x^1 < \tau(K)$.*

Proof: Define $f \equiv \pi_1^f(x^1) - K$. By the implicit function theorem,

$$\frac{d\tau(K)}{dK} = -\frac{df/dK}{df/dx^1} = \frac{1}{P' q_1^f(x^1)} < 0.$$

Therefore, $\pi_1^f(x^1) > K$ if $x^1 < \tau(K)$; $\pi_1^f(x^1) < K$ if $x^1 > \tau(K)$. The lemma follows our analysis above. ∎

With firm 1's entry rule given by Lemma 1, we are ready to move back to stage one. In stage one, firms 0 and 1 decide whether to strike a deal on good I or not, while keeping firm 1's entry rule into consideration. Denote $\Pi_i^e(p, x^1), i = 0, 1$ their profits when firm 1 is entering, and $\Pi_i^{out}(p, x^1), i = 0, 1$ their profits when firm 1 is staying out. We have

$$\begin{aligned} (\Pi_0^e(p, x^1), \Pi_1^e(p, x^1)) &= (\pi_0^e(x^1) - px^1, \pi_1^e(x^1) + (p - c)x^1 - K) \\ (\Pi_0^{out}(p, x^1), \Pi_1^{out}(p, x^1)) &= (\pi_0^{out}(x^1) - px^1, (p - c)x^1) \end{aligned} \tag{1.2}$$

We firstly explore the equilibrium profits if firm 1 enters. For ease of notation, *let $x^1 = 0$ indicate the scenario that no agreement is struck in stage one*.

Lemma 2 *In any SPNE if firm 1 enters, it must be $x^1 = 0$, or $p = c, x^1 \in (0, W_0]$. Profits for firm 0 and for firm 1 are $(\Pi_0^e, \Pi_1^e) = (\pi_0^W, \pi_1^W - K)$.*

Proof: Step one. We show that it is impossible for firms 0 and 1 to strike a deal with $p > c, x^1 > 0$. Suppose in some SPNE, firms 0 and 1 agree on $p > c, x^1 > 0$, then in stage two firm 1 enters. It must be $x^1 \in (W_0, V_0]$. Firstly, if $x^1 \in (0, W_0]$, firm 0's profit is $\Pi_0^e(p, x^1) = P(W_0 + W_1)W_0 - px^1 - c(W_0 - x^1)$. Let firm 0 deviate to $x^1 = 0$. Firm 1 still enters since its post-entry profit is invariant for $x^1 \in (0, W_0]$. Firm 0 gets $P(W_0 + W_1)W_0 - cW_0$, a strict improvement since $p > c$. A contradiction. Secondly, if $x^1 > V_0$, firm 0's profit is $P(V_0 + V_1)V_0 - px^1$, with $(x^1 - V_0)$ amount of good I left idle. It is strictly better off deviating to $x^1 = V_0$, again a contradiction. Hence it must be $x^1 \in (W_0, V_0]$. In this case, firms 0 and 1 get $\pi_0^l(x^1) - px^1, \pi_1^f(x^1) + (p - c)x^1 - K$ respectively. However, each firm can opt out of their transaction on good I. If so, firm 1 enters (by Lemma 1) and get $\pi_1^W - K$, with firm 0's payoff given by π_0^W. To ensure that none of them will deviate from $x^1 > 0$ to $x^1 = 0$, it requires

$$\Pi_0^e(p, x^1) \geq \pi_0^W, \quad \Pi_1^e(p, x^1) \geq \pi_1^W - K.$$

These two conditions together imply

$$[P(x^1 + q_1^f(x^1)) - c](x^1 + q_1^f(x^1)) \geq \pi_0^W + \pi_1^W. \tag{1.3}$$

However, $[P(Q) - c]Q$ is maximized at $Q = M_0$ by the definition of M_0. Moreover, $W_0 + W_1 > M_0$ since firm 0's Cournot reaction curve MM' has slope between $(-1, 0)$. Because $x^1 > W_0$ implies that $x^1 + q_1^f(x^1) > W_0 + W_1$ as $\frac{dq_1^f(x^1)}{dx^1} > -1$, we get $[P(x^1 + q_1^f(x^1)) - c](x^1 + q_1^f(x^1)) < [P(W_0 + W_1) - c](W_0 + W_1) = \pi_0^W + \pi_1^W$, contradicting Condition (1.3).

Step two. We show that it must be $x^1 = 0$, or $p = c, x^1 \in (0, W_0]$. By step one, it must be $p \leq c$ if firm 1 enters following $x^1 > 0$. Suppose $x^1 \in (W_0, V_0]$. Firm 1 gets $\pi_1^f(x^1) + (p-c)x^1 - K$, strictly decreasing in x^1 by (1.1) and the fact $p \leq c$. Therefore, firm 1 is better off rejecting firm 0's offer, to improve its profit to $\pi_1^W - K$. A contradiction. Hence in equilibrium $x^1 \leq W_0$. By (1.2), firm 1 gets $P(W_0 + W_1)W_1 - cW_1 + (p-c)x^1 - K$. If $p < c$, firm 1 will reject $x^1 > 0$. Thus it must be $p = c$ in order to have $x^1 > 0$. At such price both firms 0 and 1 are indifferent with $x^1 \in [0, W_0]$. In any case, each gets their Cournot-Nash profit, π_0^W for firm 0 and $\pi_1^W - K$ for firm 1. ∎

Assume w.l.o.g. that when firm 0 is indifferent between deterring or accommodating entry, it deters entry. In any SPNE if firm 0 engages in entry deterrence, we expect $p > c$ to hold since for firm 1 to accept firm 0's offer, its loss from staying out should at least be recouped through supplying firm 0 with good I. See Lemma 3.

Lemma 3 *In any SPNE, if firm 1 stays out, it must be* $\{p > c, x^1 \geq \tau\}$ *in stage one. Profits for firm 0 and firm 1 are* $(\Pi_0^{out}, \Pi_1^{out}) = ([P(q_0^{out}) - c]q_0^{out} - \pi_1^W + K,\ \pi_1^W - K)$, *with* $q_0^{out} = \max\{M_0, \tau(K)\}$.

Proof: Since firm 1 stays out, $x^1 \geq \tau$ holds by Lemma 1. Suppose $p = c$. Firm 1 by accepting firm 0's offer will stay out and gets zero profit, yet by rejecting the offer and entering it gets $\pi_1^W - K > 0$, a contradiction. Following the same logic, any $p > c$ such that $(p-c)x^1 < \pi_1^W - K$ will be rejected by firm 1. Firm 0 must guarantee firm 1 no less than $\pi_1^W - K$ in order to have firm 1 accept its offer. Firm 0's problem is

$$\max_{p,x^1} \Pi_0^{out}(p, x^1) = \begin{cases} P(M_0)M_0 - cM_0 - (p-c)x^1 & \text{if } \ x^1 \leq M_0 \\ P(x^1)x^1 - px^1 & \text{if } \ x^1 \in (M_0, V_0] \end{cases}$$
$$\text{s.t.} \quad x^1 \geq \tau(K)$$
$$(p-c)x^1 = \pi_1^W - K$$

If $\tau(K) < M_0$, any $x^1 \in [\tau(K), M_0]$ with $p = c + (\pi_1^W - K)/x^1$ solves firm 0's problem. Instead, if $\tau(K) \geq M_0$, it is solved at $x^1 = \tau, p = c + (\pi_1^W - K)/\tau$. Since $\tau(K) \leq V_0 < O_0$, total quantity for good F is $q_0^{out} = \max\{M_0, \tau(K)\}$, leading to equilibrium profits $(\Pi_0^{out}, \Pi_1^{out})$ as given in the lemma. ∎

Firm 0 can successfully deter entry by offering a large enough bribe to firm 1, so that $x^1 \geq \tau(K)$ is accepted in stage one. However, each unit of good I purchased from firm 1 entails $p - c > 0$ amount of extra burden on firm 0. Based on Lemma 2 and Lemma 3, the incentive compatibility condition for firm 0 to deter entry is

$$\Pi_0^{out} \geq \Pi_0^e,$$

rewritten as

$$[P(\max\{M_0, \tau(K)\}) - c]\max\{M_0, \tau(K)\} \geq \pi_0^W + \pi_1^W - K. \tag{1.4}$$

The intuition for this condition is clear. Since selling good F is the ultimate source of profit for both firms, (1.4) means that profit from selling good F with firm 1 producing alone at marginal cost c, conditional on firm 1's willing to stay out, should not be less than the total duopoly profits net of entry cost. If this is true, there exists surplus generated by a more concentrated final product market which can be shared between firms 0 and 1, and leaves each better off than in the case when entry occurs.

If $\tau(K) \leq M_0$, Condition (1.4) is trivially satisfied since $[P(M_0) - c]M_0 > \pi_0^W + \pi_1^W$. If $\tau(K) > M_0$, for the left-hand-side of Condition (1.4), we have

$$\frac{d[P(\tau)\tau - c\tau]}{dK} = (P'\tau + P - c)\frac{d\tau}{dK} > 0$$

because $P'\tau + P - c < 0$. The right-hand-side strictly decreases in K. Thus there exists a unique K, denote as $\bar{K}$, defined by

$$\bar{K} \equiv \{K|[P(\tau(K)) - c]\tau(K) = \pi_0^W + \pi_1^W - K\}.$$

Condition (1.4) holds if and only if $K \geq \bar{K}$.

1.3.2 Major Result

Define

$$K^M \equiv \{K|\tau(K) = M_0\}.$$

Note that $K^M < \pi_1^W$, since $M_0 > W_0$, and $\tau(K)$ is strictly decreasing in K. Also notice that if Condition (1.4) is violated at $K = \pi_1^V$ (at which $\tau(K) = V_0$), we have $\bar{K} \in (\pi_1^V, K^M)$; otherwise $\bar{K} < \pi_1^V$, implying that Condition (1.4) is always true. The following theorem states our major result.

Theorem 1 *The SPNE for $K \in [\pi_1^V, \pi_1^W)$ is as follows:*
(I) If $K \in [K^M, \pi_1^W)$, entry is strategically deterred. In any SPNE, $\{p = c + (\pi_1^W - K)/x^1, x^1 \in [\tau(K), M_0]\}$. Firm 1 stays out, and $x^2 = M_0 - x^1, q_0 = M_0$.
(II) If $K \in [\pi_1^V, K^M)$, there are two subcases:

(IIa) If $K \geq \bar{K}$, entry is strategically deterred. The unique SPNE is given by $(p, x^1) = (c + (\pi_1^W - K)/\tau(K), \tau(K))$. Firm 1 stays out, and $x^2 = 0, q_0 = \tau(K)$.

(IIb) If $K < \bar{K}$, entry is accommodated. In any SPNE, either $x^1 = 0$ or $\{p = c, x^1 \in (0, W_0]\}$. Firm 1 enters, and $x^2 = W_0 - x^1, q_0 = W_0, q_1 = W_1$.

Proof: Proof of (I). Firstly, since $K \geq K^M$, it is true that $\tau(K) \leq M_0$. Given $x^1 \in [\tau(K), M_0]$, firm 1 stays out according to Lemma 1. Since firm 0's monopoly quantity is M_0 at marginal cost c, in stage three it will expand x^1 to M_0 if $x^1 < M_0$, thus $x^2 = M_0 - x^1, q_0 = M_0$ follows. Secondly, no firm has incentive to deviate in stage one. If firm 1 deviates to rejecting firm 0's offer, it will enter with $x^1 = 0$. Its ensuing profit is $\pi_1^W - K$, the same as when it accepts firm 0's offer. Firm 1 will not deviate. On the other side, firm 0 is better off have entry deterred since Condition (1.4) holds. The combination of $\{p, x^1\}$ yields the least burden to firm 0 with entry successfully deterred. Thus firm 0 will not deviate either. The strategies in (I) constitute SPNE. Thirdly, there does not exists any other SPNE. It is easy to see that if there is no entry in stage two, SPNE strategy must be given as in (I), otherwise firm 2 will deviate. Suppose in some SPNE firm 1 enters. In this case, the ensuing profits are given by Lemma 2. However, by Lemma 3 and Condition (1.4), total profit is higher with entry deterred. With firm 1's profit pinned down at $\pi_1^W - K$, firm 0 strictly prefers to deter entry. Therefore, entry must be deterred in stage two.

Proofs to (IIa) and (IIb) are similar, hence omitted here. ∎

Figure 1.3 illustrates the SPNE strategy when K varies along the axis. To have the whole picture, here we investigate the cases $K \geq \pi_1^W$ and $K < \pi_1^V$. For the former case, no real entry threat exists. Firm 0 will offer $p = c$ if $x^1 > 0$. The equilibrium quantity produced by firm 0 is M_0, with $x^1 \in [0, M_0]$. For the latter case, entry is unavoidable. Firm 1 will accept $x^1 > W_0$ only if p is large enough to recoup its loss by acting as a follower in the market of good F. However, due to the same argument as for Lemma 2, such a high price always drives firm 0 to turn to the competitive market. Thus $x^1 \leq W_0$, and $p = c$ must hold if $x^1 > 0$. The downstream competition yields Cournot quantities, leading to profits π_0^W and $\pi_1^W - K$.

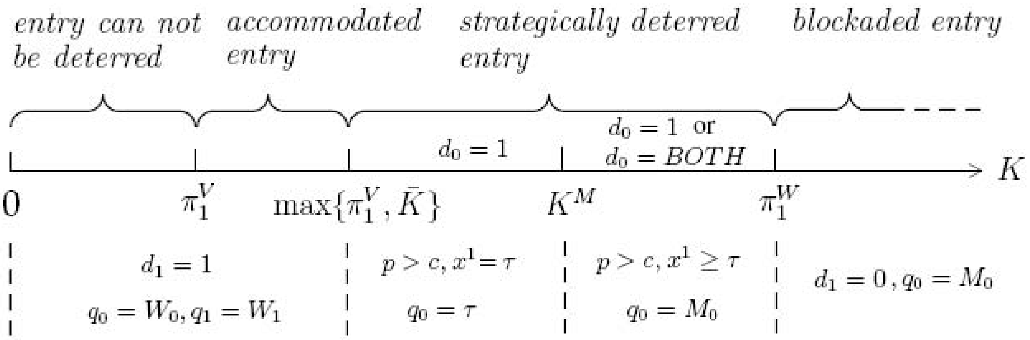

Figure 1.3: SPNE

There can be a continuum of SPNE for our model in some range of K, yet SPNE entry strategy is always unique. Moreover, when entry is strategically deterred, SPNE sourcing pattern always admits $x^1 > 0$, although with $K \in [K^M, \pi_1^W)$, it can be that firm 0 sources partly to firm 1 and partly to the competitive market.

Consider a linear example. Suppose $P = \max\{0, a - Q\}$, with $a > c > 0, Q = q_0 + q_1$. The following values are easily calculated:

$$M_0 = \frac{a-c}{2}; \quad W_0 = W_1 = \frac{a-c}{3}, \pi_0^W = \pi_1^W = \frac{(a-c)^2}{9};$$

$$V_0 = \begin{cases} \frac{a+c}{3} & \text{if } c < \frac{a}{2} \\ \frac{a}{2} & \text{o.w.} \end{cases} \qquad V_1 = \begin{cases} \frac{a-2c}{3} & \text{if } c < \frac{a}{2} \\ 0 & \text{o.w.} \end{cases}$$

$$\pi_0^V = \begin{cases} \frac{(a+c)(a-2c)}{9} & \text{if } c < \frac{a}{2} \\ \frac{a(a-2c)}{4} & \text{o.w.} \end{cases} \qquad \pi_1^V = \begin{cases} \frac{(a-2c)^2}{9} & \text{if } c < \frac{a}{2} \\ 0 & \text{o.w.} \end{cases}$$

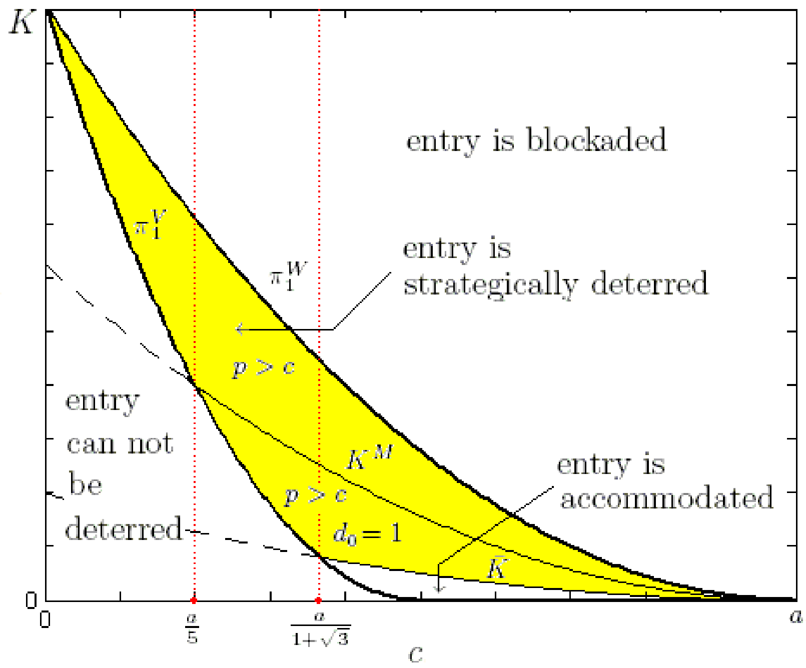

Figure 1.4: A Linear Example

Let $K \in [\pi_1^V, \pi_1^W)$, where entry may be strategically deterred. Given p and $x^1 \in (W_0, V_0]$, firm 1 will stay out if and only if

$$(a - x^1 - q_1^f(x^1))q_1^f(x^1) - cq_1^f(x^1) \leq K.$$

The value of τ is solved at equality as $\tau = a - c - 2\sqrt{K}$. Solving $\tau = M_0$ gives $K^M = \frac{(a-c)^2}{16}$. The requirement for entry deterrence is given by Condition (1.4):

$$\frac{1}{9}(a^2 - 2ac - 2c^2) \geq \pi_0^W + \pi_1^W - K.$$

If $c \leq \frac{a}{1+\sqrt{3}}$, it always holds for any $K \geq 0$, implying that entry is strategically deterred. Instead, if $c > \frac{a}{1+\sqrt{3}}$, $\bar{K}$ is solved at its equality as $\bar{K} = \frac{2}{27}(2-\sqrt{3})(a-c)^2$. Entry is strategically deterred for $K > \bar{K}$, otherwise is accommodated.

Figure 1.4 illustrates the equilibrium outcome with the example. When entry is deterred, if $K \leq K^M$, firm 0 sources solely to firm 1 with $x^1 = \tau(K) \geq M_0$, then produces $q_0 = \tau(K)$. If $K > K^M$, firm 0 either sources to both firm 1 and the competitive market or exclusively to firm 1, with $q_0 = M_0$. If $K < \pi_1^V$ and lies between π_1^V and $\bar{K}$, entry is profitable deterred for each firm, yet firm 0's sourcing to firm 1 ceases its commitment value, yielding entry in equilibrium.

1.3.3 Social Welfare Analysis

This section investigates the impact on social welfare of strategic sourcing which deters firm 1's entry, where social welfare is measured as the sum of consumer's surplus and firms' profits. The range of K that matters to our analysis is $K \in [\max\{\pi_1^V, \bar{K}\}, \pi_1^W)$, so that entry is strategically deterred in equilibrium and firm 0 is sourcing to firm 1. We shall focus on K in this range.

If firm 1 is not providing good I at all, the unique equilibrium is that firm 1 enters and Cournot quantity $W_0 + W_1$ is produced for good F. In this benchmark case, social welfare is

$$W^C = \int_0^{W_0+W_1} [P(q) - c]dq - K.$$

When there is strategic sourcing leading to entry deterrence, social welfare is

$$W^\tau = \int_0^{\max\{M_0,\tau(K)\}} [P(q) - c]dq.$$

Thus

$$\begin{aligned} W^\tau - W^C &= K - \int_{\max\{M_0,\tau(K)\}}^{W_0+W_1} [P(q) - c]dq \\ &\equiv K - A(K) \end{aligned}$$

measures the distortion of strategic sourcing on social welfare. Here $A(K)$ gives the society's loss due to strategic sourcing, net of the entry cost K. $A(K)$ is strictly increasing in K for $K < K^M$ and becomes a constant thereafter. The relationship of $A(K)$ and K is shown in Figure 1.5 using a linear demand function.

There are two effects of strategic sourcing on social welfare. The first one is its impact on firms' profits through the saving of K and the distortion on production of good F. Since strategic sourcing exists only when both firms are left no worse off, clearly each firm must get no less than what it would get without strategic sourcing. We have that the first effect is social-welfare improving. The second one is the change of consumers' welfare, which can be negative or positive, solely depending on the quantity produced of good F. If $\tau(K)$ is so large that $\tau(K) > W_0 + W_1$, consumers' welfare can also improve. The total effect is generally ambiguous, yet under a large range of parameters it is positive, see area A, B and C in Figure 1.5, where strategic sourcing which deters entry strictly improves social welfare.

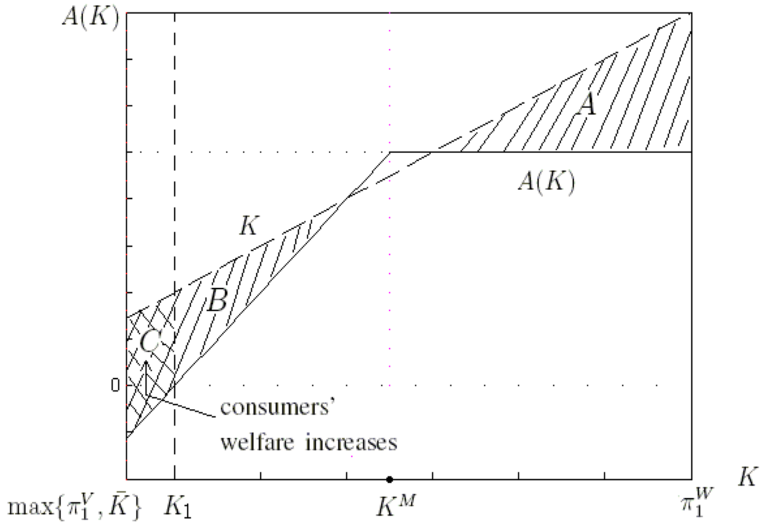

Figure 1.5: Social Welfare Effect of Strategic Outsourcing

In area A, K is large so that the industry avoids an inefficient entry, whereas quantity produced for good F is $M_0 < W_0 + W_1$, thus consumers are strictly worse off. The first effect dominates the second effect hence social welfare is higher. Instead, in area B, the value of K is so small that to deter entry, firm 0 has to source $\tau(K)$ close to the duopoly quantity. The upshot is, the second effect is negative but small, so it is dominated by the first effect. Denote $K_1 \equiv \{K|\tau(K) = W_0 + W_1\}$. $K < K_1$ holds in area C, implying that $\tau(K) > W_0 + W_1$. In this area, not only both firms are better off, but also consumers' welfare is improved, since a quantity larger than the duopoly quantity is sold for good F .

1.4 Model Variations

We use the following model variations to check the robustness of our basic finding.

1.4.1 When Post Entry Competition Is à la Bertrand

Keep everything the same as in the baseline model, except that if firm 1 enters, firms 0 and 1 produce differentiated good F and compete by setting prices r_0, r_1. Demand functions for firms 0 and 1, given by $q_0(r_0, r_1)$, $q_1(r_0, r_1)$ are well-behaved, yielding a unique interior solution for the price competition.

Firms 0 and 1's reaction functions in the post-entry game are of Dixit (1980) type, illustrated by Figure 1.6. RR' is firm 1's reaction curve after its entry. There are two reference lines for firm 0: OO' is its reaction curve with marginal cost zero, and MM' is its reaction curve with marginal cost c. If firm 0 sources $x^1 > 0$ in stage two, its reaction function is kinked and connecting OO' to MM', with the part between given by $q_0(r_0, r_1) = x^1$. When x^1 increases, $q_0(r_0, r_1) = x^1$ shifts up, driving the intersection of firms 0 and 1's reaction functions to shift along WV to the left. By manipulating x^1, firm 0 can have any point along segment WV as the post-entry equilibrium, thus is again granted a limited leadership along segment WV.

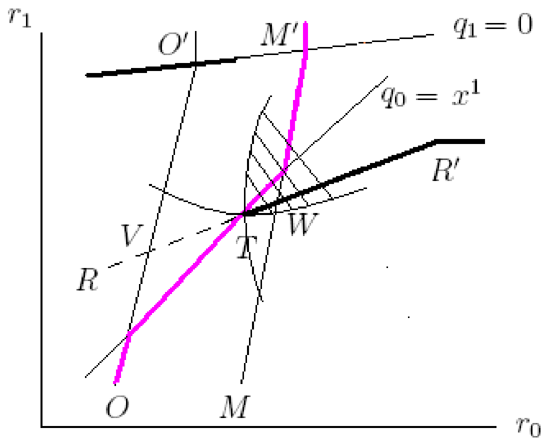

Figure 1.6: Post-entry Reaction Functions under Bertrand Competition

With a positive entry cost K, firm 1's reaction function is kinked and coincides with $q_1(r_0, r_1) = 0$ for r_0 low enough. In the figure, $R'TO'$ shows its reaction curve when firm 1 optimally stays out if $r_0 \leq T_0$. As long as point T is on segment WV, firm 0 by sourcing x^1 big enough, can convince firm 1 that its future price satisfies $r_0 \leq T_0$, therefore having entry deterred.

Denote firm $i, i = 0, 1$'s profit at point W, V as ξ_i^W, ξ_i^V respectively. Similar as in Cournot competition, entry is blockaded for $K \geq \xi_1^W$ and can not be deterred for $K < \xi_1^V$. Assume $K \in [\xi_1^V, \xi_1^W)$, where entry can be deterred by a large enough x^1.

In any SPNE, if firm 1 stays out, it must be $x^1 > 0$ under $p > c$ since firm 0 must compensate firm 1's loss by staying out. Instead, if firm 1 enters, no sourcing can occur between firms 0 and 1 with $p > c$. To see this, notice that the necessary condition for $\{p > c, x^1 > q_0(W_0, W_1)\}$ is, total profit generated under such transaction is larger than $\xi_0^W + \xi_1^W$, their autarky duopoly profits. However, $x^1 > q_0(W_0, W_1)$ implies that their post-entry reaction functions intersect at a point left to W. In this case, point W improves both firms' profits. As an example, Figure 1.6 gives the iso-profit curves at point T, with the hatched area being a Pareto improvement for the industry from T. Thus the necessary condition is violated, and we must have $x^1 \le q_0(W_0, W_1)$, leading to Bertrand-Nash point W as the equilibrium outcome. Moreover, if $x^1 > 0$, $p = c$ must hold. Firms 0 and 1's profits are ξ_0^W and $\xi_1^W - K$ whether firm 0 sources to firm 1 or not.

Denote the threshold of x^1 which makes firm 1 indifferent between entering or not as τ^B. The necessary and sufficient condition for entry to be deterred is

$$[\min\{q_0^{-1}(\tau^B, 0), M_0'\} - c] \max\{\tau^B, q_0(M_0', \infty)\} \ge \xi_0^W + \xi_1^W - K,$$

which is trivially satisfied if $q_0^{-1}(\tau^B, 0) \ge M_0'$. In this case, entry is strategically deterred with $K \in [\xi_1^V, \xi_1^W)$. When $q_0^{-1}(\tau^B, 0) < M_0'$, there exists a threshold of K solved at equality of the expression above, denoted as $\bar{K}^B$, such that entry is strategically deterred as long as $K \in [\max\{\xi_1^V, \bar{K}^B\}, \xi_1^W)$.

Since Bertrand competition is harsher than Cournot competition and leads to lower duopoly profit, firm 0 has less incentive to accommodate entry, yielding $\bar{K}^B < \bar{K}$. Moreover, $\xi_1^V < \pi_1^W$. We expect less entry with Bertrand competition.

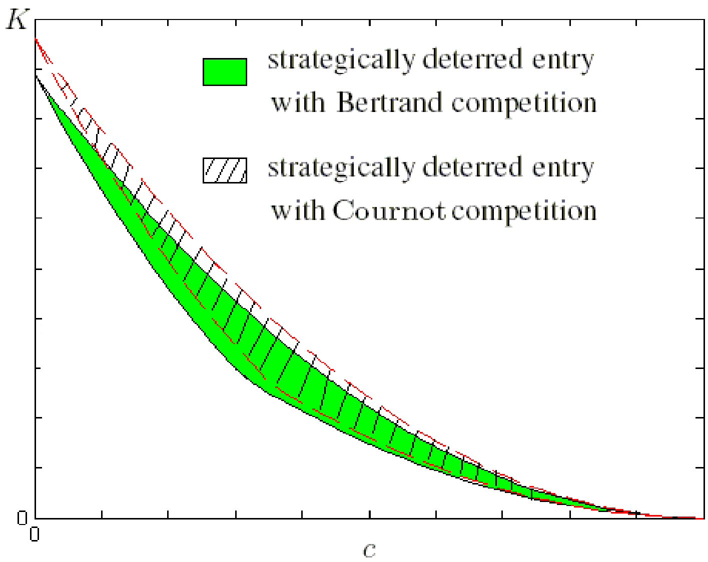

Figure 1.7: Cournot vs. Bertrand

A numerical example illustrates this idea. Suppose market demand for good F is

$$q_i = \begin{cases} 10 - r_i + \frac{1}{2} r_j & \text{if } \; r_j < 10 + \frac{r_i}{2} \\ 15 - \frac{3}{4} r_i & \text{o.w.} \end{cases} \qquad i, j = 0, 1, i \ne j$$

which can be converted into inverse demand function

$$p_i = 20 - \frac{4}{3} q_i - \frac{2}{3} q_j, \quad i, j = 0, 1, i \ne j.$$

Using this example, Figure 1.7 shows that $\max\{\xi_1^V, \bar{K}^B\}$ lies below $\max\{\pi_1^V, \bar{K}\}$ for each value of c.

1.4.2 When Good I Is Ordered in Stage One

In real world there are cases where firm 0 has to order good I ahead of the final product competition with a certain period, for reasons including that transportation of good I or production preparation of good F requires time. One may wonder if our basic finding continues to hold when firm 0 must order all of good I in stage one.

This section investigates a different timing of the basic model. Suppose in stage one, firm 0 makes a take-it-or-leave-it offer $\{p, x^1\}$ to firm 1, then firm 1 decides either to accept or reject. After observing firm 1's decision, firm 0 orders quantity x^2 from the competitive market, which is unobservable to firm 1. In stage two, firm 1 decides to enter or stay out. If it enters, in stage three, firms 0 and 1 determines quantities q_0, q_1 for good F, otherwise firm 0 decide q_0 alone. Assume firm 1 can not observe the value of x^2 either prior to or after its entry.

The solution concept to the modified game is sequential equilibrium. As in the benchmark model, entry is blockaded if $K \geq \pi_1^W$ and can not be deterred if $K < \pi_1^V$. We focus on $K \in [\pi_1^V, \pi_1^W)$.

If Firm 1 Is Not Providing good I

Let us first consider a simplified case when firm 1 does not provide good I at all. Firm 1 knows that firm 0 orders from the competitive market at price c.

Lemma 4 *For $K \in [\pi_1^V, K^M)$, a unique sequential equilibrium exists: $x^2 = q_0 = W_0$; firm 1 believes $x^2 = W_0$, then enters to produce $q_1 = W_1$. Equilibrium profits are π_0^W for firm 0 and $\pi_1^W - K$ for firm 1.*

Proof: Firstly, we show the strategies and belief above constitute an equilibrium. Given firm 1's belief, its optimal strategy is to enter and produce $q_1 = W_1$; given firm 1's strategy, firm 0 is optimal producing $q_0 = W_0$, thus it should source $x^2 = W_0$.

Secondly, we show that there does not exist any other equilibrium. For $K \in [\pi_1^V, K^M)$, we have $\tau(K) \in (M_0, V_0]$. Suppose that firm 1 believes that firm 0 has sourced $x^2 \geq \tau(K)$ hence stays out. However, given firm 1's strategy, firm 0 should source only $x^2 = M_0$, to reap its monopoly profit. Thus firm 1 should not believe that firm 0 is deterring entry, and it should enter. Given that firm 1 enters, the only intersection of their reaction functions in the final-product market is point W, at which each has no incentive to deviate. Hence the strategies together with firm 1's belief specified by the lemma is the unique equilibrium. ∎

Lemma 5 *For $K \in [K^M, \pi_1^W)$, when there is no sourcing between firms 0 and 1, two pure strategy equilibria exist:*
i. $x^2 = q_0 = W_0$; firm 1 believes $x^2 = W_0$, then enters to produce $q_1 = W_1$. Equilibrium profits are π_0^W for firm 0 and $\pi_1^W - K$ for firm 1;
ii. $x^2 = q_0 = M_0$; firm 1 believes $x^2 = M_0$, then stays out. Equilibrium profits are π_0^M for firm 0 and zero for firm 1.
There is also one mixed strategy equilibrium:
iii. $x^2 = W_0$ with probability θ^* and $x^2 = M_0$ with probability $1 - \theta^*$; firm 1 believes in $(\theta^*, 1 - \theta^*)$, and enters to produce $q_1 = W_1$ with probability γ^*, stays out with probability $1 - \gamma^*$, with $\theta^*, \gamma^* \in (0, 1)$. Firm 1's expected profit is zero.

Proof: It is easy to see that i, ii constitute two pure strategy equilibria and there does not exist other pure strategy equilibrium. Firm 0 chooses between $x^2 = W_0$ and $x^2 = M_0$; firm 1 chooses between entering or staying out. If firm 0 is accommodating entry yet firm 1 stays out, the outcome is that firm 0 is a monopolist which produces $q_0 = W_0$. Instead, if firm 0 is deterring entry yet firm 1 enters, firm 0 will produce $q_0^x \equiv \min\{M_0, V_0\}$, since its reaction function in stage three is OO'. Total payoff for each firm is given below:

0 / 1	(γ) Enter	$(1-\gamma)$ Stays out
(θ) W_0	$\pi_0^W, \pi_1^W - K$	$[P(W_0) - c]W_0, 0$
$(1-\theta)$ M_0	$[P(q_0^x + W_1) - c]q_0^x, [P(q_0^x + W_1) - c]W_1 - K$	$[P(M_0) - c]M_0, 0$

For firm 0, given that firm 1 enters, $\pi_0^W = [P(W_0+W_1)-c]W_0 > [P(q_0^x+W_1)-c]q_0^x$ since $M_0 > W_0, V_0 > W_0$ holds; given that firm 1 stays out, $[P(M_0) - c]M_0 > [P(W_0) - c]W_0$. For firm 1, given that $q_0 = W_0$, $\pi_1^W - K > 0$; given that $q_0 = M_0$, $[P(q_1^x + W_1) - c]W_1 - K < 0$. The reason for the second inequality is, if $q_0^x = M_0$, then $[P(M_0+W_1)-c]W_1 < [P(M_0+M_1)-c]M_1 \leq K$ since $K \geq K^M$; if $q_0^x = V_0$, then $[P(V_0 + W_1) - c]W_1 < [P(V_0 + V_1) - c]V_1 < K$ since $K > \pi_1^V$. There must exist a mixed strategy equilibrium, with firm 1's expected profit being zero. ∎

If Firm 1 Is Providing good I

Now we back to game Γ_2. Through our analysis above, if firm 1 is not providing good I for firm 0, its equilibrium profit is either $\pi_1^W - K$ or zero. The restriction to firm 0, that is, it must order all good I in stage one, in fact gives firm 0 extra power to commit to its future strategy. We have a lemma below.

Lemma 6 *For $K \in [\pi_1^V, \pi_1^W)$, in any equilibrium, if $K < K^M$, entry is strategically deterred through $x^1 = \tau, p > c$ as long as Condition (1.4) holds, and is accommodated otherwise. If $K \geq K^M$, there exists two pure strategy equilibrium. In one of* *them, entry is strategically deterred through $x^1 \in [\tau, M_0], p > c$ and $q_0 = M_0$; in the other one, $x^1 = 0, x^2 = M_0$, firm 1 stays out, and $q_0 = M_0$.*

Proof: Proof is straightforward and is omitted. ∎

Proposition 1 *The equilibrium entry decision for the benchmark game is robust for the modified game. I.e. firm 1 enters only when $K < \max\{\pi_1^V, \bar{K}\}$.*

Proof: It is straightforward since Condition (1.4) is satisfied when $K \geq \bar{K}$. ∎

1.4.3 When There Are Economies of Scale for Producing I

Two reasons make strategic sourcing when economies of scale prevail for producing good I be interesting. The first reason is practical. Our model applies to the worldwide outsourcing, and one important incentive for outsourcing is to pursue economies of scale. The second reason is, the existence of economies of scale incurs complicated strategic consideration, which may make our former prediction ambiguous.

Under economies of scale, firm 1 has incentive to attract firm 0's order if it is going to enter in spite of the follower's disadvantage, because the units it produces for firm 0 helps to decrease its future production cost. On the other side, firm 0 will be cautious about sourcing to firm 1, since by doing so, firm 1 may be convinced to enter as an entrenched competitor of good F. When such considerations dominate, firm 0 may no longer source to firm 1. In this case, it must be that firm 0 orders solely from one firm in the competitive market to utilize economies of scale.

However, we find that the basic argument for the benchmark game applies here with economies of scale, leaving the qualitative part of our major conclusion intact. Under quite general assumptions, the incentive for firm 0 to source to firm 1 in order to deter entry is well preserved, and it may dominate other strategic considerations and lead to a sourcing contract between firms 0 and 1.

Suppose firms $1, ..., n$'s production cost $C(q)$ for good I satisfies $C'(q) > 0, C''(q) < 0$ for any unit of $q \geq 0$. To guarantee the existence and uniqueness of a pure-strategy Nash equilibrium between firms 0 and 1 after firm 1's entry, assume

$$P''(Q)q_1 + P' - C'' < 0, \tag{1.5}$$

where $Q = q_0 + q_1$. Condition (1.5) requires that the cost concavity for good I can not be too large. As an example, consider the case with linear demand $P(Q) = \max\{0, a - Q\}$ and quadratic cost $C(q) = cq - vq^2$. Then Condition (1.5) implies that $v < \frac{1}{2}$. Moreover, let the price of good I required by firms $2, ..., n$ as $p_2, ..., p_n$ in stage three. All other issues are the same as in the baseline model.

The validity of strategic sourcing aimed at entry-deterrence is still present. To see this, note that if x^1 is large but not too large, firm 1 knows that after its entry firm 0's quantity of good F is given by x^1. The reason is, on the one side, ordering a little bit more from any other provider entails a high price for firm 0 hence is not profitable; on the other side firm 0 has no incentive to leave any of x^1 unused since its cost is sunk. Thus firm 1's optimal choice is to accommodate the value of x^1 by producing the follower's quantity upon its entry. Its optimal profit after entry is

$$\pi_1^f(x^1) = P(x^1 + q_1^f(x^1))q_1^f(x_1) - C(x^1 + q_1^f(x^1)).$$

By envelope theorem,

$$\frac{d\pi_1^f(x^1)}{dx^1} = P'q_1^f(x_1) - C' < 0.$$

Again it is possible for firm 0 to drive down firm 1's post-entry profit to zero through sourcing to it a large enough quantity.

Moreover, having firm 1 stay out by constructing a buyer-seller relationship can be profitable for both firms 0 and 1. The argument is as follows. When firm 0 is a monopolist, the profit it can reap from the market of good F is bigger than the total profit of duopolists, even if firm 1 supplies firm 0 with a decreasing average cost. In this case, firms 0 and 1 can find an appropriate price at which the payment from firm 0 to firm 1 is enough to remedy firm 1's loss by staying out, and at the same time leave firm 0 no worse off than in a duopoly market.

To have a closed-form solution under economies of scale, assume that market demand for good F and the marginal cost for good I are both linear. More precisely, assume $P(Q) = \max\{0, a - Q\}$, marginal cost of good I is decreasing linearly in q, with production cost be

$$C(q) = \begin{cases} cq - vq^2 & \text{for } q \leq \dfrac{c}{2v} \\ \dfrac{c^2}{4v} & \text{for } q > \dfrac{c}{2v} \end{cases}$$

Assume the parameters satisfy

$$0 < c < a \leq \frac{c}{2v}. \tag{1.6}$$

The last inequality of Condition (1.6) guarantees that in equilibrium, any quantity for good F produced by firm 1 entails positive marginal cost. Notice that $v < \frac{1}{2}$ by (1.6). Timing is the same as for the benchmark model.

Define

$$\pi_1^W \equiv \frac{(1-v)^3(a-c)^2}{(3-6v+2v^2)^2}, \;\; \pi_1^V \equiv (1-v)[\frac{(1+2v)a-2c}{3-2v}]^2;$$

$$\tau(K) \equiv \frac{a-c-2\sqrt{K(1-v)}}{1-2v}, \quad M_0 \equiv \frac{a-c}{2(1-v)}, \quad \tilde{x}^1 = \frac{(1-2v)(a-c)}{3-2v}.$$

Also define

$$\bar{K} \equiv \frac{(a-c)^2(1+2v)^2}{4(1-v)(3-2v)^2}, \quad \hat{K} \equiv \frac{(a-c)^2}{16(1-v)^3}.$$

It is true that $\pi_1^V < \pi_1^W, \bar{K} < \hat{K}$. Our finding is shown by the theorem below, also illustrated by Figure 1.8.

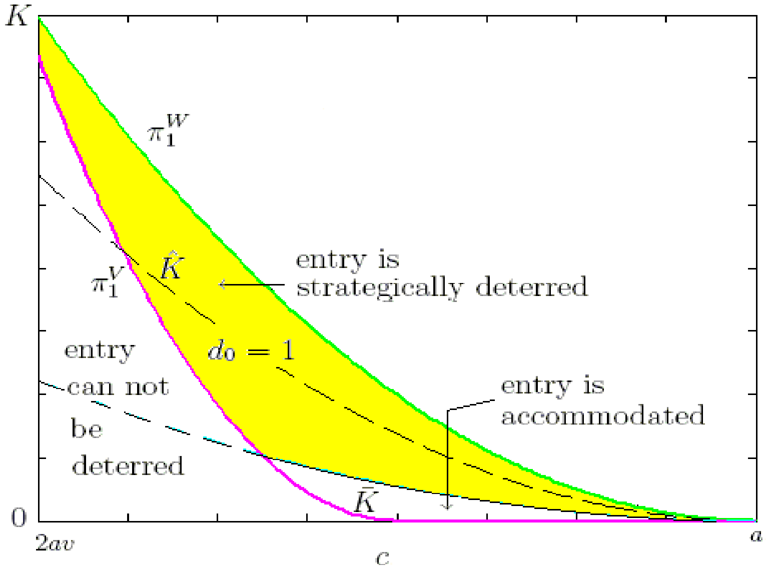

Figure 1.8: Major result of $\Gamma(a, c, v)$ with $a = 10, v = 0.1$.

Theorem 2 *(Figure 1.8) SPNE for $K \in [\pi_1^V, \pi_1^W)$ is given below.*
(I) If $K \geq \bar{K}$, entry is strategically deterred. There are two cases:

(Ia) If $K < \hat{K}$, the unique SPNE is depicted by $(p, x^1) = ((\pi_1^W - K + C(\tau(K))/\tau(K), \tau(K))$, $d_1 = 0, x^2 = 0, q_0 = \tau(K)$;

(Ib) If $K \geq \hat{K}$, the unique SPNE is depicted by $(p, x^1) = ((\pi_1^W - K + C(M_0)/M_0, M_0)$, $d_1 = 0, x^2 = 0, q_0 = M_0$.

(II) If $K < \bar{K}$, entry is accommodated. The unique SPNE is $(p, x^1) = ((\pi_1^W - K + C(\tilde{x}^1)/\tilde{x}^1, \tilde{x}^1), d_1 = 1, x^2 = 0, q_0 = \tilde{x}^1, q_1 = q_1^f(\tilde{x}^1)$.

Proof: Proof is omitted and is available upon request. ∎

There is only one important change in our finding compared to the baseline model. When entry is strategically deterred, the outsourcing pattern is unique: firm 0 always outsources exclusively to firm 1. This phenomenon of course is driven by firm 0's incentive to pursue scale economies.

1.4.4 When Firm 1 Can Deter Entry through Capacity Construction

As pointed out by Dixit (1980), capacity construction can deter entry in virtue of its commitment value. In our model, if firm 0 can also profitably deter entry by constructing its own production capacity of good I, one may wonder which strategy, sourcing or capacity building, will be employed in order to have entry blocked.

In the real world, building capacity then producing good I may incur relatively high cost for firm 0, compared to the scenario when it sources to providers located in low-cost areas, as is usually the case with outsourcing. If this is true, sourcing to firm 1 becomes more profitable for firm 0 as a means of deterring entry.

More saliently, the power of commitment achieved through capacity construction is sensitive to the assumption of the capacity's perfect observability and zero observation cost. Bagwell (1995) points out that, the value of commitment may vanish if the incumbent's capacity is observed with some noise, regardless of how small that noise is. Várdy (2004) found that if observation incurs some cost to the entrant, then the incumbent's commitment loses entirely its value, irrespective of the size of the cost. These findings may explain to some extent the lack of empirical evidence for excess capacity aimed at entry deterrence (Hilke (1984), Lieberman(1987)). Instead, in case of sourcing, it is the quantity ordered directly from the entrant which has the commitment value. There is no space for imprecision on quantity observation or extra observation cost to arise, even ignoring the fact that entry deterrence is in the incumbent's interests. Thus sourcing strategy can offer a precise and effective commitment to the incumbent's future quantity.

Even if firm 0 can build up capacity then produce good I equally efficient as firm 1, and its capacity is perfectly observable to firm 1 at zero observation cost, we find that, firm 0 may still deter entry by ordering certain amount from firm 1 at $p > c$, instead of building capacity then producing the same amount at cost c.

Suppose firm 0's unit cost for building up capacity is $(1-\alpha)c$, then within such capacity it can produce good I at average cost $\alpha c, \alpha \in [0,1]$. Thus its total average cost for sourcing good I inside is c. We extend the baseline model in a way that, in stage one, firm 0 chooses either to build up its own capacity, or offer firm 1 a sourcing contract $\{p, x^1\}$. If it chooses to build up capacity, its capacity is observed by firm 1.

In the case when firm 0 chooses capacity building then producing within its established capacity, its marginal cost in the downstream competition with firm 1 is αc, instead of c. Thus firm 0's post-entry reaction function is shifted from RR' to CC', which enables firm 0 to commit to a quantity as large as B_0. Yet CC' lies to the left of OO' unless $\alpha = 1$, See Figure 1.9. Therefore, unless capacity building can fully sunk firm 0's production cost of good I in stage three (the case when $\alpha = 1$), firm 0's commitment power is weaker compared to what it is if it sources to firm 1.

For K small so that a quantity larger than B_0 should be committed in order to deter entry, the only effective vehicle firm 0 can employ is sourcing to firm 1. On the other side, for K large so that committing to a quantity no larger than B_0 is enough to foreclose entry, firm 0 will always build up its own capacity since sourcing to firm 1 incurs average cost $p > c$.

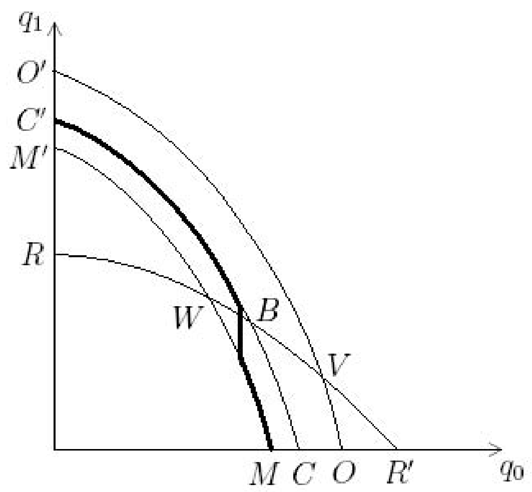

Figure 1.9: CC' is firm 0's post-entry reaction curve in case of capacity building.

Figure 1.10 shows firm 0's equilibrium strategy in terms of capacity building or sourcing to firm 1. For a given value of K, if α is small, that is, most of the cost of sourcing good I inside goes to capacity building, firm 0 will choose to build up its own capacity to deter entry. Instead, if α is large so that commitment power is weak for firm 0 by building up capacity, it will source to firm 1 aimed at entry deterrence, even though producing inside is cheaper.

1.4.5 Multiple Entrants

Consider the situation where there exists $N > 1$ potential entrants, denoted as firms $1, ..., n$. Each faces entry cost $K \geq 0$. Without strategic entry deterrence by firm 0, all of them enters with a positive profit $\pi_i^{(N+1)} - K > 0, i = 1, ..., n$. The original game is modified as below.

In stage one, firm 0 proposes simultaneously $\{p^i, x^i\}$ to firm $i, i = 1, ..., n$, and firm i decides to accept or not. In stage two, each of firm $i, i = 1, ..., n$, without observing the outcome from stage one between firm 0 and firm $j, j \neq i$, decides simultaneously to enter or not. In stage three, observing the outcome in stage two, firms in the market of good F choose quantity simultaneously.

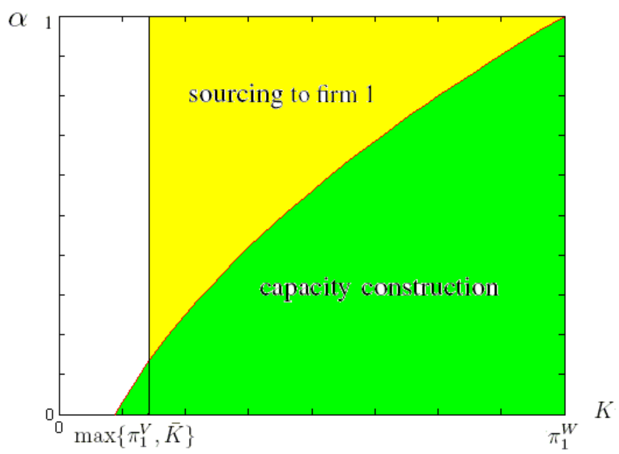

Figure 1.10: Sourcing vs. capacity building

We explore Perfect Bayesian Nash equilibrium in pure strategy for this game. Since firms $1, ..., n$ are symmetric, w.l.o.g., whenever firm 0 sources to a unique supplier out of $1, ..., n$, we put it as firm 1. The main finding is summarized below

Proposition 2 *With multiple entrants, whenever Condition (4) is true, it is an equilibrium that* $p^1 > c, x^1 \in [\tau(K), \max\{\tau(K), M_0\}]$, $x^i = 0$ *for* $i \neq 1$, *then* $q_0 = \max\{\tau(K), M_0\}$ *with all firms* $1, ..., n$ *staying out.*

Suppose $N = 2$ and $P = \max\{0, a - Q\}$ with $a > 2c > 0$. The standard Cournot profit with three incumbents at marginal cost c is $\pi_i^{(3)} = \frac{(a-c)^2}{16}, i = 0, 1, 2$. Instead, if firm 0 faces 0 marginal cost, the Cournot profit is $\pi_0^{3V} = \frac{(a+2c)(a-2c)}{16}$ for firm 1, and $\pi_i^{3v} = \frac{(a-2c)^2}{16}, i = 1, 2$ for firms 1 and 2. Given that there are three incumbents, firms 0, 1 and 2, with firm 0 a Stakelberg leader and both firms 2 and 3 Stackelberg followers, the optimal quantity for firm 1 to produce is $q_1^f(q_0)$ and its profit is $\pi_1^f(q_0)$. The solution to $\pi_1^f(x^1) = K$ is $\tau' = a - c - 3\sqrt{K}$.

Denote $q^c = \frac{a-c}{3}$, the standard Cournot quantity with two incumbents. In the scenario where only firms 0 and 2 are incumbent and firm 0 is a Stackelberg leader, denote firm 2's Stackelberg follower's quantity as $q_2^f(q_0)$. The incentive compatibility condition for firm 0 to deter the entry of firm 1 whereas accommodate firm 2's entry is

$$P(\max\{q^c, \tau'\}) - c - \min\{q_2^f(\tau'), q^c\}) \max\{q^c, \tau'\} \geq \pi_0^{(3)} + \pi_1^{(3)} - K.$$

If $\max\{q^c, \tau'\} = q^c$, the condition is trivially satisfied. Instead, when $\tau' > q^c$, the condition above holds for $K \geq \bar{K}' \equiv \frac{11-6\sqrt{2}}{196}(a-c)^2$. Thus for $K \in [\max\{\pi_1^{(3V)}, \bar{K}'\}, \pi_1^{(3)})$, there exists a second equilibrium entry pattern, that is, firm 1's entry is deterred through $x^1 \in [\tau', \max\{\tau, q^c\}], p^1 > c$; and firm 2 enters. The equilibrium is shown by the following figure.

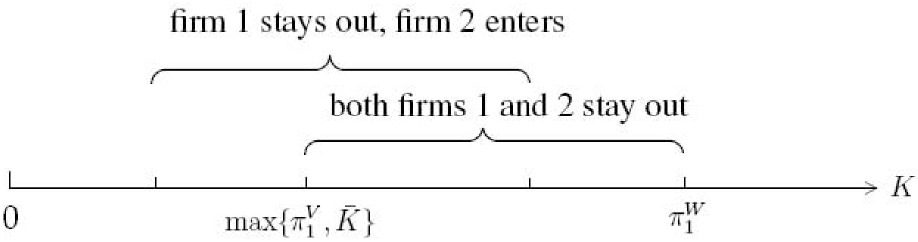

Figure 1.11: $N = 2$

1.5 Discussion

As has been shown, our finding is robust under a set of variations of the baseline model. Furthermore, our finding remains intact if firm 0 does not have full power in determining the transaction clauses. Since both firms 0 and 1 are no worse off whenever outsourcing occurs between them, any bargaining procedure (for example, Nash bargaining) which allows them to share the surplus from entry deterrence admits the same result — i.e., there will be outsourcing from firm 0 to firm 1 with $p > c$. The only thing that changes is how they share the total surplus.

One assumption important to our model is the confidentiality of the sourcing contract, which implies that firm 0's order from the competitive market is unobservable to firm 1. Although secrecy clauses are routine in the real world (for example, Ravenhill (2003); *BusinessWeek* online (2005)), it is worth considering an alternative assumption. Suppose instead that firm 0's order from any firm is publicly observable. This will only strengthen the effect of sourcing on entry deterrence, as firm 0's order of good I achieves a commitment value regardless of who the supplier is. Consider a model where firm 0 orders good I ahead of firm 1's entry decision. In this case, firm 1 is always a Stackelberg follower in the post-entry competition. Now firm 0 can deter entry simply by ordering a large enough amount of supplies, thus eradicating the necessity of ordering specifically from firm 1. Firm 0 is thus burdened less by deterring entry since it no longer needs to pay firm 1 a higher price. The upshot is that more entry is deterred in equilibrium, whereas with entry deterrence, we can have $x^1 = 0$. However, many other real business concerns may entice firm 0 to continue to outsource to firm 1 — for example, the value of a long-run cooperation. More importantly, firm 0 has an incentive to keep its order confidential. In the case of outsourcing, firms generally want to keep issues secret, including to whom and how much they outsource. One reason is that job loss due to outsourcing is a public concern (*BusinessWeek* 2005).

1.6 Conclusion

The major point of this work is to bring to light the unnoticed function of sourcing in entry deterrence. To summarize, a final-product producer can deter entry by ordering its key input from the entrant. Additionally, both players are better off than in the scenario where entry is actually carried out. Moreover, in connection with strategic sourcing, a large quantity of the final product is usually produced. As a result, the social welfare and even the consumers' welfare can be improved with entry deterred.

Our finding actually suggests that trade liberalization may lead to a more concentrated market. Moreover, our analysis indicates that, firms who can successfully enter the final-product market may willingly stay as intermediate-product providers, without really carrying out entry. Therefore, if governments have the intention of fostering domestic industry for the final-product, they may need to deliberately regulate the industry's behavior, in order to have a successful penetration in the final product market.

When the incumbent itself can produce the intermediate product efficiently, our work offers insights concerning the observed phenomenon known as *partial outsourcing*, which refers to the situation where a firm diversifies its components demand between outside suppliers and inside production. Shy and Stenbacka (2005) explain partial outsourcing in terms of a trade off between savings in production costs and increases in monitoring costs. Complementarily, our work suggests that there can exist strategic considerations underlying the same phenomenon. According to our finding, an incumbent could find it optimal to outsource a certain amount of its demand to an outside provider solely because of the incentive of deterring future entry, while producing the rest of its demand inside.

Although our major point is robust under many variations of assumptions, it has been assumed throughout that each player has complete information. An interesting future work might consider a scenario where the potential entrant has incomplete information concerning the properties of either the incumbent or the final-product market. In this case, the quantity outsourced to the entrant will also convey valuable information, and it is interesting to check how well its commitment value can be preserved. Also it would be interesting to consider the case where there exists multiple entrants or sequential entry, or where outsourcing is under a long-run contract.

Chapter 2

Outsourcing Induced by Strategic Competition

2.1 Introduction

One of the principal concerns of any firm is to configure the supply of intermediate goods essential to its production. Of late, with the liberalization of trade and the lowering of barriers to entry, supply chain configurations have assumed global proportions. Indeed, in several industries, it has become the trend for firms to cut across national boundaries and outsource their supplies "offshore", provided the economic lure is strong enough. Many diverse factors influence firms' decisions. First, of course, there is the immediate cost of procuring the goods which—other things being equal—firms invariably seek to minimize. Then there is the question of risk: a firm may be unwilling to commit itself to a single party and instead spread its orders among others, even if they happen to be costlier, in order to ensure a steady flow of inputs. Sometimes a firm may tie up with a broad spectrum of suppliers so as to increase its access to the latest technological innovation, which could be forthcoming from any one of them. There can arise situations when a firm is impelled to select suppliers that will be strategic allies in its endeavor to penetrate newly emerging markets. For the analyses of these and other factors, and how they impinge on firms' decisions, see, e.g., Jarillo (1993), Spiegel (1993), Vidal and Goetschalkx (1997), Domberger (1998), Aggarwal (2003), Shy and Stenbacka (2003), Chen et al. (2004).

One intriguing possibility that has been alluded to, but not much explored, is that *strategic incentives* may arise in an oligopoly which outweigh other considerations and play the pivotal role in firms' selection of suppliers. Instances of this are presented by Jarillo (1993) and Domberger (1998), of which we recount only two.

The first case comes from Germany. AEG[1] used to be a traditional supplier to both BMW[2] and Mercedes Benz. At some point, with a view to vertical integration, Mercedes Benz acquired AEG. This caused BMW to look for a different supplier, despite the inevitable extra costs of the switch (see p. 67, Jarillo, 1993).

The second case involves General Electric (GE) in the United States. In the early 1980's, GE investigated the possibility of outsourcing its lower brand microwave ovens from outside, since these had become too costly to manufacture at its factory in Maryland. Discussions were first held with, and even trial orders given to, Matsushita which happened to be a major rival of GE and also the world leader for this product in terms of both volume and technology. But ultimately GE turned to Samsung, then a small company with little experience in microwaves. The strategy entailed additional costs, such as sending American engineers to Korea, but it worked well for GE (see, pp. 84-86, Jarillo, 1993; and also Case Study 6.2, p. 108, Domberger, 1998).

Such case studies clearly point to the need for a game-theoretic analysis. In this paper we bring to light a scenario in which the outsourcing patterns emerge out of the strategic competition between firms. We find that it is typically *not* the case that a firm will outsource supplies to its rivals. There are two distinct reasons for this. The first is based on increasing returns to scale: if a firm places a sizeable order with its rival, it significantly lowers the rival's costs on account of the increasing returns, and this stands to its detriment in the ensuing competition on the final product. Thus the firm is led to outsource to others who may be costlier but, being out of the final product market, do not pose the threat of future competition. The second reason is more subtle and persists even in the case of constant returns to scale (i.e., linear costs)—indeed, it comes to the fore in this case. It is the main focus of this paper.

To be precise, suppose there are many firms $\mathcal{N}$ competing in the market for a final product α. Intermediate goods η are critical to the production of α, but only *some* of the firms $\mathcal{I} \subset \mathcal{N}$ have the competence to manufacture η at reasonable cost. The other firms $\mathcal{J} \equiv \mathcal{N} \backslash \mathcal{I}$ must obtain η from elsewhere. One possibility is to outsource η to their rivals in $\mathcal{I}$. But there is also a fringe of firms $\mathcal{O}$ on the "outside" which can manufacture η. What distinguishes $\mathcal{O}$ from $\mathcal{I}$ is that no firm in $\mathcal{O}$ can enter the market for the final product α. (This could be because it lacks the technology to convert η to α, or else faces high set-up costs—and, possibly, other barriers to entry—in the market α.[3]) To keep matters simple, we consider a purely

[1]Allgemeine Deutsche Electricitätsgesellschaft

[2]Bayerische Motoren Werke (or, Bavarian Motor Works)

[3]In particular, think of the following set-up. The market for α is concentrated in the "developed world". The firms in $\mathcal{O}$, on the other hand, are located offshore in the "developing world" and can manufacture η but lack the (advanced) technology for converting η to α. Even if some of them were to make the technological breakthrough, they would face not just the standard set-up costs for penetrating the market α, but further barriers to entry that pertain to foreign firms. This international setting perhaps makes our hypothesis of an outside fringe $\mathcal{O}$ more viable. But we do not need it, and

linear model, i.e., in which the costs of production for both η and α are linear; as is the market demand for α.

Our main result is that, in this scenario, strategic considerations can come into play that will cause the firms in $\mathcal{J}$ to outsource η (outside) to $\mathcal{O}$ rather than (inside) to $\mathcal{I}$, *even if the costs of manufacturing η are higher in $\mathcal{O}$ than in $\mathcal{I}$*, so long as they are not much higher.

The intuition goes roughly as follows and is best seen with just three firms. Suppose (i) $\mathcal{I}$ and $\mathcal{J}$ are Cournot duopolists which compete in the market for the final product α; (ii) $\mathcal{I}$ and $\mathcal{O}$ can produce the intermediate good η, but $\mathcal{J}$ cannot; and (iii) $\mathcal{O}$ cannot enter the market for α. Thus $\mathcal{J}$ is confronted with the decision of how much η to outsource to $\mathcal{I}$ and how much to $\mathcal{O}$, all of which it will convert to α. It turns out that the optimal course of action for $\mathcal{J}$ is to outsource *exclusively* to either $\mathcal{I}$ or $\mathcal{O}$, never to both. Now if $\mathcal{J}$ outsources to $\mathcal{I}$, then $\mathcal{I}$ immediately knows the amount outsourced. This has the effect of establishing $\mathcal{J}$ as leader in the Stackelberg game that ensues in the market for α, in which $\mathcal{I}$ is forced to become the follower. In contrast, if $\mathcal{J}$ outsources to $\mathcal{O}$ then—thanks to the sanctity of the secrecy clause[4]—$\mathcal{I}$ will only know that $\mathcal{J}$ has struck a deal with $\mathcal{O}$ but not the quantity that $\mathcal{J}$ has ousourced. Thus $\mathcal{I}$ and $\mathcal{J}$ will remain Cournot duopolists in the ensuing game on market α.

If costs for manufacturing η do not vary too much between $\mathcal{I}$ and $\mathcal{O}$, then $\mathcal{I}$ will earn less as a Stackelberg follower than as a Cournot duopolist. This will tempt $\mathcal{I}$ to push $\mathcal{J}$ towards $\mathcal{O}$ by quoting so high a price for the intermediate good η that, inspite of the premium that $\mathcal{J}$ is willing to pay for the privilege of being the leader, $\mathcal{J}$ prefers to go to $\mathcal{O}$. The temptation can only be resisted if it is feasible for $\mathcal{I}$ to provide η at such an exorbitant price that it can recoup as provider what it loses as follower. But such an exorbitant price can be undercut by $\mathcal{O}$, as long as $\mathcal{O}$'s costs are not too much higher than $\mathcal{I}$'s. The upshot is that in any subgame-perfect Nash equilibrium[5] (SPNE) of the game, $\mathcal{J}$ will outsource to $\mathcal{O}$.

To complete the intuitive argument, we must still show that $\mathcal{J}$'s outsourcing orders will be exclusive. If $\mathcal{J}$ intends to produce no more than its Cournot quantity of α, then its rival $\mathcal{I}$'s output of α is invariant of who $\mathcal{J}$ outsources η to, and so $\mathcal{J}$ would do best to outsource η from whichever of $\mathcal{I}$ or $\mathcal{O}$ is charging the lower price. On the other hand, if $\mathcal{J}$ intends to produce more than its Cournot quantity, then it is best for $\mathcal{J}$ to fully take advantage of its leadership and outsource the Stackelberg amount to $\mathcal{I}$.

all we formally postulate is the existence of this fringe.

[4]The secrecy clause is crucial to our analysis. It can be upheld on the simple ground that it is routinely seen in practice (see, e.g., Ravenhill, 2003; Clarkslegal and Kochhar, 2005). But, as we argue in Section 6, there are a variety of settings in which it can be shown to hold endogenously in equilibrium.

[5]*Throughout* we confine ourselves to pure strategies.

The actual argument is more intricate and the exact result is presented in Section 3. As was said, there are no economies of scale or cost advantages for the outside firm $\mathcal{O}$. In fact, we suppose that $\mathcal{O}$ has a higher cost than $\mathcal{I}$ for manufacturing η. Our main result states that, if $\mathcal{O}$'s cost does not exceed a well-defined threshold, $\mathcal{J}$ will outsource to $\mathcal{O}$ in any SPNE.

Worthy of note is the fact that it is *not* $\mathcal{J}$ who has the "primary" strategic incentive to outsource to $\mathcal{O}$. This incentive resides with $\mathcal{I}$ who is anxious to ward off $\mathcal{J}$ and force $\mathcal{J}$ to turn to $\mathcal{O}$. The anxiety gets played out when $\mathcal{O}$ does not have a severe cost disadvantage compared to $\mathcal{I}$. Otherwise, $\mathcal{I}$ is happy to strike a deal with $\mathcal{J}$ since it can get high provider prices that compensate it for becoming a follower. Which subgame gets played between $\mathcal{I}$ and $\mathcal{J}$ on market α—Cournot or Stackelberg—is thus not apriori fixed, but *endogenous* to equilibrium. This is all the more striking since, in our overall game, the option is open for firm $\mathcal{J}$ to outsource to both $\mathcal{I}$ and $\mathcal{O}$ and to thus bring any "mixture" of the Stackelberg and Cournot games into play. The logic of the SPNE rules out mixing and shows that only one of the two pure games will occur along the equilibrium play.

It should also be mentioned that our game involves simultaneous moves at various junctures, first at the very start, when firms $\mathcal{I}$ and $\mathcal{O}$ independently quote prices at which they are willing to supply η, and later in those subgames which follow after $\mathcal{J}$'s decision to outsource positive amounts of η to $\mathcal{O}$. Thus we are far from having perfect information in our game, and it is not a priori clear that SPNE will even exist in *pure* strategies. We prove that, in fact, there is a continuum of pure strategy SPNE, across which the outputs—both individual and aggregate—of the firms differ, but the outsourcing pattern is nevertheless invariant.

Economies of scale can easily be incorporated into our model. But then, as was said, a new strategic consideration arises, though it does not affect the tenor of our results (see Section 5.1 and, for full details, see[6] Chen (2007)). The primary strategic incentive to outsource to $\mathcal{O}$ can shift from $\mathcal{I}$ to $\mathcal{J}$. For now $\mathcal{J}$ must worry that if it outsources η to $\mathcal{I}$, then $\mathcal{I}$ will develop a cost advantage on account of economies of scale. In other words, $\mathcal{I}$ will be able to manufacture η for itself at an average cost that is significantly lower than what it charges to $\mathcal{J}$. This might outweigh any leadership advantage that $\mathcal{J}$ obtains by going to $\mathcal{I}$. So, foreseeing a competitor in $\mathcal{I}$ that is fierce inspite of being a follower, $\mathcal{J}$ would prefer to outsource to $\mathcal{O}$ as long as $\mathcal{O}$'s price is not too much above $\mathcal{I}$'s. This, in turn, will happen if $\mathcal{O}$'s costs are not significantly higher than $\mathcal{I}$'s. But then, if $\mathcal{J}$ is outsourcing to $\mathcal{O}$, economies of scale can drive $\mathcal{I}$ to outsource to $\mathcal{O}$ as well!

These two strategic considerations, the first impelling $\mathcal{I}$ to push $\mathcal{J}$ towards $\mathcal{O}$ and the second impelling $\mathcal{J}$ to turn away from $\mathcal{I}$ on its own and to seek out $\mathcal{O}$, are intermingled in the presence of economies of scale. It is hard to disentangle them and say precisely when one fades out, leaving spotlight on the other. But by

[6]See also Chen (2004) for a related model.

eliminating economies of scale altogether, we are here able to focus on just the first scenario, wherein the game turns essentially on the informational content of the strategies.

Our analysis indicates that firms which position themselves on the "outside", by *not* entering the market for the final product, are more likely to attract orders for intermediate goods. There is some evidence that this can happen in practice. By the mid-1980's (see Ravenhill, 2003), US companies in the electronics industry were looking "to diversify their sources of supply" in order to fare better against their Japanese competitors. Malaysia and Singapore made a strong bid to get the US business. A key feature of the government policies of both nations was that "they were not attempting to promote national champions in the electronic industry", but the objective was rather "to build a complementary supply base, not to create local rivals that might displace foreign producers". Their success in becoming major supply hubs for electronic components is well documented. Of course it is true that they had the advantage of low-cost skilled labor. But what we wish to underscore is their deliberate and well-publicized *abstention* from markets for the final products. According to our analysis, the abstention by itself gave Malaysia and Singapore a competitive edge: even if their costs were to rise and exceed those in Japan, US firms would still favor them as suppliers, since the Japanese firms are entrenched rivals on the final product.

The paper is organized as follows. We discuss the related literature in Section 2. We present the model in Section 3, stripped down to its bare minimum, and with just three firms. The main result is stated in Section 4 and its proof is in Section 5. In Section 6, we indicate how our result is robust to various extensions of the model. Finally, in Section 7 we give an intuitive justification for the presence of the secrecy clause in the contract between firm $\mathcal{J}$ and firm $\mathcal{O}$.

2.2 Related Literature

There is considerable literature on endogenous Stackelberg leadership.[7] The paper most closely related to ours, and inviting immediate comparison, is Baake, Oechssler and Schenk (1999). They consider a duopoly model to examine what they call "cross-supplies" within an industry—in our parlance, this is the phenomenon that a firm outsources to its rival. The "endogenous Stackelberg effect" is indeed pointed out by them: firm A, upon accepting the order outsourced by its rival B, automatically becomes a Stackelberg follower in the ensuing game on the final markets. But there are set-up costs of production in their model, and provided these costs are high enough, A can charge B a sufficiently high price so as to be com-

[7]E.g., Hamilton and Slutsky (1990), Robson (1990), Mailath (1993), Pal (1993), van Damme and Hurkens (1999)—in all of which the timing of entry by firms is viewed as strategic.

pensated for being a follower. The upshot is that cross-supplies can be sustained in SPNE.

There are several points of difference between their model and ours. First, their argument relies crucially on the presence of sufficiently strong economies of scale (set-up costs). If these are absent or weak, there is no outsourcing in SPNE in their model. In contrast, in our model, outsourcing occurs *purely* on account of the endogenous Stackelberg effect (recall that we have constant returns to scale[8]). Second, outsourcing occurs only in *some* of their SPNE: there always coexist other SPNE where it does not occur. In our model, the outsourcing is invariant across *all* SPNE. In short, they show that outsourcing *can* occur, while we show that it *must*. Third, it is critical for their result that there be no outside suppliers.[9] Such suppliers would generate competition that would make it infeasible for A to charge a high price to B, invalidating their result. In our model, the situation is different. We allow for both kinds of suppliers: those that are inside as potential rivals and others that are outside. It turns out that increasing the number of either type leaves our result intact (see Section 5). Finally—and this, to our mind, is the most salient difference—the economic phenomena depicted in Baake et al. and here are different, indeed almost complementary. In Baake et al., the issue is to figure out when a firm will outsource to a vertically integrated rival (VIR).[10] Here we consider precisely the opposite scenario and pinpoint conditions under which a firm will turn away from a VIR and outsource instead to an outsider, even if the outsider happens to have a costlier technology.[11] The fact that both models take cognizance of the endogenous Stackelberg effect is a technical—albeit interesting—point. What is significant is that this effect is embedded in disparate models and utilized to explain complementary economic phenomena.

The phenomenon we focus on—that firms may be reluctant to order intermediate goods from a VIR—is, of course, susceptible to alternative explanations. In an interesting paper, Heavner (2004) considers a situation in which, once a firm F orders from a VIR, the VIR acquires *ex post* monopoly power over the intermediate good. This is because there is no possibility for F to *ex ante* contract on the quality of the supply: if F chooses the VIR as supplier, it must do so without any qualifications whatsoever, and be at the mercy of the VIR as to the quality of the supply forthcoming. But then, since F is its future competitor in the final product market, the VIR has ex-post incentive to reduce the quality of its supply to F. Anticipating this, F does not contract with the VIR at all.

[8]Though, as was said, outsourcing is further boosted by economies of scale in our model.

[9]Recall that these are suppliers who are not present as rivals in the final product market.

[10]As is also the case in Chen (2001).

[11]Our analysis thus suggests that the current widespread trend of outsourcing to offshore locations can well persist for strategic reasons, even if offshore costs were to rise, so long as the offshore companies abstain from the final product markets of their clients.

In contrast, in our model, suppliers engage in cut-throat ex ante competition over the price at which they are willing to contract; if a deal is made, they have to abide by the price that was contracted ex ante, and supply as much as is demanded at that price. There is indeed no ex post choice available at all to the VIR: it is bound to execute the terms of its ex ante contract. Thus our model explores a scenario which is "orthogonal" to Heavner (2004) and where the force identified by him is absent.

Pursuing the logic of incomplete contracts further, contingencies may arise which are not covered by the contract. In these situations, the VIR may have ex post incentive to "hold back" on its supply to F in order to reap monopolistic profits on the final product market (see, e.g., Carlton (1979)). Fearing the hold-back, firm F will not contract with the VIR and look to an outside upstream firm for its supply.

Finally, one could consider a model in which competing downstream firms differentiate their products in order to mitigate price competition (as in Shaked and Sutton (1982)). But then the intermediate goods each firm needs must be tailor-made to its specific final brand. If suppliers are unable to cater to more than one final brand, then downstream firms inevitably end up ordering from dedicated upstream suppliers.

Our analysis does not militate against any of the foregoing explanations. It simply steers clear of them. The model we work with is deliberately stripped down to eliminate economies of scale, product differentiation and price competition, incompleteness of ex ante contracts etc. The other explanations alluded to above cannot therefore come into play, and we are left with the pure strategic incentives that are the focus of this paper.

2.3 The Model

For ease of notation, we substitute 0, 1, 2 for $\mathcal{O}$, $\mathcal{I}$, $\mathcal{J}$. As was said, firms 1 and 2 are duopolists in the market for a final good α. An intermediate good η is required to produce α. Firm 1 can manufacture η, but 2 cannot. There is an "outside" firm 0 which can also manufacture η. What distinguishes 0 from 1 is that 0 cannot enter the market for the final good α. Firm 0's sole means of profit is the manufacture of good η for the "inside" firms 1 and 2.

The inverse market demand for good α is given by $P = \max\{0, a - Q\}$, where Q denotes the total quantity of α produced by firms 1 and 2, and P denotes the price of α. The constant marginal cost of production of good η is c_0 for 0 and c_1 for 1. Furthermore both 1 and 2 can convert x units of good η into x units of good α at the (for simplicity) same constant marginal cost, which w.l.o.g we normalize to zero. We assume

$$0 < c_1 < c_0 < (a + c_1)/2 \tag{2.1}$$

The condition $c_1 < c_0$ gives a cost disadvantage to the outside firm 0 and loads the dice against good η being sourced to it. The inequality $c_0 < (a + c_1)/2$ prevents 1 from automatically becoming a monopolist in the market for good α.

The extensive form game between the three firms is completely specified by the parameters c_1, c_0, a and so we shall denote it $\Gamma(c_0, c_1, a)$. It is played as follows. For $i \in \{0, 1\}$ and $j \in \{1, 2\}$, put

$$q_j^i \equiv \text{ quantity of good } \eta \text{ outsourced by firm } j \text{ to firm } i$$

(and put $q \equiv \{q_j^i\}_{j=1,2}^{i=0,1}$). In the first stage of the game, firms 0 and 1 simultaneously and publicly announce prices p_0 and p_1 at which they are ready to provide good η. Seeing these prices, firm 2 then chooses q_2^0, q_2^1. Firm 1 observes q_2^1 but not q_2^0, since q_2^0 is part of the secret contract between 0 and 2. Finally[12] firm 1, also knowing the prices, decides how much q_1^1 to produce on its own and how much q_1^0 to outsource to 0, making sure that $x_1(q) \equiv q_1^1 + q_1^0 - q_2^1 \geq 0$ so that it is able to honor its commitment to supply q_2^1 units of η to 2. Denote $x_2(q) \equiv q_2^0 + q_2^1$. Thus $x_1(q)$ and $x_2(q)$ are the outputs produced by 1 and 2 in the market α.

It remains to describe the payoffs of the three firms at the terminal nodes of the game tree. Any such node is specified by $p \equiv (p_0, p_1)$ and $q = \{q_j^i\}_{j=1,2}^{i=0,1}$. The payoff to firm i is $\Pi_i(p, q)$ where

$$\begin{aligned}
\Pi_0(p, q) &= p_0(q_1^0 + q_2^0) - c_0(q_1^0 + q_2^0) \\
\Pi_1(p, q) &= (a - x_1(q) - x_2(q))x_1(q) + p_1 q_2^1 - p_0 q_1^0 - c_1 q_1^1 \\
\Pi_2(p, q) &= (a - x_1(q) - x_2(q))x_2(q) - p_0 q_2^0 - p_1 q_2^1
\end{aligned}$$

This completes the description of the game $\Gamma(c_0, c_1, a)$.

2.4 The Main Result

By an SPNE of $\Gamma(c_0, c_1, a)$, we shall mean a *subgame perfect Nash equilibrium in pure strategies* of the game $\Gamma(c_0, c_1, a)$.

Our main result asserts that, if the the cost disadvantage of the outside firm 0 is not too significant (i.e. $c_0 - c_1$ is not too large), then 2 will outsource good η to 0 in any SPNE.

Put $c^* = \frac{13}{14}c_1 + \frac{1}{14}a$ and observe that (2.1) implies $c_1 < c^* < \frac{a+c_1}{2}$. Our result is summarized in Figure 1 below, in which c_0 is varied on the horizontal axis, holding a and c_1 fixed (and is even allowed to fall below c_1).

[12] We *could* have supposed that firm 1 must place its order with 0 before finding out the quantity q_2^1. This would alter the game somewhat but not our conclusion (Theorem 1 in Section 3 will hold without any change). But the timing that we have given seems more natural to us. There is a fundamental asymmetry of information between firms 1 and 2. Firm 1 always has the option of waiting to see how much q_2^1 firm 2 will outsource to it before approaching 0 to outsource its own q_0^1. In contrast, firm 2 can never know whether firm 1 has gone to 0 or not, so it cannot plan to wait until 1 has outsourced to 0 before placing its order with 1.

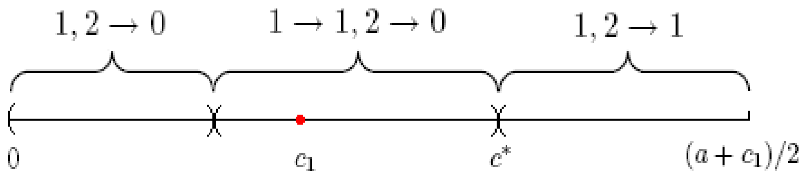

Figure 2.1: The Outsourcing Pattern ($j \to i \equiv j$ outsources η exclusively to i)

Notice that the interval (c_1, c^*) is of particular interest because here firm 0 has a cost disadvantage compared to firm 1, yet 2 outsources η to 0 rather than from 1. Strategic considerations dominate firms' behavior here. Below this interval, when $c_0 \le c_1$, 0 has a cost advantage over 1 and so 2 even more readily outsourced to 0; in fact, for small enough c_0, *both* 1 and 2 outsource to 0. We shall ignore this easy case where firm 0 becomes additionally attractive on account of its lower cost. To keep strategic incentives in the foreground, we shall suppose throughout that $c_0 > c_1$.

For a precise statement of our result, define the function $\tau : [c_1, (a + c_1)/2] \to R_+$ by

$$\tau(p_0) = \frac{(3 - 2\sqrt{2})(a + c_1)}{6} + \frac{2\sqrt{2}p_0}{3}; \tag{2.2}$$

and, for any interval $[u, v] \subseteq [c_1, (a + c_1)/2]$, define

$$(\text{Graph } \tau)[u, v] \equiv \{(p_0, \tau(p_0)) | p_0 \in [u, v]\}$$

and abbreviate

$$\text{Graph } \tau \equiv (\text{Graph } \tau)[c_1, (a + c_1)/2]$$

Since $\tau(c_1) > c_1$ and $\tau((a+c_1)/2) = (a+c_1)/2$, Graph τ is a straight line contained in the square $[c_1, (a + c_1)/2]^2$ (see Figure 2).

We are now ready to state our main result.

Theorem. (I) *In any SPNE of* $\Gamma(c_0, c_1, a)$*, firm* 1 *never outsources to firm* 0*, i.e.*, $q_1^0 = 0$.

(II) *If* $c_0 \in (c_1, c^*)$, *there is a continuum of SPNE of* $\Gamma(c_0, c_1, a)$, *indexed by supplier prices* $(p_0, p_1) \in$ *(Graph* τ*)*$[c_0, c^*]$; *and, in every SPNE, firm* 2 *outsources* η *to the outside firm* 0.

(III) *If* $c_0 \in (c^*, (a + c_1)/2)$, *there is a continuum of SPNE of* $\Gamma(c_0, c_1, a)$, *indexed by supplier prices* $(p_0, p_1) \in$ *(Graph* τ*)*$[c_1, c_0]$; *and, in every SPNE, firm* 2 *outsources* η *to the inside firm* 1.

(IV) *Finally, if* $c_0 = c^*$, *there are two SPNE of* $\Gamma(c_0, c_1, a)$ *with the same provider prices* $(p_0, p_1) = (c^*, \tau(c^*))$, *but firm* 2 *outsources* η *to* 0 *in the first SPNE and to* 1 *in the second SPNE.*

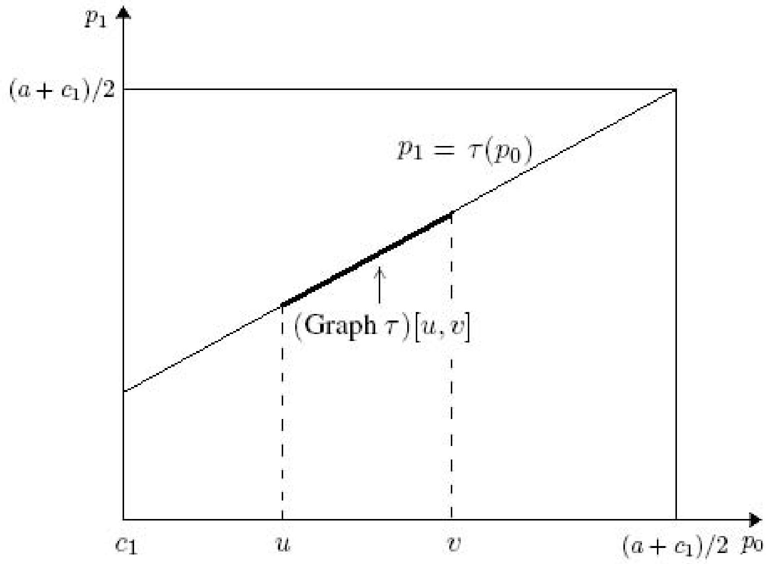

Figure 2.2: (Graph τ)$[u, v]$

2.5 Proof

2.5.1 Preparatory Lemmas

Throughout the lemmas below, c_1 and a are fixed and (recall) $c_0 \in (c_1, (a + c_1)/2)$.

Lemma 1. *In any SPNE of* $\Gamma(c_0, c_1, a)$, *we must have* $p_0 \geq c_1$.

Proof. Suppose $p_0 < c_1$. Since $c_1 < c_0$ (by assumption, see (1)), firm 0 makes $(p_1 - c_0) < 0$ dollars per unit of the total outsourced order $q_1^0 + q_2^0$ that it receives. If it could be shown that $q_1^0 + q_2^0 > 0$, there would be an immediate contradiction, because firm 0 can in fact ensure zero payoff by deviating from p_0 to some sufficiently high p_0' (any p_0' higher than the maximum price a in market α will do), at which price neither firm will outsource anything to it.

To complete the proof, we now show that $q_1^0 + q_2^0 > 0$.

Let $q_2^0 = 0$ (otherwise we are done). If 2 produces a positive amount, it must outsource to 1, i.e., $q_2^1 > 0$. Then, since $p_0 < c_1$, 1 will pass on this order to 0, i.e., $q_1^0 > 0$.

If 2 produces nothing then, as is easily verified, firm 1 will make a positive sale of α, i.e., $q_1^0 + q_1^1 > 0$. But the cost of producing $q_1^0 + q_1^1$ is $p_0 q_1^0 + c_1 q_1^1$. Since $p_0 < c_1$, optimality requires that $q_1^1 = 0$, so we conclude that $q_1^0 > 0$. ■

In view of Lemma 1, we will *assume* $p_0 \geq c_1$ *throughout the rest of this section.*

Let $G(p_0, p_1, q_2^1)$ denote the subgame between 1 and 2, after (p_0, p_1) and q_2^1 are announced. In this subgame, 1 and 2 simultaneously choose (q_1^0, q_1^1) and q_2^0, with $q_1^0 + q_1^1 \geq q_2^1$. Denote $z \equiv q_1^0 + q_1^1$. If $p_0 > c_1$ then, in order to produce z, it is a strictly dominant strategy for firm 1 to set $q_1^0 = 0$ and $q_1^1 = z$ (i.e., to produce all of z at the lower cost c_1); if $p_0 = c_1$, then firm 1 is indifferent on the split of z. In either case, firm 1 procures η at cost c_1.

We may suppress η and think of $G(p_0, p_1, q_2^1)$ as a game involving only good α, in which 1 produces $x_1 \equiv z - q_2^1 \equiv q_1^0 + q_1^1 - q_2^1$ at cost c_1 and 2 produces q_2^0 at cost p_0; and in which 2 has an "endowment" q_2^1 procured before entering the game at price p_1. The payoffs of 1 and 2 in $G(p_0, p_1, q_2^1)$ are given by

$$\Pi_1(x_1, q_2^0) = (a - q_2^1 - x_1 - q_2^0)x_1 - c_1 x_1 + (p_1 - c_1)q_2^1$$

$$\Pi_2(x_1, q_2^0) = (a - q_2^1 - x_1 - q_2^0)(q_2^1 + q_2^0) - p_0 q_2^0 - p_1 q_2^1$$

(The terms $(p_1 - c_1)q_2^1$ and $p_1 q_2^1$, involving good η, can be viewed as constants that are given from the past, before the game $G(p_0, p_1, q_2^1)$ is played.)

Lemma 2. *$G(p_0, p_1, q_2^1)$ has a unique NE.*

Proof. Let $(q_1^{\mathcal{C}}(p_0), q_2^{\mathcal{C}}(p_0))$ denote the quantities of firms 1 and 2 in the unique NE of the Cournot game $G(p_0, p_1, 0)$. As is well-known

$$(q_1^{\mathcal{C}}(p_0), q_2^{\mathcal{C}}(p_0)) = \begin{cases} ((a - 2c_1 + p_0)/3, (a + c_1 - 2p_0)/3) & \text{if } p_0 \leq (a + c_1)/2 \\ ((a - c_1)/2, 0) & \text{if } p_0 \geq (a + c_1)/2 \end{cases} \tag{2.3}$$

Let $[y]_+ \equiv \max\{0, y\}$ for any $y \in R$. It is easy to check that the NE of $G(p_0, p_1, q_2^1)$ is unique and, indeed as follows.

(i) if $0 \leq q_2^1 \leq [q_2^{\mathcal{C}}(p_0)]_+$, then 2 produces $q_2^{\mathcal{C}}(p_0) - q_2^1$ and 1 produces $q_1^{\mathcal{C}}(p_0)$ (as before);

(ii) if $[q_2^{\mathcal{C}}(p_0)]_+ < q_2^1 \leq a - c_1$, then 2 produces zero and 1 produces $(a - c_1 - q_2^1)/2$;

(iii) if $q_2^1 > a - c_1$, then both produce zero. ■

Lemma 3. *Suppose $p_0 \geq (a + c_1)/2$. Then the NE of $G(p_0, p_1, q_2^1)$ is invariant of p_0. Hence w.l.o.g. we may restrict $p_0 \leq (a + c_1)/2$.*

Proof. If $p_0 \geq (a + c_1)/2$, then $q_2^{\mathcal{C}}(p_0) = 0$ by (2.3) and then (from (i), (ii), (iii) in the proof of Lemma 2) $q_2^0 = 0$. Since $c_1 < (a + c_1)/2$, we have $p_0 > c_1$ and hence $q_1^0 = 0$ as well. So firm 0 receives no order from anyone when $p_0 \geq (a + c_1)/2$. The lemma follows. ■

We now move one step back in the game tree of $\Gamma(c_0, c_1, a)$ and denote by $G(p_0, p_1)$ the game that ensues after the simultaneous announcement of p_0 and p_1.

In looking for SPNE of $G(p_0, p_1)$, it suffices to consider the problem in which firm 2 chooses q_2^1 and then the unique NE of $G(p_0, p_1, q_2^1)$ is played.

First imagine two games between firms 1 and 2 in the market α. In both games, the inverse demand for α is fixed at $P = \max\{0, a-Q\}$ and the (constant, marginal) cost of firm 1 (to produce α) is fixed at c_1. The constant marginal cost $c \in [c_1, (a+c_1)/2]$ of firm 2 (to produce α) is considered variable and hence the game depends on c. Let $\mathcal{S}^{21}(c)$ be the Stackelberg duopoly with 2 as the leader and 1 the follower and let $\mathcal{C}(c)$ be the Cournot duopoly between 1 and 2. These games have unique SPNE.[13] Let $f(c)$ and $\ell(c)$ denote the profits of 1 (follower) and 2 (leader) in the SPNE of $\mathcal{S}^{21}(c)$. Let $\kappa_1(c)$ and $\kappa_2(c)$ denote the corresponding profits in $\mathcal{C}(c)$. Finally, let $q_1^{\mathcal{S}}(c)$ and $q_2^{\mathcal{S}}(c)$ denote the output produced by 1 and 2 in the SPNE of $\mathcal{S}^{21}(c)$ (and recall $q_1^{\mathcal{C}}(c)$ and $q_2^{\mathcal{C}}(c)$ are the corresponding outputs in $\mathcal{C}(c)$). It is well known that

$$(q_1^{\mathcal{S}}(c), q_2^{\mathcal{S}}(c)) = \begin{cases} (0, (a-c)/2) \text{ if } c \leq [2c_1 - a]_+, \\ (0,\ a-c_1) \text{ if } [2c_1 - a]_+ \leq c \leq [(3c_1 - a)/2]_+, \\ ((a-3c_1+2c)/4,\ (a+c_1-2c)/2) \text{ if } [(3c_1-a)/2]_+ \leq c \leq (a+c_1)/2, \\ ((a-c_1)/2,\ 0) \text{ if } c \geq (a+c_1)/2. \end{cases} \tag{2.4}$$

Lemma 4 below characterizes the SPNE of $G(p_0, p_1)$ as (p_0, p_1) varies. To state it, we need to partition the price space $[c_1, (a+c_1)/2] \times [0, \infty)$ of (p_0, p_1) into four regions $R_{\mathcal{M}}$, $R_{\mathcal{S}}$, $R_{\mathcal{C}}$ and Graph τ (see Figure 3). Recall that $[y]_+ \equiv \max\{0, y\}$ for any $y \in R$, and put

$$R_{\mathcal{M}} = \{(p_0, p_1) \in R_+^2 | c_1 \leq p_0 \leq (a+c_1)/2, 0 \leq p_1 \leq [(3c_1 - a)/2]_+\}$$

$$R_{\mathcal{S}} = \{(p_0, p_1) \in R_+^2 | c_1 \leq p_0 \leq (a+c_1)/2, [(3c_1 - a)/2]_+ < p_1 < \tau(p_0)\}$$

$$R_{\mathcal{C}} = \{(p_0, p_1) \in R_+^2 | c_1 \leq p_0 \leq (a+c_1)/2, p_1 > \tau(p_0)\}$$

Also, let us use the phrase "in SPNE" to mean "in the play induced by the SPNE". We are now ready to state Lemma 4.

Lemma 4. (Figure 3) *Suppose* $p_0 \leq (a+c_1)/2$. *Let* $x_1(q) \equiv q_1^0 + q_1^1 - q_2^1$ *and* $x_2(q) \equiv q_2^0 + q_2^1$ *be the quantities sold by firms* 1 *and* 2 *in the market* α.

(i) *In any SPNE of* $G(p_0, p_1)$, $q_2^0 q_2^1 = 0$ *and w.l.o.g.* $q_1^0 = 0$ (*so that* $x_1(q) = q_1^1 - q_2^1$ *and* $x_2(q) = max\{q_2^0, q_2^1\}$).

(ii) *If* $(p_0, p_1) \in R_{\mathcal{M}}$, $G(p_0, p_1)$ *has a unique SPNE with* $q_2^0 = 0$, $x_2(q) = q_2^1 > 0$ *and* $x_1(q) = 0$. *Firms* 0, 1, 2 *have zero, negative, positive payoffs respectively and firm* 2 *is a monopolist.*

(iii) *If* $(p_0, p_1) \in R_{\mathcal{S}}$, $G(p_0, p_1)$ *has a unique SPNE in which* $q_2^0 = 0$ *and the ensuing game is* $\mathcal{S}^{21}(p_1)$ *where* $x_2(q) = q_2^1 = q_2^{\mathcal{S}}(p_1)$ *and* $x_1(q) = q_1^{\mathcal{S}}(p_1)$. *Firms* 0, 1 *and* 2 *earn zero,* $F(p_1) \equiv f(p_1) + (p_1 - c_1)q_2^{\mathcal{S}}(p_1)$ *and* $\ell(p_1)$ *respectively.*

[13] In the Cournot game $\mathcal{C}(c)$, SPNE is just NE.

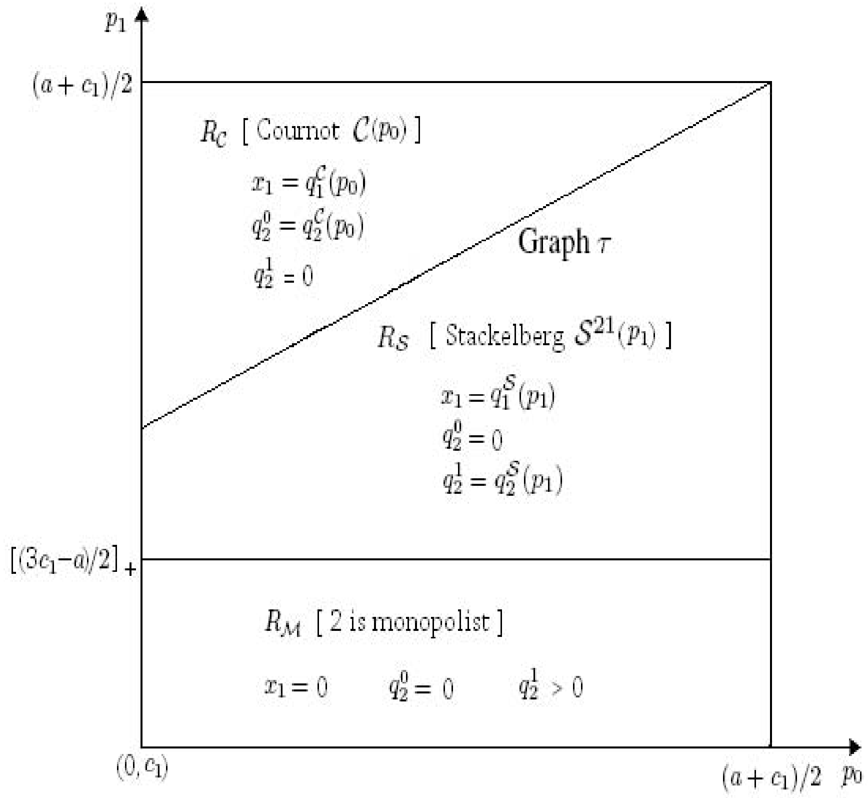

Figure 2.3: SPNE of $G(p_0, p_1)$

(iv) *If* $(p_0, p_1) \in R_{\mathcal{C}}$, $G(p_0, p_1)$ *has a unique SPNE in which* $q_2^1 = 0$ *and the ensuing game is* $\mathcal{C}(p_0)$ *where* $x_2(q) = q_2^0 = q_2^{\mathcal{C}}(p_0)$ *and* $x_1(q) = q_1^1 = q_1^{\mathcal{C}}(p_0)$. *Firms* 0, 1 *and* 2 *earn* $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$, $\kappa_1(p_0)$ *and* $\kappa_2(p_0)$ *respectively.*

(v) *If* $(p_0, p_1) \in$ *Graph* τ, $G(p_0, p_1)$ *has exactly two SPNE. In the first SPNE,* $q_2^0 = 0$ *and the ensuing game is* $\mathcal{S}^{21}(p_1)$ *where* $x_2(q) = q_2^1 = q_2^{\mathcal{S}}(p_1)$ *and* $x_1(q) = q_1^{\mathcal{S}}(p_1)$; *firms* 0, 1 *and* 2 *earn zero,* $F(p_1) \equiv f(p_1) + (p_1 - c_1)q_2^{\mathcal{S}}(p_0)$ *and* $\ell(p_1)$ *respectively. In the second SPNE,* $q_2^1 = 0$ *and the ensuing game is* $\mathcal{C}(p_0)$ *where* $x_1(q) = q_1^1 = q_1^{\mathcal{C}}(p_0)$ *and* $x_2(q) = q_2^0 = q_1^{\mathcal{C}}(p_0)$; *firms* 0, 1 *and* 2 *earn* $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$, $\kappa_1(p_0)$ *and* $\kappa_2(p_0)$ *respectively.*

Proof. We first argue that w.l.o.g. $q_1^0 = 0$. Recall $p_0 \geq c_1$. If $p_0 > c_1$, it is obvious that $q_1^0 = 0$. If $p_0 = c_1$, there is an irrelevant multiplicity of optimal choices for firm 1: it is indifferent between all pairs (q_1^0, q_1^1) such that $q_1^0 + q_1^1$ is a given constant z. But no matter how 1 breaks the tie, this has no effect on the rest of the game, i.e., on the choice (q_2^0, q_2^1) of firm 2, or on the price of α, or on the payoffs of 1 and 2. Thus we may take[14] $q_1^0 = 0$ and $q_1^1 = z$.

The rest of the proof is again a matter of straightforward calculation. From (i), (ii), (iii) in the proof of Lemma 2, we can compute the payoffs $\Pi_i(p_0, p_1, q_2^1)$ at the terminal node of the game $\Gamma(c_0, c_1, a)$ that is reached by the unique NE of the subgame $G(p_0, p_1, q_2^1)$. (Note that Π_1 and Π_2 include the sunk cost $p_1 q_2^1$ incurred by 2 and concomitant gain $(p_1 - c_1)q_2^1$ of 1, prior to reaching the node (p_0, p_1, q_2^1) in Γ.) These are as follows (recalling the Cournot quantities $q_i^{\mathcal{C}}(p_0)$ from (2.3)).

$$\Pi_2(p_0, p_1, q_2^1) = \begin{cases} (q_2^{\mathcal{C}}(p_0))^2 + (p_0 - p_1)q_2^1 & \text{if } 0 \leq q_2^1 \leq q_2^{\mathcal{C}}(p_0) \\ (a + c_1 - 2p_1 - q_2^1)q_2^1/2 & \text{if } q_2^{\mathcal{C}}(p_0) < q_2^1 \leq a - c_1 \\ (a - q_2^1)q_2^1 - p_1 q_2^1 & \text{if } a - c_1 < q_2^1 < a \\ -p_1 q_2^1 & \text{if } q_2^1 \geq a \end{cases} \tag{2.5}$$

$$\Pi_1(p_0, p_1, q_2^1) = \begin{cases} (q_1^{\mathcal{C}}(p_0))^2 + (p_1 - c_1)q_2^1 & \text{if } 0 \leq q_2^1 \leq q_2^{\mathcal{C}}(p_0) \\ (a - c_1 - q_2^1)^2/4 + (p_1 - c_1)q_2^1 & \text{if } q_2^{\mathcal{C}}(p_0) < q_2^1 \leq a - c_1 \\ (p_1 - c_1)q_2^1 & \text{if } q_2^1 > a - c_1 \end{cases} \tag{2.6}$$

Next we move one step back in the game tree Γ and consider the maximization problem faced by firm 2 at the start of the game $G(p_0, p_1)$. The set of its optimal choices is

$$\beta(p_0, p_1) = \arg\max_{q_2^1 \geq 0} \Pi_2(p_0, p_1, q_2^1)$$

It can be verified, using (2.5), that

$$\beta(p_0, p_1) = \begin{cases} \{q_2^{\mathcal{S}}(p_1)\} & \text{if } p_1 < \tau(p_0) \\ \{0, q_2^{\mathcal{S}}(p_1)\} & \text{if } p_1 = \tau(p_0) \\ \{0\} & \text{if } p_1 > \tau(p_0) \end{cases} \tag{2.7}$$

The lemma follows from (i), (ii), (iii) (in the proof of Lemma 2) and (2.7). ■

Lemma 5. *Suppose $p_1 \geq (a + c_1)/2$. Then the SPNE of $G(p_0, p_1)$ are invariant of p_1. Hence w.l.o.g. we may restrict $p_1 \leq (a + c_1)/2$.*

Proof. When $p_1 \geq (a + c_1)/2$, we are in the region $R_{\mathcal{C}}$. So $q_2^1 = 0$ by (iv) of Lemma 4, proving the result. ■

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[14] To be very formal, when $p_0 = c_1$, the choices $(q_1^0, q_1^1) \in R_+^2$ of firm 1 may be partitioned into "equivalence classes" $\Lambda(z)$, $z \in R_+$, where $\Lambda(z) = \{(q_1^0, q_1^1) \in R_+^2 | q_1^0 + q_1^1 = z\}$. The game $G(p_0, p_1)$, and in particular the set of its SPNE, is unaffected by which element firm 1 picks in $\Lambda(z)$. In other words, when $p_0 = c_1$, firm 1 may be viewed as choosing only z (and it is irrelevant which point in $\Lambda(z)$ it actually picks to "effect" z).

Furthermore note that when we go a step back to the root of the tree $\Gamma(c_0, c_1, a)$, it can in fact never happen in any SPNE of $\Gamma(c_0, c_1, a)$ that $p_0 = c_1$ and that $q_1^0 > 0$ (see Lemma 7).

Let us recall (from Lemma 4) the payoff $F(p_1)$ of firm 1, when 1 is the follower in $\mathcal{S}^{21}(p_1)$ and charges p_1 to firm 2, i.e., $F(p_1) \equiv f(p_1) + (p_1 - c_1)q_2^{\mathcal{S}}(p_0)$.

Lemma 6. *F is strictly increasing on* $[c_1, (a + c_1)/2]$.

Proof. Straightforward computation, using (2.4). ∎

Lemma 7. *In any SPNE of* $\Gamma(c_0, c_1, a)$, *the following hold*:

(i) $p_1 > [(3c_1 - a)/2]_+$

(ii) $p_0 < (a + c_1)/2$

(iii) $q_2^0 + q_2^1 > 0$

(iv) *if* $p_0 < c_0$, *then* $q_1^0 = q_2^0 = 0$.

(v) *if* $q_2^1 > 0$, *then* $(p_0, p_1) \in$ (*Graph* τ)$[c_1, c_0]$.

Proof. (i) Suppose $p_1 \leq [(3c_1 - a)/2]_+$. Then we are in the region $R_{\mathcal{M}}$ and, by (ii) of Lemma 4 (see Figure 3) $q_2^1 > 0$. Since $0 < c_1 < a$ (by (1)), we have $[(3c_1 - a)/2]_+ < c_1$ and so $p_1 < c_1$. Thus firm 1's payoff is $(p_1 - c_1)q_1^1 < 0$. But 1 can deviate and set a sufficiently high price (any price above a will do) to ensure that firm 2 does not outsource to it, and thus 1 can earn a non-negative payoff, a contradiction.

(ii) By Lemma 3, we may suppose $p_0 \leq (a + c_1)/2$. So if the claim is false, $p_0 = (a + c_1)/2$. By Lemma 5 and (i) above, $p_1 \in ([(3c_1 - a)/2]_+, (a + c_1)/2]$. If $p_1 < (a + c_1)/2$, then $(p_0, p_1) \in R_{\mathcal{S}}$ and so, by (iii) of Lemma 4, firm 1 gets payoff $F(p_1)$. Since F is strictly increasing, we must have $p_1 = (a + c_1)/2$ (otherwise 1 can improve its payoff by increasing p_1). We conclude that $p_0 = p_1 = (a + c_1)/2$. Then $(p_0, p_1) \in$ Graph τ and, by (v) of Lemma 4, there are two possible SPNE of $G(p_0, p_1)$. No matter which prevails, $q_2^0 = 0$ (by (2.3) and (2.4)) and hence firm 0 gets zero payoff. Let 0 deviate by changing p_0 to p_0' where $c_0 < p_0' < (a + c_1)/2$. But $(p_0', p_1) \in R_{\mathcal{C}}$ and $q_2^0 > 0$ (by Lemma 4 and (2.3)), so firm 0 earns positive payoff as a result of the deviation, a contradiction.

(iii) Denote $I_0 \equiv [c_1, (a + c_1)/2)$ and $I_1 \equiv ([(3c_1 - a)/2]_+, (a + c_1)/2]$. Then we have $(p_0, p_1) \in I_0 \times I_1$ by Lemma 5 and (i) and (ii) above.

First consider $p_1 = (a + c_1)/2$. Then $(p_0, p_1) \in R_{\mathcal{C}}$ for any $p_0 \in I_0$ and in any SPNE of $G(p_0, p_1)$, we have $q_2^0 = q_2^{\mathcal{C}}(p_0)$ by Lemma 4. Since $q_2^{\mathcal{C}}(p_0) > 0$ for $p_0 \in I_0$ (by (2.3)), we have $q_2^0 > 0$.

Next consider $p_1 \in I_1 \backslash \{(a + c_1)/2\}$. Then it follows from Lemma 4 that in any SPNE of $G(p_0, p_1)$, $q_2^0 + q_2^1$ equals either $q_2^{\mathcal{C}}(p_0)$ or $q_2^{\mathcal{S}}(p_1)$. Since $q_2^{\mathcal{C}}p_0) > 0$ for $p_0 \in I_0$ (by (2.3)) and $q_2^{\mathcal{S}}(p_1) > 0$ for $p_1 \in I_1 \backslash \{(a + c_1)/2\}$ (by (2.4)), the result follows.

(iv) When $p_0 < c_0$, firm 0 gets $(p_0 - c_0) < 0$ dollars for every unit that is outsourced to it. If $q_1^0 + q_2^0 > 0$, then 0 gets negative payoff. But 0 can deviate and set a sufficiently high price to ensure that no firm outsources to it and 0 can thus guarantee zero payoff, a contradiction.

(v) By (i) and (ii) above and by Lemma 5, $c_1 \leq p_0 < (a+c_1)/2$ and $[(3c_1-a)/2]_+ < p_1 \leq (a+c_1)/2$. But then, by Lemma 4, $q_2^1 > 0$ implies $(p_0, p_1) \in [R_{\mathcal{S}} \cup \text{ Graph } \tau]$. If $(p_0, p_1) \in R_{\mathcal{S}}$, i.e., $p_1 < \tau(p_0)$, then (again by Lemma 4) firm 1 earns $F(p_1)$. Since F is strictly increasing (by Lemma 6), 1 can improve its payoff by raising its price to $p_1 + \varepsilon < \tau(p_0)$, a contradiction. This proves $(p_0, p_1) \in \text{ Graph } \tau$.

It remains to show that $p_0 \leq c_0$. Suppose $p_0 > c_0$. Then since $c_0 > c_1$ by assumption, we have $p_0 > c_1$ which immediately implies that $q_1^0 = 0$. Since $q_2^1 > 0$, we also have $q_2^0 = 0$ by (i) of Lemma 4. So firm 0 gets no order and earns zero payoff. Let firm 0 reduce p_0 to $p_0 - \varepsilon > c_0$. Since $(p_0, p_1) \in \text{ Graph } \tau$ as shown in the previous paragraph, $(p_0 - \varepsilon, p_1) \in R_{\mathcal{C}}$ (see Figure 3) and so firm 2 will outsource a positive amount to 0 after 0's deviation (by Lemma 4 and (2.3)). Thus 0 earns a positive payoff after its deviation, a contradiction. ■

Recall the Stackelberg and Cournot duopoly games, $\mathcal{S}^{21}(c)$ and $\mathcal{C}(c)$ in which the cost of firm 1 is fixed at c_1 while that of its rival firm 2 is a variable c. The function $\kappa_1(c)$ simply gives the standard Cournot profit of firm 1. In contrast, $F(c) = f(c) + (c - c_1)q_2^{\mathcal{S}}(c)$ lumps together the profit $f(c)$ that 1 makes as the follower in $\mathcal{S}^{21}(c)$ as well as the revenue $(c - c_1)q_2^{\mathcal{S}}(c)$ that 1 earns by supplying 2 its Stackelberg-leader output $q_2^{\mathcal{S}}(c)$ at price c.

The following lemma compares F and κ_1. First define

$$\tilde{c} = 55c_1/62 + 7a/62 \tag{2.8}$$

and observe that $c_1 < \tilde{c} < (a + c_1)/2$ by (1).

Lemma 8. (Figure 4) κ_1 *is strictly increasing on* $[c_1, (a+c_1)/2]$. *Moreover*, $F < \kappa_1$ *on* $[c_1, \tilde{c})$, $F > \kappa_1$ *on* $(\tilde{c}, (a + c_1)/2)$, $F(\tilde{c}) = \kappa_1(\tilde{c})$ *and* $F((a + c_1)/2) = \kappa_1((a + c_1)/2)$.

Proof. Straightforward computation using the explicit formulae for κ_1 and F that follow from (2.3) and (2.4). ■

For any c, we shall define $\lambda(c)$ to be the minimum cost of firm 2 at which 1 is willing to switch from the Cournot game $\mathcal{C}(c)$ to being follower in the Stackelberg game $\mathcal{S}^{21}(\lambda(c))$. Precisely

$$\lambda : [0, \tilde{c}] \to [0, \tilde{c}]$$

is given by

$$\lambda \equiv F^{-1} \circ \kappa_1.$$

The function λ is well-defined, strictly increasing and $\lambda(\tilde{c}) = \tilde{c}$.

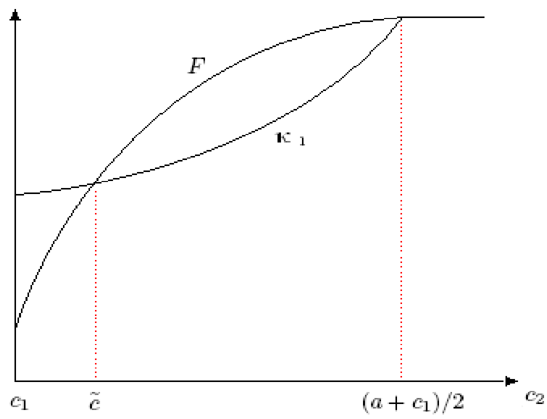

Figure 2.4: The Functions κ_1 and F

Lemma 9. (Figure 5) *Let* $c \in [c_1, \tilde{c}]$. *Then* $\kappa_1(c) = F(\lambda(c))$, $F(y) < \kappa_1(c_2)$ *for* $y < \lambda(c)$ *and* $F(y) > \kappa_1(c)$ *for* $y > \lambda(c)$.

Proof. The proof follows from lemmas 6, 8 and the definition of λ. ■

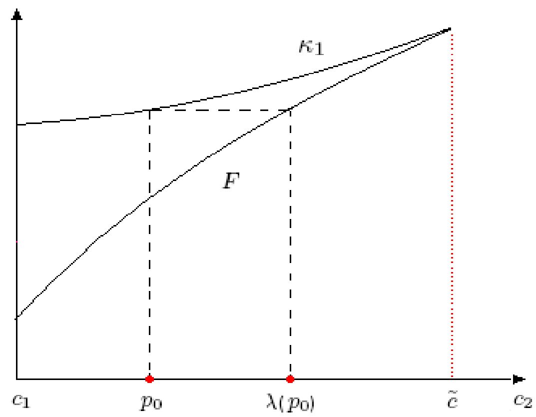

Figure 2.5: The Function λ

The next lemma compares the functions τ and λ. Define

$$c^* = 13c_1/14 + a/14 \tag{2.9}$$

and observe from (1) and (2.8) that

$$c_1 < c^* < \tilde{c}. \tag{2.10}$$

Lemma 10. (Figure 6) *Let* $c \in [c_1, \tilde{c}]$. *Then* $\lambda(c^*) = \tau(c^*)$, $\tau(c) < \lambda(c)$ *for* $c \in [c_1, c^*)$, $\tau(c) > \lambda(c)$ *for* $c_2 \in (c^*, \tilde{c}]$.

Proof. Straightforward computation using the explicit formula for τ in (2.2) and the explicit formulae for κ_1 and F that follow from (2.3) and (2.4). ∎

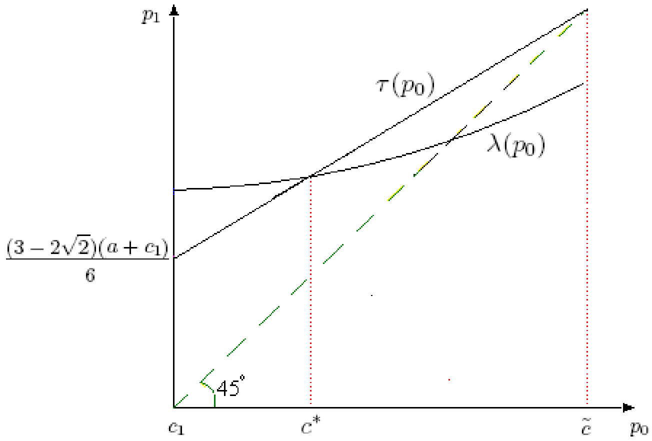

Figure 2.6: $\tau(p_0)$ and $\lambda(p_0)$

Lemma 11. $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$ *is increasing in* p_0 *for* $p_0 \in [c_1, \tilde{c}]$.

Proof. A simple calculation shows that $(p_0 - c_0)q_2^{\mathcal{C}}(p_0) = (p_0 - c_0)(a + c_1 - 2p_0)/3$ for $p_0 \in [c_1, (a + c_1)/2]$ from which the result follows. ∎

Lemma 12. *In any SPNE of* $\Gamma(c_0, c_1, a)$, *if* $q_2^0 > 0$, *then* $(p_0, p_1) \in$ *(Graph*τ*)*$[c_1, c^*]$.

Proof. In step 1 we show that $p_0 \in [c_1, c^*]$ and in step 2 we show that $p_1 = \tau(p_0)$.

Step 1: By Lemma 1, we have $p_0 \geq c_1$. So it suffices to show that $p_0 \leq c^*$.

Since $q_2^0 > 0$ we have, by (iv) and (v) of Lemma 4, that $(p_0, p_1) \in [R_{\mathcal{C}} \cup$ Graph $\tau]$ and that the payoff of firm 1 is $\kappa_1(p_0)$. In what follows, we show that if $p_0 > c^*$, firm 1 can earn more than $\kappa_1(p_0)$ by setting a price $p_1' < \tau(p_0)$, a contradiction establishing step 1.

Recall from (2.10) that $c_1 < c^* < \tilde{c}$. First suppose that $\tilde{c} \leq p_0 < (a + c_1)/2$. By (ii) of Lemma 7, we must have $p_0 < (a + c_1)/2$. Let firm 1 change p_1 to $p_1' \equiv \tau(p_0) - \varepsilon > 0$. Then $(p_0, p_1') \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 1's payoff is $F(p_1')$. Since $\tau(p_0) > p_0$ for $p_0 < (a + c_1)/2$, we have $p_1' > p_0$ for small enough ε, implying that $F(p_1') > F(p_0)$ (since F is strictly increasing—see Lemma 6). Since $p_0 \geq \tilde{c}$, it follows from Lemma 8 that $F(p_0) \geq \kappa_1(p_0)$. Hence $F(p_1') > \kappa_1(p_0)$, showing that firm 1 has made a gainful deviation, a contradiction. So we must have $p_0 < \tilde{c}$.

Now suppose that $c^* < p_0 < \tilde{c}$. Then $\lambda(p_0) < \tau(p_0)$ by Lemma 10. Let firm 1 change p_1 to $p_1' \equiv \lambda(p_0) + \varepsilon$ where ε is small enough to ensure that $\lambda(p_0) + \varepsilon <$

$\tau(p_0)$. Then $(p_0, p_1') \in R_{\mathcal{S}}$ and 1 gets the payoff $F(p_1')$ (by (iii) of Lemma 4). Since F is strictly increasing, $F(p_1') > F(\lambda(p_0))$. By the definition of λ, we have $F(\lambda(p_0)) = \kappa_1(p_0)$. Hence $F(p_1') > \kappa_1(p_0)$, showing that firm 1 has made a gainful deviation, a contradiction. This proves that $p_0 \in [c_1, c^*]$.

Step 2: Since $q_2^0 > 0$, we must have $(p_0, p_1) \in [R_{\mathcal{C}} \cup$ Graph $\tau]$ and the payoff of firm 0 is $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$ (by (iv) and (v) of Lemma 4).

By Lemma 5, we may suppose that $p_1 \leq (a+c_1)/2$. We have already shown that $p_0 \in [c_1, c^*]$. Since $c^* < (a + c_1)/2$, we have $\tau(p_0) < (a + c_1)/2$. If $(p_0, p_1) \in R_{\mathcal{C}}$, then $p_1 \in (\tau(p_0), (a + c_1)/2]$. Let 0 deviate and set a price $p_0' \equiv p_0 + \varepsilon < \tilde{c}$ where ε is sufficiently small to ensure that $p_0' < \tilde{c}$ and $p_1 > \tau(p_0')$. Then $(p_0', p_1) \in R_{\mathcal{C}}$ and firm 0 will earn $(p_0' - c_0)q_2^{\mathcal{C}}(p_0')$ after the deviation. Then by Lemma 11, it follows that $(p_0' - c_0)q_2^{\mathcal{C}}(p_0') > (p_0 - c_0)q_2^{\mathcal{C}}(p_0)$. This shows that when $(p_0, p_1) \in R_{\mathcal{C}}$, firm 0 can make a gainful deviation. Hence we must have $(p_0, p_1) \in$ Graph τ which, together with $p_0 \in [c_1, c^*]$, proves that $(p_0, p_1) \in$ (Graph $\tau)[c_1, c^*]$. ■

2.5.2 Proof of the Theorem

Proof of (I) This has been proved as (i) of Lemma 4 and (iv) if Lemma 7.

Proof of (II) Consider $c_0 < c^*$. First we show that $q_2^1 = 0$ in any SPNE. For if $q_2^1 > 0$, we must have $(p_0, p_1) \in$ (Graph $\tau)[c_1, c_0]$ by (v) of Lemma 7. Then $p_1 = \tau(p_0) < (a + c_1)/2$ and, by (v) of Lemma 4, firm 1 gets payoff $F(p_1) = F(\tau(p_0))$. Let 1 deviate and choose $p_1' \in (\tau(p_0), (a + c_1)/2]$. Then $(p_0, p_1') \in R_{\mathcal{C}}$ and, by (iv) of Lemma 4, 1 gets payoff $\kappa_1(p_0)$. Since $p_0 \leq c_0 < c^*$, Lemma 10 implies that $\tau(p_0) < \lambda(p_0)$. By the strict monotonicity of F (Lemma 6), it follows that $F(\tau(p_0)) < F(\lambda(p_0))$. By the definition of λ, $F(\lambda(p_0)) = \kappa_1(p_0)$. Since $F(p_1) = F(\tau(p_0))$, we conclude that $\kappa_1(p_0) > F(p_1)$, showing that firm 1 has improved after the deviation, a contradiction.

By (iii) of Lemma 7, $q_2^0 + q_2^1 > 0$. We have just shown that $q_2^1 = 0$. Hence we must have $q_2^0 > 0$ in any SPNE. Then it follows from Lemma 12 that $(p_0, p_1) \in$ (Graph $\tau)[c_1, c^*]$ in any SPNE. By (iv) of Lemma 7, we must have $p_0 \geq c_0$. Since $c_0 < c^*$, the interval $[c_0, c^*]$ is non-empty, hence $(p_0, p_1) \in$ (Graph $\tau)[c_0, c^*]$.

It remains to show that for any $(p_0, p_1) \in$ (Graph $\tau)[c_0, c^*]$ we do get an SPNE with $q_2^0 > 0$.

First consider firm 2. Since $(p_0, p_1) \in$ Graph τ we see (by (v) of Lemma 4) that firm 2 has exactly two optimal choices, which involve exclusive orders from either 0 or 1. Since it is already choosing the former, it cannot profit by a unilateral deviation.

Next consider firm 0. Its payoff is $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$, which is non-negative since $p_0 \geq c_0$. But $(p_0, p_1) \in$ Graph τ, i.e., $p_1 = \tau(p_0)$. If 0 reduces its price from p_0

to $p_0' < c_1$, then (since $c_1 < c_0$), 0 gets at most zero payoff. If 0 reduces its price from p_0 to $p_0' \geq c_1$, then $\tau(p_0') < \tau(p_0) = p_1$. So $(p_0, p_1) \in R_{\mathcal{C}}$ implying (by (iv) of Lemma 4) that 0 will get $(p_0' - c_0)q_2^{\mathcal{C}}(p_0')$. Observe that $(p_0' - c_0)q_2^{\mathcal{C}}(p_0') < (p_0 - c_0)q_2^{\mathcal{C}}(p_0)$ (by Lemma 11 and the fact that $c^* < \tilde{c}$), so again the deviation is not gainful. If 0 increases its price from p_0 to p_0', then $\tau(p_0') > \tau(p_0) = p_1$ and $(p_0, p_1) \in R_{\mathcal{S}}$. Then, by (iii) of Lemma 4, 0 gets zero payoff, again gaining nothing.

Finally consider firm 1. Its payoff is $\kappa_1(p_0)$. Recall that $p_1 = \tau(p_0)$. If 1 raises its price to $p_1' > p_1 = \tau(p_0)$, then $(p_0, p_1') \in R_{\mathcal{C}}$ and, by (iv) of Lemma 4, 1 will still get $\kappa_1(p_0)$. If 1 lowers its price to $p_1' < p_1 = \tau(p_0)$, then $(p_0, p_1') \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 1 will get $F(p_1')$. By the strict monotonicity of F, we have $F(p_1') < F(p_1) = F(\tau(p_0))$. Since $p_0 \leq c^*$, we have $\tau(p_0) \leq \lambda(p_0)$ (Lemma 10), so that $F(\tau(p_0)) \leq F(\lambda(p_0))$. By the definition of λ, $F(\lambda(p_0)) = \kappa_1(p_0)$. Hence we conclude that $F(p_1') < \kappa_1(p_0)$, showing that 1 cannot improve by any unilateral deviation. This completes the proof of part (II).

Proof of (III) Consider $c^* < c_0 < (a + c_1)/2$. If $q_2^0 > 0$, then (a) $p_0 \geq c_0$ (by (iv) of Lemma 7) and (b) $p_0 \leq c^*$ (by Lemma 12). Since $c^* < c_0$, both (a) and (b) cannot hold. So we must have $q_2^0 = 0$. Since $q_2^0 + q_2^1 > 0$ (by (iii) of Lemma 7), we conclude that $q_2^1 > 0$. Then, by (v) of Lemma 7, it follows that $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c_0]$.

It remains to show that for any $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c_0]$ we do get an SPNE with $q_2^0 > 0$.

First consider firm 2. We can argue exactly as in the proof of (I) that it cannot make a gainful unilateral deviation.

Next consider firm 0. Its payoff is zero. Since $p_0 \in [c_1, c_0]$, by lowering its price to $p_0' < p_0 \leq c_0$, it can get at most zero payoff. Since $(p_0, p_1) \in \text{Graph } \tau$, $p_1 = \tau(p_0)$. If 0 it raises its price to $p_0' > p_0$, then $\tau(p_0') > \tau(p_0) = p_1$. Hence $(p_0', p_1) \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 0 continues to get zero payoff.

Finally consider firm 1. Its payoff is $F(p_1) = F(\tau(p_0))$. If 1 lowers its price to $p_1' < p_1 = \tau(p_0)$, then $(p_0, p_1') \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 1 gets $F(p_1')$. By the monotonicity of F, $F(p_1') < F(p_1)$ and so 1 does not profit. If 1 raises its price to $p_1' > p_1 = \tau(p_0)$, then $(p_0, p_1') \in R_{\mathcal{C}}$ and, by (iv) of Lemma 4, 1 gets $\kappa_1(p_0)$. Consider two cases. If $p_0 \geq \tilde{c}$, we have $F(p_0) \geq \kappa_1(p_0)$ by Lemma 8. Since $p_0 < (a + c_1)/2$, we have $\tau(p_0) > p_0$ (see Figure 2) so that $F(\tau(p_0)) > F(p_0)$. Hence $F(\tau(p_0)) > \kappa_1(p_0)$, so 1 does not gain. If $p_0 < \tilde{c}$, we have $\tau(p_0) > \lambda(p_0)$ by Lemma 10, so $F(\tau(p_0)) > F(\lambda(p_0))$. By the definition of λ, $F(\lambda(p_0)) = \kappa_1(p_0)$ and we have $F(\tau(p_0)) > \kappa_1(p_0)$, so once again 1 does not gain. This completes the proof of part (III).

Proof of (IV). The argument is as in parts (II) and (III), hence omitted. ■

2.6 Variations of the model

Our model can be varied in many ways, but the essential theme remains intact: if $\mathcal{O}$'s costs are not much higher than $\mathcal{I}$'s, $\mathcal{J}$ will outsource to $\mathcal{O}$. The overall analysis follows the outline of the proof of Theorem 1, but the details can get more complicated, and we omit them here.

2.6.1 Economies of scale

Keeping the rest of the model fixed as before, now suppose that there are increasing, instead of constant, returns to scale in the manufacture of the intermediate good η, i.e., the average cost $c_i(q)$ of manufacturing q units of η falls (as q rises) for both $i = 0, 1$. For simplicity, suppose $c_i(q)$ falls linearly and that $c_0(q) = \lambda c_1(q)$ for some positive scalar[15] λ. It can then be shown that there exists a threshold $\lambda^* > 1$ such that if $\lambda < \lambda^*$:

(i) firm 2 outsources to firm 0 in any SPNE,

(ii) both firms 1 and 2 outsource to firm 0 in any SPNE when economies of scale are not too small.

This result is established in Chen (2007). (We already gave the intuition for it in the introduction.)[16]

2.6.2 Multiple firms of each type

Suppose there are n_0, n_1, n_2 replicas of firms 0, 1, 2. The timing of moves is assumed to be as before, with the understanding that all replicas of a firm move simultaneously wherever that firm had moved in the original game. Restricting attention to type-symmetric SPNE, Theorem 1 again remains intact with a lower threshold.

2.6.3 Only Outside Suppliers

The strategic incentives that we have analyzed can arise in other contexts. Suppose, for instance, that 1 and 2 both need to outsource the supply of the intermediate good η to outsiders $\mathcal{O} = \{O_1, O_2, \ldots\}$. If 2 goes first to $\mathcal{O}$ and 1 knows which O_i has received 2's order, then 1 will have incentive to outsource to some O_j that is distinct

[15]Thus $c_1(q) = \max\{0, c - bq\}$ and $c_0(q) = \lambda \max\{0, c - bq\}$ for positive scalars b, c, λ.

[16]It is needed here that the economies of scale be not too pronounced, otherwise pure strategy SPNE may fail to exist. More precisely, for the average cost function $c_1(q)$ =max$\{0, c - bq\}$, it is assumed that $0 < b < c/2a$ to guarantee (i) the existence of pure strategy SPNE and (ii) in equilibrium, the quantity produced entails positive marginal cost.

from O_i, even if O_j's costs are higher than O_i's, so long as they are not much higher. For if 1 went to O_i, it might have to infer the size of 2's orders and thus be obliged to become a Stackelberg follower (e.g., because O_i has limited capacity and can attend to 1's order only after fully servicing the prior order of 2). Alternatively, even if 1 does not know who 2 has outsourced to, or indeed if 2 has outsourced at all, it may be safer for 1 to spread its order among several firms in $\mathcal{O}$ so that it minimizes the probability of becoming 2's follower. We leave the precise modeling and analysis of such situations for future research.

2.7 The Secrecy Clause

It is crucial to our analysis that the quantity outsourced by 2 to 0 cannot be observed by 1. This is not an unrealistic assumption. Many contracts, in practice, do incorporate a confidentiality or secrecy clause (see, e.g., Ravenhill, 2003; Clarkslegal and Kochhar, 2005. See also Hart and Tirole (1990) for further justification of the secrecy clause).

But the secrecy clause can often be *deduced* to hold endogenously in equilibrium (in appropriately "enlarged" games).

Indeed suppose that the quantity q outsourced by 2 to 0 can be made "public" (and hence observable by 1) or else kept "secret" between 2 and 0. We argue that a public contract can never occur (be active) at an SPNE, as long as the game provides sufficient "strategic freedom" to its various players. For suppose it did occur : 1 knew that 2 buys q units of η from 0 at price p_0. Thus 1 is a Stackelberg follower in the final market α, regardless of whom 2 chooses to outsource η to. It would be better for 1 to quote a lower price $p_0 - \varepsilon$ for η. This would be certain to lure 2 to outsource to 1. But $p_0 \geq c_0$, since 0 could not be making losses at the presumed SPNE; hence $p_0 - \varepsilon > c_1$ for small enough ε (recall $c_0 > c_1$). By manoeuvering 2's order to itself, firm 1 thus earns a significant profit on the manufacture of η. It does lose a little on the market for α, because 2 has a lower cost $p_0 - \varepsilon$ of η (compared to the p_0 earlier), but the loss is of the order of ε. Thus 1 has made a profitable unilateral deviation, contradicting that we were at an SPNE.

Note that our argument relies on the fact that 1 has the strategic freedom to "counter" the public contract. If, furthermore, 0 also has the freedom to reject the public contract and counter it with a secret contract, then—foreseeing the above deviation by firm 1—firm 0 will only opt for secret contracts.

The most simple instance of such an enlarged game is obtained by inserting an initial binary move by 0 at the start of our game Γ. This represents a declaration by 0 as to whether its offer to 2 is by way of a public or a secret contract. The game Γ follows 0's declaration. It is easy to verify that any SPNE of the enlarged game must have 0 choosing "secret", followed by an SPNE of Γ. Of course, more complicated enlarged games can be thought of. For example, after the simultaneous announcement of p_0 and p_1 in our game Γ, suppose firm 2 has the option to choose “Public q” or ”Secret q” in the event that it goes to 0, followed by “Accept” or “Reject” by 0. Clearly 1 finds out q only if “Public q” and “Accept” are chosen. On the other hand, if 0 chooses “Reject” we (still having to complete the definition of the enlarged game) could suppose that 2’s order of η is automatically directed to 1. This game is more complex to analyze, but our argument above still applies and shows that a public contract will never be played out in any SPNE.

We thus see that the secrecy clause can often emerge endogenously from strategic considerations, even though—for simplicity—we postulated it in our model. It has been pointed out already by Clarkslegal and Kochhar that the firm placing orders (firm 2 in our model) may demand secrecy in order to protect sensitive information from leaking out to its rivals and destroying its competitive advantage. Our analysis reveals that the firm *taking* the orders (i.e., firm 0) may also—for more subtle strategic reasons—have a vested interest in maintaining the secrecy clause.

Chapter 3

Outsourcing under Strategic Competition and Economies of Scale

3.1 Introduction

In Chen, Dubey and Sen (2005), we show that the second mover's disadvantage incurred through outsourcing for the supplier when outsourcing occurs between competitors, leads to a higher price charged by an inside provider (a firm who is competing the purchaser in their final products) compared to an outside provider (a firm who is not competing the purchaser). The upshot is that in equilibrium outsourcing occurs only between non-competitors, with the inside provider producing by itself.

Suppose there are two duopolists for a final product, one of which (referred to as the inside provider) can also produce the required intermediate product, while the other one can not (referred to as firm A). There also exists a firm who can only produce the intermediate product (the outside provider). The inside and outside providers are price competing to provide firm A in the outsourcing market, after that firm A and the inside provider Cournot compete in their final products. Chen, Dubey and Sen (2005) show that, the strategic reason for firm A to go to the outside provider for the intermediate product, is that the inside provider is charging a price high enough to drive it away, in order to avoid the second-mover's disadvantage.

However, with economies of scale exist for the production technology in the intermediate good, new strategic consideration arises and things become unclear. The strategic role of outsourcing stated above is still there, while economies of scale impose different incentives to different players in the outsourcing game. For the inside provider, now it is more willing to provide firm A, since by doing so its average cost can be driven down, which strengthens itself for the final product competition. However, firm A has less incentive to outsource to the inside provider, fearing that by doing so it will face a stronger competitor in the future. For the outside provider, economies of scale may give it incentive to set its price so low, if then both firm A and the inside provider would outsource to it. The total effect from economies of scale on the equilibrium outsourcing strategy, is ambiguous and not yet investigated.

Since economies of scale are big reasons for outsourcing, it is important to explore the strategic role of outsourcing under strictly concave technologies. This model shows that, when both the inside and the outside providers produce the intermediate good with scale economies, firm A still outsource to the outside provider. Moreover, economies of scale will drive the inside provider to outsource also to the outside provider rather than producing, and this is true in any equilibrium if the economies of scale are not too small.

Following are two real observations. In 1980's, General Electric (GE) wanted to outsource its manufacture of microwave ovens because its production cost was higher than its Japanese competitors. Discussions were held with Matsushita, the world technology leader in microwave ovens. However, due to the fact that Matsushita is one of GE's major competitors, it turned out that at last GE outsourced to Samsung, a small Korean company at that time with limited experience in microwave production. In order to guarantee the quality, GE had to sent out American engineers to Korea to help (Domberger(1998)). GE's choice can not be easily justified only from the cost side. It is the competition between GE and other lower cost microwave oven producers which drives GE turn to another firm (Samsung), who was at that time outside of GE's final product market.

Such a strategic consideration occurs in another case, given by Jarillo(1993). Sharp has tried to tout its video cameras by fitting them with a color visor, which the other manufactures can not produce economically and are buying from Sharp. By doing so, Sharp will jeopardize its strong leadership in selling the color displays, the activity which can make money. That is, by competing those manufacturers in their final products, Sharp may be inadvertently zapping its competitiveness and may also strongly encouraging other competitors to enter the color display business.

In the following part of this work, Section 2 is the benchmark model. Section 3 depicts our major findings, then Section 4 gives all proofs. Section 5 checks the robustness of the major findings. Section 6 gives a modified game together with the major conclusions. Section 7 concludes.

3.2 A Game with Imperfect Information

3.2.1 The Model

Two Cournot duopolists, firm 1 and firm 2, are competing in the final product denoted as good F. The unique intermediate component for producing good F is good

I, which firm 1 can also produce inside but firm 2 can not.[1] To produce good F, firm 2 can either outsource to firm 1 for good I, or it can outsource to firm 0, a provider of good I who is not producing good F. We call firm 1 as an *inside provider* for good I and firm 0 an *outside provider* for good I.

Assume one unit of good I can be converted into one unit of good F. Firm 1 and firm 2 have the same linear technology in producing good F from good I, with their average cost for this procedure w.l.o.g normalized to zero. The inverse demand for good F is $P = a - Q$ for $Q \leq a$. Assume that firm 0 and firm 1 are symmetric in producing good I, with their cost function given by

$$C_i(q) = \begin{cases} bq - cq^2 & \text{for } q \leq \dfrac{b}{2c} \\ \dfrac{b^2}{4c} & \text{for } q > \dfrac{b}{2c} \end{cases} \qquad i = 0, 1$$

Therefore, they both have economies of scale for a limited quantity. We make the following assumption:

$$A1. \quad 0 < b < a < \frac{b}{2c}.$$

$A1$ guarantees that a monopolist with the above cost function is profitable to produce positive quantity. Moreover, with $a < \frac{b}{2c}$, the production of good I is always under a strictly concave technology. Also note that $A1$ implies

$$0 < c < \frac{b}{2a} < \frac{1}{2}, \tag{3.1}$$

which is sufficient for the existence and uniqueness of a Cournot-Nash equilibrium between firm 1 and firm 2.[2]

The game consists of three stages:

In stage one, firm 0 and firm 1 simultaneously announce prices $\{p_0, p_1\}$ for providing good I.

In stage two, firm 2 chooses a provider between firm 0 and firm 1, together with the quantity to outsource. To simplify, assume that firm 2 will outsource to either firm 0 or firm 1, but not to both. Binding contracts are signed between firm 2 and the chosen provider.

In stage three, firm 1 determines either to produce inside or to outsource to firm 0, together with its corresponding quantity. Assume that firm 1 will either produce fully inside or outsource fully to firm 0.

Note that since firm 1 is competing firm 0 in providing firm 2, it will wait until firm 2 has made its decision about to which one to outsource. After that firm 1 will

[1] Assuming that firm 2 can also produce I but with a very high cost compared to firm 1 will not change anything.

[2] See ‘Oligopoly pricing, old ideas and new tools’, by Xavier Vives.

choose between either producing inside or outsourcing to firm 0. If firm 2 outsources to firm 1, firm 1 willy-nilly observes the quantity outsourced by firm 2. Instead, if firm 2 outsources to firm 0, firm 1 knows that it has turned to firm 0 for good I, but does not know firm 2's quantity. In the first case, firm 2's quantity observed by firm 1 acts as a commitment so that firm 1 has to accommodate when setting its own quantity, which grants firm 2 a leader's status. Therefore, a Stackelberg leader-follower relationship arises. However, in the second case when firm 2 outsources to firm 0, there is a Cournot competition for good F between firm 1 and firm 2, since neither of them observes the other's quantity in advance. In other words, firm 1 has imperfect information when firm 2 outsources to firm 0. Denote this game as $\Gamma^{imp}(a, b, c)$.

Let *outsourcing decision* in particular denote the choice of firms 1 and 2 about to which one to outsource. Denote $d_i, i = 1, 2$ as the outsourcing decision such that

$$d_i = \begin{cases} 0, & \text{if firm i outsources to firm 0} \\ 1, & \text{if firm i outsources to firm 1} \end{cases} \qquad i = 1, 2.$$

Note that firm 1 outsources to firm 1 means that firm 1 produces inside. The combination of firm 1 and firm 2's outsourcing decisions is called as *outsourcing pattern*, represented by

$$D = \{d_1 d_2\}.$$

For example, $D = 10$ means that firm 1 is producing inside and firm 2 is outsourcing to firm 0. There are four outsourcing patterns, i.e. $D \in \{11, 10, 00, 01\}$. In each outsourcing pattern, denote the quantity produced for good F for each firm as $q_i^D, i = 1, 2$, and the profit for each firm as $\pi_i^D, i = 0, 1, 2$.

We employ subgame perfect Nash equilibrium (SPNE) to solve for this game, under the proviso that no firm will use dominated strategies in equilibrium.

From firm 0's respect, its sole means of winning non-negative profit is through providing firm 1 or (and) firm 2 the intermediate product. With firm 1 competing it in the outsourcing market, firm 0 will charge p_0 low, but no lower than the level that through providing the intermediate product its profit is negative. Moreover, with economies of scale for good I, it can be more profitable for firm 0 to decrease p_0 to a level such that both firms 1 and 2 outsource to firm 0, instead of providing firm 2 only with a higher p_0.

For firm 1, there are two elements which affect its decision on p_1. On the one side, providing firm 2 incurs a follower's disadvantage for firm 1, therefore firm 1 has incentive to charge a high price in order to drive firm 2 away. On the other side, with economies of scale, providing firm 2 can help firm 1 to achieve a lower average cost, which in turn strengthens its competitive status for the final product. These two effects offset each other and the total effect is ambiguous.

Therefore, firm 2's outsourcing decision will not be clear. On the one side, firm

2 is attracted by the leader's advantage it can get by outsourcing to firm 1. On the other side, firm 2 can possibly get a lower price for the intermediate product if it outsources to firm 0.

With linear cost of the intermediate product, we have shown that in equilibrium firm 2 outsources to firm 0 (Chen, Dubey and Sen, (2006)). The sole reason there is that firm 1 will charge a high enough p_1 to ward firm 0 off, in order to avoid the follower's disadvantage. With economies of scale for both firms 0 and 1, we find that this effect is still there, moreover, when economies of scale is not too small, the incentive to fully explore economies of scale will drive both firms 1 and 2 to outsource to firm 0. Our major findings are listed out in Section 3.

3.2.2 Model Analysis

Backward Induction in Each Outsourcing Pattern

Figure 3.1 gives the reduced extensive form of $\Gamma^{imp}(a, b, c)$ in stage two and three, according to different outsourcing patterns. If firm 2 outsources to firm 1, i.e. $D = 11$ or $D = 01$, quantities are set by a Stackelberg sequential play; if firm 2 outsources to 0, i.e. $D = 00$ or $D = 10$, quantities are set by a Cournot simultaneous play. Optimal quantities in each outsourcing pattern, $q_i^D(p_0, p_1), i = 1, 2$, are solved by first order conditions and illustrated by Figure 3.2. Details are in the appendix.

The optimal profits for firm 1 and firm 2 in each outsourcing pattern are graphically shown by Figure 3.3 (calculation is in the appendix). When $D = 11$, p_1 is the relevant price for good I. If p_1 is so low that $p_1 \leq p_{1z}$ holds, firm 1 will drop out from the market for good F and focus in providing firm 2. If $p \geq \bar{p}_1$, firm 2 will drop out and firm 1 becomes a monopoly in the market for good F. Only when $p_{1z} < p_1 < \bar{p}$, both firm 1 and firm 2 are active in producing good F. Let firm 1 and firm 2's optimal profits when both of them are active in producing good F be $\pi_{1f}^{11}(p_1)$ and $\pi_{2l}^{11}(p_1)$, respectively, with f and l referring to their follower and leader's status. We have that $\pi_{1f}^{11}(p_1)$ is increasing and strictly concave in p_1; $\pi_{2l}^{11}(p_1)$ is decreasing and strictly convex in p_1. Moreover, when $p_1 \leq p_{1z}$, firm 1's optimal profit $\pi_1^{11}(p_1)$ is negative and strictly increasing in p_1.

When $D = 10$, denote $\pi_{1c}^{11}(p_1)$ and $\pi_{2c}^{11}(p_1)$ as firm 1 and firm 2's optimal profits when they are both active for producing good F. The subscript c refers to their Cournot competition in this outsourcing pattern. Furthermore, $\pi_{1c}^{10}(p_0)$ is convex and increasing in p_0; $\pi_{2c}^{10}(p_0)$ is concave and decreasing in p_0.

When $D = 00$, firm 1 and firm 2 are symmetric and engage in Cournot competition. They are producing positive quantities for good F as long as $p < a$, with corresponding optimal profits $\pi_{1c}^{00}(p_0) = \pi_{2c}^{00}(p_0) = \pi_c^{00}(p_0)$. Here $\pi_c^{00}(p_0)$ is strictly concave and decreasing in p_0.

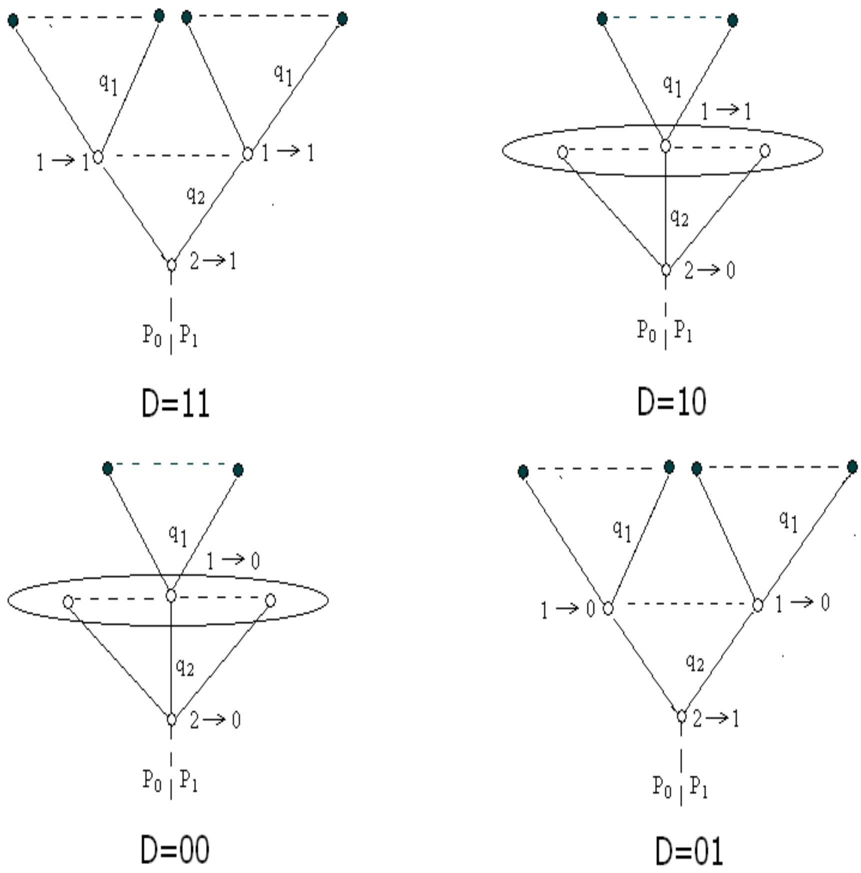

Figure 3.1: Reduced extensive form game for each outsourcing pattern

When $D = 01$, both p_0 and p_1 are relevant to determine firm 1 and firm 2's profits. The regime of $\{p_0, p_1\}$ for both of them to be active in producing good F is given by $z(p_0) < p_1 < h(p_0)$, see appendix. Denote the firms' optimal profits in this regime as $\pi_{1f}^{01}(p_0, p_1)$ and $\pi_{2l}^{01}(p_0, p_1)$, respectively. As long as $p_0 < a$, it is true that $\pi_{1f}^{01}(p_0, p_1)$ is concave and increasing in p_1, convex and decreasing in p_0; $\pi_{2l}^{01}(p_0, p_1)$ is convex and increasing in p_0, convex and decreasing in p_1.

Intuitively, $D = 01$ is unlikely to be in any SPNE. Since firm 2 would not like to strengthen firm 1, its final-product competitor, through outsourcing to firm 1 and in turn helping firm 1 to decrease firm 1's average cost, the reason for firm 2 to outsource to firm 1 must be that p_1 is sufficiently low. However, firm 1 outsources to firm 0 instead of producing inside means that p_0 is even lower. Therefore firm 2 should outsource to firm 0, instead of firm 1. Lemma 7 shows that this intuition is correct and is proved in appendix.

Lemma 7 *Under $A1$, $D = 01$ can not be in any SPNE.* □

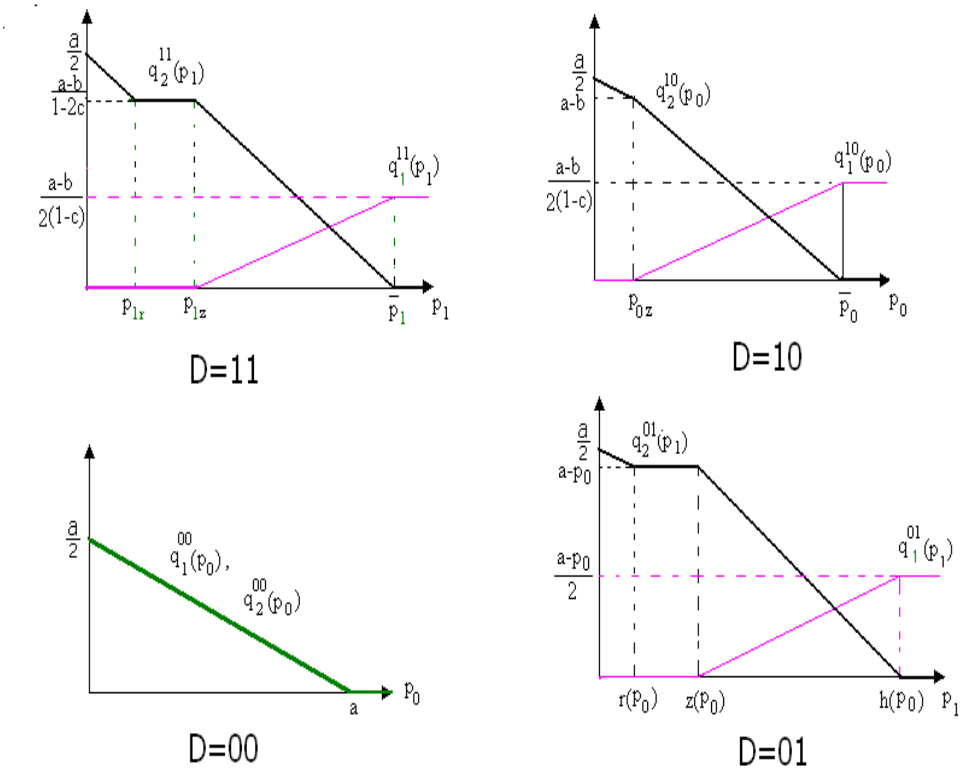

Figure 3.2: Optimal quantities for firm 1 and firm 2 in each outsourcing pattern

Firm 0's Profit in Each Outsourcing Pattern

The sole means for firm 0 to achieve a positive profit is through providing either firm 1 or firm 2. According to Lemma 7, firm 0 can get a positive profit only with either $D = 00$ or $D = 10$. Its corresponding profits in these outsourcing patterns, given by $\pi_0^{10}(p_0)$ and $\pi_0^{00}(p_0)$, are shown by Figure 3.4. Basic observations are collected below, with p_0^{10} the value of p_0 maximizing $\pi_0^{10}(p_0)$, p_0^{00} the value of p_0 maximizing $\pi_0^{00}(p_0)$.

Observation 1 $\pi_0^{10}(p_0) < 0$ *for* $p_0 < \underline{p}_0$, $\pi_0^{10}(\underline{p}_0) = 0$, $\pi_0^{10}(p_0) = 0$ *for* $p_0 \geq \bar{p}_0$. *Moreover,* $\pi_0^{10}(p_0)$ *is strictly concave and increasing in* $p_0 \in [\underline{p}_0, p_0^{10}]$.

Observation 2 $\pi_0^{00}(p_0) < 0$ *for* $p_0 < \underline{\underline{p}}_0$, $\pi_0^{00}(\underline{\underline{p}}_0) = 0$, $\pi_0^{00}(p_0) = 0$ *for* $p_0 \geq a$. *Moreover,* $\pi_0^{00}(p_0)$ *is strictly concave and increasing in* $p_0 \in [\underline{\underline{p}}_0, p_0^{00}]$.

Observation 3 $a > p_0^{00} > \bar{p}_0 > p_0^{10} > \underline{p}_0 > \underline{\underline{p}}_0 > 0$.

Here, $\underline{p}_0$ solves $\pi_0^{10}(p_0) = 0$ and is given by

$$\underline{p}_0 = \frac{2ac^2 + 3b - 5bc - ac}{2c^2 - 6c + 3};$$

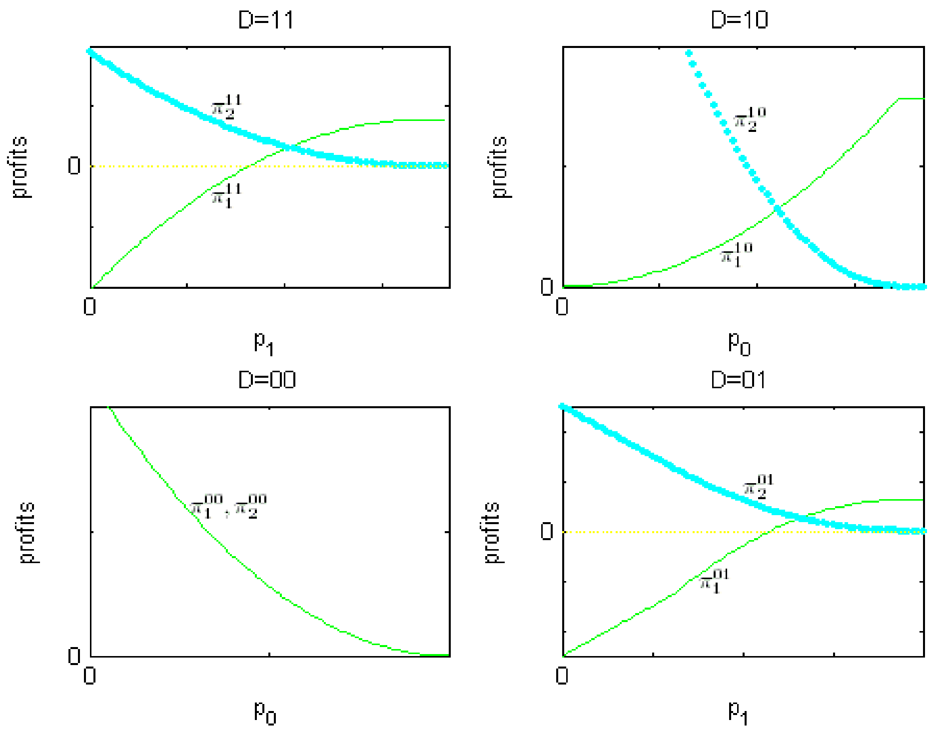

Figure 3.3: Optimal profits for firms 1 and 2 in each outsourcing pattern. The graph for $D = 01$ is drawn according to a fixed p_0.

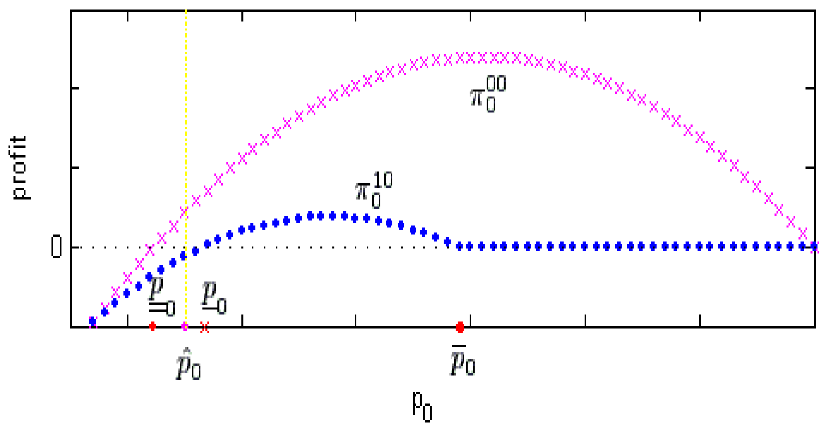

Figure 3.4: Firm 0's profits when $D = 00$ and $D = 10$

$\underline{\underline{p}}_0$ solves $\pi_0^{00}(p_0) = 0$, given as

$$\underline{\underline{p}}_0 = \frac{3b - 2ac}{3 - 2c}.$$

3.3 Main Results

Define

$$\hat{p}_0 \equiv \frac{3a(1-\sqrt{1-c})+6b\sqrt{1-c}-4ac}{3\sqrt{1-c}+3-4c}$$

such that $\pi_{1c}^{10}(\hat{p}_0) = \pi_c^{00}(\hat{p}_0)$. Thus, $\hat{p}_0$ gives the value of p_0 at which firm 1 is indifferent between $d_1 = 1$ and $d_1 = 0$, when it is expecting $d_2 = 0$ in the last stage.

The value of p_1 at which firm 2 is indifferent between $d_2 = 0$ when $D = 00$ ensues, and $d_2 = 1$ when $D = 11$ ensues, is given by a function $\beta(p_0)$. Define

$$\Omega_0^\beta \equiv [\underline{\underline{p}}_0, \hat{p}_0], \quad \Omega_1^\beta \equiv [p_{1z}, \bar{p}_1],$$

and

$$\beta(p_0) \equiv \pi_2^{11^{-1}} \circ \pi_2^{00}(p_0) \equiv \frac{3(a+b-2ac)-2(a-p_0)\sqrt{2(1-c)}}{6(1-c)}.$$

Both $\pi_2^{00}(p_0)$ and $\pi_2^{11}(p_1)$ are monotonic, besides, $\pi_2^{11}(\bar{p}_1) < \pi_2^{00}(\hat{p}_0), \pi_2^{11}(p_{1z}) > \pi_2^{00}(\underline{\underline{p}}_0)$. Therefore $\beta(p_0)$ is well defined.

The value of p_1 at which firm 2 is indifferent between $d_2 = 0$ when $D = 10$ ensues, and $d_2 = 1$ when $D = 11$ ensues, is given by a function $\alpha(p_0)$. Define

$$\Omega_0^\alpha \equiv [\hat{p}_0, \bar{p}_0], \quad \Omega_1^\alpha \equiv [p_{1z}, \bar{p}_1].$$

$\pi_2^{11}(p_1)$ is strictly decreasing in Ω_1^α, $\pi_2^{10}(p_0)$ is strictly decreasing in Ω_0^α. Moreover, $\pi_2^{11}(p_{1z}) > \pi_2^{10}(\hat{p}_0), \pi_2^{11}(\bar{p}_1) = \pi_2^{10}(\bar{p}_0)$. Therefore, $\alpha(p_0)$ is well-defined:

$$\alpha(p_0) \equiv \pi_2^{11^{-1}} \circ \pi_2^{10} \equiv \frac{a+b-2ac}{2(1-c)} - \frac{2(a+b-2ac-2p_0+2cp_0)\sqrt{2(1-c)}}{2(3-4c)(1-c)}.$$

Also define

$$\underline{c} = 0.07010997262, \tilde{c} \equiv 0.1808334279.$$

Theorem 3 *Under assumption A1, there exist SPNE of $\Gamma^{imp}(a,b,c)$ indexed by E^{00} and E^{10}.*
I. In E^{00}, provider prices satisfy

$$\{(p_0, p_1 = \beta(p_0)) : p_0 \in [\underline{\underline{p}}_0, \hat{p}_0)\}$$

or

$$\begin{cases} (\hat{p}_0, p_1 \in [\beta(\hat{p}_0), p_1^c]) & \textit{if} \quad c < \tilde{c} \\ (\hat{p}_0, p_1 \geq \beta(\hat{p}_0)) & \textit{o.w.} \end{cases}$$

and both firms 1 and 2 outsource to firm 0.
II. In E^{10}, provider prices satisfy

$$\{(p_0, p_1 = \alpha(p_0)) : p_0 \in [p_0^c, \tilde{p}_0^{10}]\},$$

for $c \leq \underline{c}$, and firm 2 outsources to firm 0 while firm 1 produces inside. □

In the above theorem, p_0^c lies between $\underline{p}_0$ and $\tilde{p}_0^{10}$ for $c < \underline{c}$, and $p_0^c = \tilde{p}_0^{10}$ at $c = \underline{c}$. Moreover, $p_1^c = \alpha(p_0^c)$.

Corrolary 1 *Under assumption A1, there does not exist any SPNE in which firm 1 and firm 2 both outsource to firm 1.* □

Corrolary 2 *Under assumption A1, there does not exist other SPNE for $\Gamma^{imp}(a, b, c)$ than E^{00} and E^{10}.* □

The SPNE are shown by the thick lines in Figure 3.5 in the regime diagram.

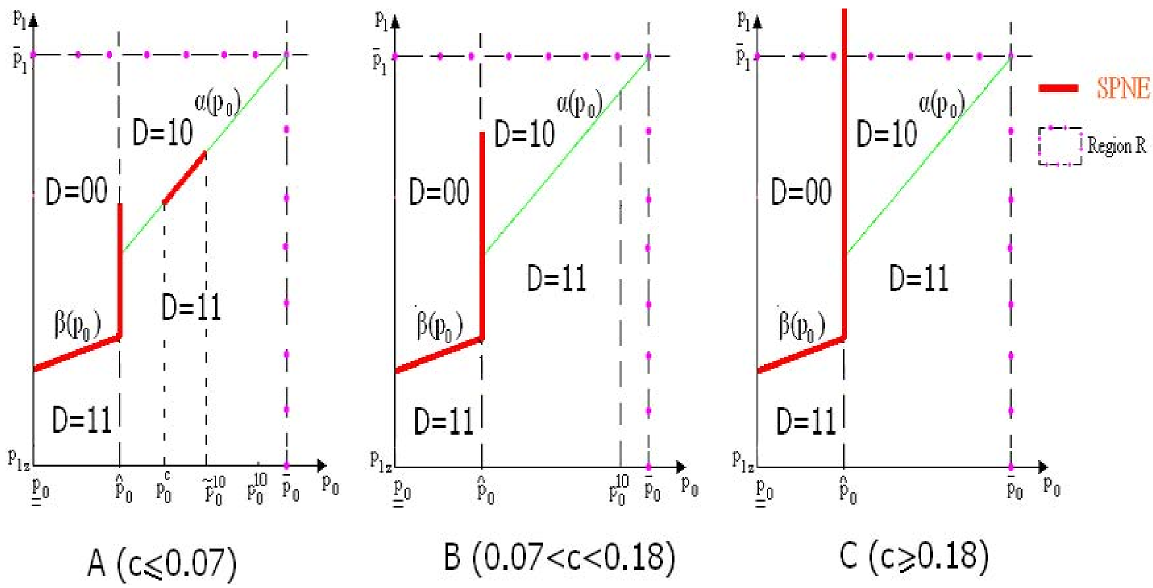

Figure 3.5: SPNEs for $\Gamma^{imp}(a, b, c)$

Although our conclusion above is derived from the assumption that firm 0 and firm 1 have the same cost for the intermediate product, it is still true even when firm 0 has moderate cost disadvantage compared to firm 1. Assume that 0's cost function is

$$C_0(q) = \lambda C_1(q), \quad \text{with } \lambda \in [1, \frac{a}{b}). \tag{3.2}$$

By assuming $\lambda < \frac{a}{b}$, A1 is true with firm 0's new cost function. Denote this new game in which firm 0 has cost disadvantage as $\Gamma^{imp}(a, b, c, \lambda)$.

When firm 0 has a higher cost, the lower bound of p_0 for firm 0 to be willing to provide will increase. Now the lowest p_0 for firm 0 to provide in any SPNE depends on the value of λ, denoted by $\underline{p}_0(\lambda)$ for E^{10} and $\underline{\underline{p}}_0(\lambda)$ for E^{00}, both are increasing in λ. On the other side, $p_0^c(\lambda)$ is decreasing in λ, and $\hat{p}_0, \tilde{p}_0^{10}$ are fixed by firm 1's strategy. Define

$$\lambda^{00} \equiv \frac{3a\sqrt{1-c} - 6b\sqrt{(1-c)} - 3a + 4ac}{4ac\sqrt{1-c} - 3b\sqrt{1-c} - 3b + 4bc - 4bc\sqrt{1-c}},$$

$$\lambda^{10} \equiv \frac{12ac^2 - 12ac - 14bc + a + 13b}{2(1-c)(7b - 4bc - 2ac)}.$$

We have

$$\hat{p}_0 \geq \underline{\underline{p}}_0(\lambda) \Leftrightarrow \lambda \leq \lambda^{00}, \tilde{p}_0^{10} \geq \underline{p}_0(\lambda) \Leftrightarrow \lambda \leq \lambda^{10}.$$

λ^{00} is increasing in c whereas λ^{10} is decreasing in c. Furthermore, we have

$$\lim_{c\to 0} \lambda^{00} = 1, \ \lim_{c \to 0} \lambda^{10} = \frac{a + 13b}{14b},$$
$$\lambda^{10}|_{c=\hat{c}} = 1, \ \lambda^{00}|_{c=\frac{b}{2a}} < \frac{a}{b}.$$

At a given $\lambda > 1$, by comparing $\pi_0^{10}(\tilde{p}_0^{10}, \lambda)$ and $\pi_0^{00}(\hat{p}_0, \lambda)$, we can find the upper bound of c as $\underline{c}(\lambda)$, at which these two are equal, as the necessary condition for $D = 10$ to be in SPNE. Define $\tilde{\lambda} \equiv \underline{c}^{-1}(\lambda)$. At a given value of c, $\tilde{\lambda}$ solves

$$\pi_0^{10}(\tilde{p}_0^{10}) = \pi_0^{00}(\hat{p}_0).$$

Since λ^{00} solves

$$\pi_0^{10}(\underline{\underline{p}}_0) = \pi_0^{00}(\hat{p}_0),$$

λ^{10} solves

$$\pi_0^{10}(\tilde{p}_0^{10}) = \pi_0^{00}(\underline{p}_0),$$

these three cures, $\lambda^{00}, \lambda^{10}, \tilde{\lambda}$, intersect at the same value of c. See Figure 3.6.

Theorem 4 *(Figure 3.6)Under A1 and (3.3), as long as $\lambda < \max\{\lambda^{10}, \lambda^{00}\}$, firm 2 outsources to firm 0 in any SPNE for $\Gamma^{imp}(a, b, c, \lambda)$.* □

3.4 Proof

3.4.1 Backward Induction Cross Patterns

Profits of firm 1 and firm 2 are fixed in each outsourcing pattern by backward induction. For a given combination of $\{p_0, p_1\}$, firm 2 compares its profits between

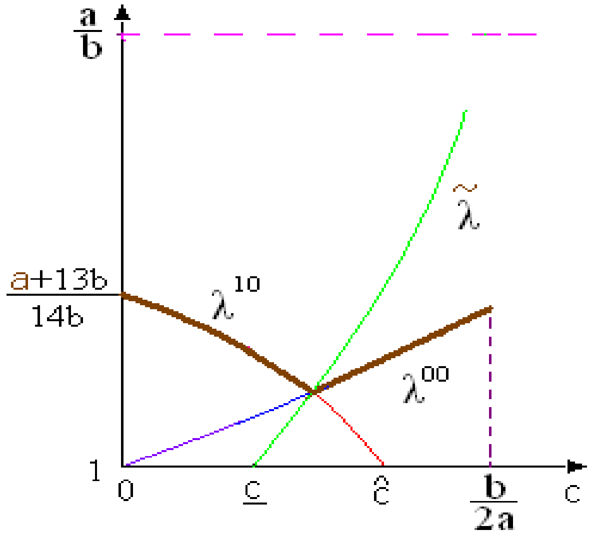

Figure 3.6: λ^{00} and λ^{10}.

$d_2 = 0$ and $d_2 = 1$ to determine its optimal strategy, while correctly anticipating firm 1's future response. After that, firm 1 compares its profits between $d_1 = 0$ and $d_1 = 1$, then picks up the one which yields it a higher profit. Therefore, firm 1 and firm 2 are doing backward induction cross different outsourcing patterns. Since $D = 01$ is off equilibrium, there are three outsourcing patterns as the candidates for SPNE. We begin from the last stage.

Strategies in Stage three

If $d_2 = 1$, by Lemma 7, it must be $d_1 = 1$ for any SPNE. If $d_2 = 0$, firm 1 compares $\pi_1^{10}(p_0)$ and $\pi_1^{00}(p_0)$ in order to make its outsourcing decision. We have that $\pi_{1c}^{10}(p_0)$ is strictly increasing in p_0 and $\pi_c^{00}(p_0)$ is strictly decreasing in p_0, and their unique intersection occurs at $p_0 = \hat{p}_0$. Define

$$\hat{p}_0 \equiv \frac{3a(1-\sqrt{1-c}) + 6b\sqrt{1-c} - 4ac}{3\sqrt{1-c} + 3 - 4c}$$

such that $\pi_{1c}^{10}(\hat{p}_0) = \pi_c^{00}(\hat{p}_0)$.

Lemma 8 *When $d_2 = 0$, in any SPNE $d_1 = 1$ if $p_0 > \hat{p}_0$, $d_1 = 0$ if $p_0 < \hat{p}_0$. At $p_0 = \hat{p}_0$, firm 1 is indifferent between $d_1 = 0$ and $d_1 = 1$.*

Proof: If $p_0 \geq \bar{p}_0$, $\pi_1^{10}(p_0) = \pi_{1M}^{10}$, $\pi_1^{00}(p_0) = \pi_c^{00}(p_0) \leq \frac{(a-b)^2}{36(1-c)^2} < \pi_{1M}^{10}$. Firm 1 strictly prefers $d_1 = 1$. If $p_0 \leq p_{0z}$, $\pi_1^{10}(p_0) = 0$, $\pi_1^{00}(p_0) = \pi_{1c}^{10}(p_0) > 0$, firm 1 strictly prefers $d_1 = 0$. If $p_{0z} < p_0 < \bar{p}_0$, $\pi_1^{10}(p_0) = \pi_{1c}^{10}(p_0)$, $\pi_1^{00}(p_0) = \pi_c^{00}(p_0)$.

By definition of $\hat{p}_0$, $\pi_c^{00}(p_0) > \pi_{1c}^{10}(p_0)$ when $p_0 < \hat{p}_0$, and firm 1 prefers $d_1 = 0$; $\pi_c^{00}(p_0) < \pi_{1c}^{10}(p_0)$ when $p_0 > \hat{p}_0$, and firm 1 prefers $d_1 = 1$. When $p_0 = \hat{p}_0$, firm 1 is indifferent. ∎

The relationship between $\pi_{1c}^{10}(p_0)$ and $\pi_c^{00}(p_0)$ is shown by See Figure 3.7; Lemma 8 is illustrated by Figure 3.8.

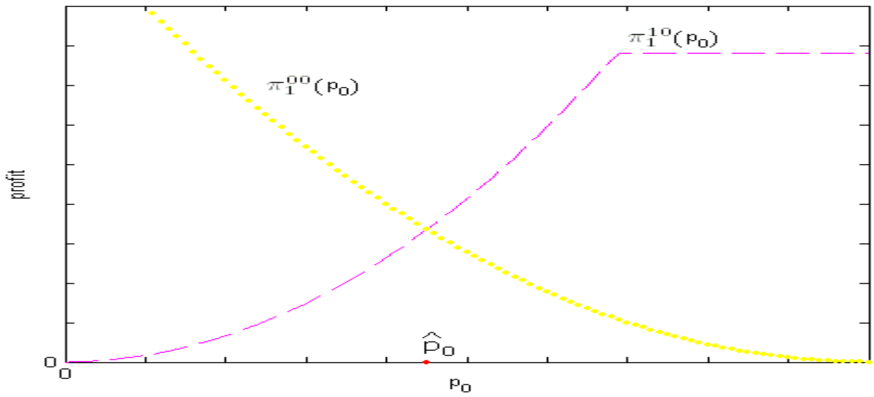

Figure 3.7: Parameters are set as $a = 10, b = 5, c = 0.2$.

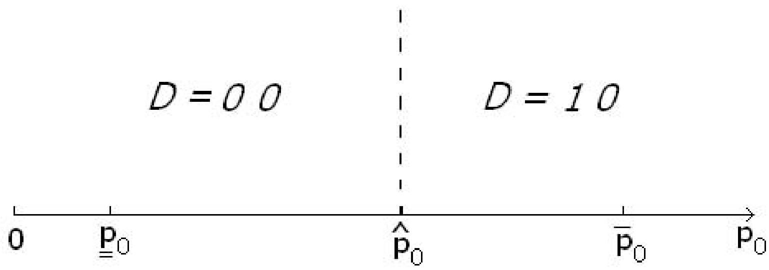

Figure 3.8: Lemma 8.

The intuition of Lemma 8 is clear. Given that firm 2 has outsourced to firm 0, firm 1 will also outsource to firm 0 only when p_0 is low enough. The threshold at which firm 1 is indifferent between $d_1 = 1$ and $d_1 = 0$ is given by $\hat{p}_0$. Therefore, for $p_0 < \hat{p}_0$, $D = 00$ and $D = 11$ are the candidates for SPNE; for $p_0 > \hat{p}_0$, $D = 10$ and $D = 11$ are the candidates. Furthermore, with the observation below, we have Lemma 9 to give the value of p_0 and the value of p_1 which are dominated.

Observation 4 $\underline{p}_0 > \hat{p}_0 > \underline{\underline{p}}_0 > p_{0z}$.

Lemma 9 *The following statements are true for any SPNE:*
i. $p_0 < \underline{\underline{p}}_0$ *is weakly dominated for firm 0;*

ii. $p_0 > \bar{p}_0$ *is weakly dominated for firm 0;*
iii. $p_1 < p_{1z}$ *is weakly dominated for firm 1;*
iv. Restricting p_1 *to* $p_1 \leq \bar{p}_1$ *does not affect the SPNE outsourcing pattern.* □

Define region R by $R \equiv \{(p_0, p_1) | \underline{p}_0 \leq p_0 \leq \bar{p}_0, p_{1z} \leq p_1 \leq \bar{p}_1\}$. Looking for undominated SPNE means that we need to check combinations of $\{p_0, p_1\}$ which falls into this region. From now on our analysis will focus on region R.

Strategies in Stage Two

Firm 2's is maximizing its profit by choosing its outsourcing decision and the corresponding quantity, while foreseeing firm 1's strategy in stage three.

■ $p_0 \in [\underline{p}_0, \hat{p}_0)$

SPNE candidates are $D = 00$ and $D = 11$. For any announcement of $\{p_0, p_1\}$, it is evident that firm 2 sets $d_2 = 0$ if $\pi_2^{00}(p_0) > \pi_2^{11}(p_1)$ and $d_2 = 1$ if $\pi_2^{00}(p_0) < \pi_2^{11}(p_1)$. When equality occurs, firm 2 is indifferent between $d_2 = 0$ and $d_2 = 1$, and $\beta(p_0)$ gives the value of p_1 which makes firm 2 indifferent. In other words, for any p_0 incurred as its cost in the Cournot game with $D = 00$, $\beta(p_0)$ is the maximum cost p_1 that firm 2 is willing to bear in order to switch to the Stackelberg game with $D = 11$, in which firm 2 acts as a leader but at the same time firm 1's average cost is lower. The situation is depicted in Figure 3.9 and spelled out below.

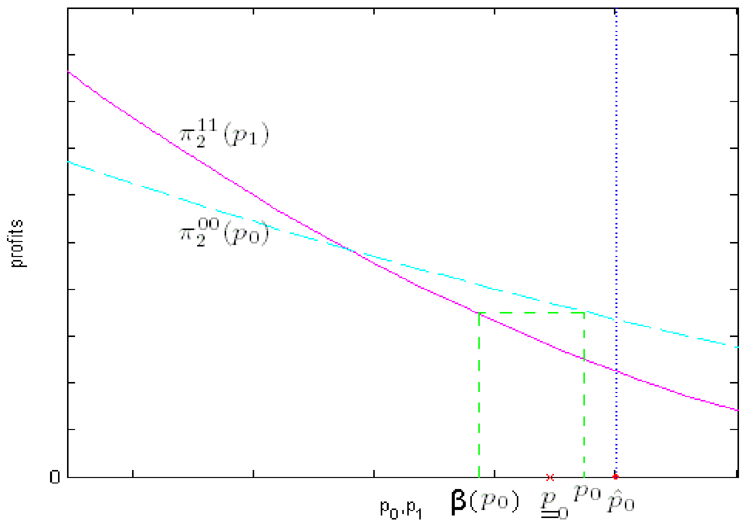

Figure 3.9: $\pi_2^{00}(p_0)$ and $\pi_2^{11}(p_1)$

Define

$$\Omega_0^\beta \equiv [\underline{p}_0, \hat{p}_0], \quad \Omega_1^\beta \equiv [p_{1z}, \bar{p}_1],$$

and

$$\beta(p_0) \equiv \pi_2^{11^{-1}} \circ \pi_2^{00}(p_0) \equiv \frac{3(a+b-2ac) - 2(a-p_0)\sqrt{2(1-c)}}{6(1-c)}.$$

Both $\pi_2^{00}(p_0)$ and $\pi_2^{11}(p_1)$ are monotonic, besides, $\pi_2^{11}(\bar{p}_1) < \pi_2^{00}(\hat{p}_0), \pi_2^{11}(p_{1z}) > \pi_2^{00}(\underset{=0}{p})$. Therefore $\beta(p_0)$ is well defined.

Lemma 10 *In region R, $p_1 = \beta(p_0)$ is a strictly increasing function from Ω_0^β to Ω_1^β. Moreover, for all $p_0 \in \Omega_0^\beta$,*
i. $\pi_2^{11}(\beta(p_0)) = \pi_2^{00}(p_0)$;
ii. $\pi_2^{11}(p_1) > \pi_2^{00}(p_0)$ if $p_1 < \beta(p_0)$ and $\pi_2^{11}(p_1) < \pi_2^{00}(p_0)$ if $p_1 > \beta(p_0)$;
iii. $d_2 = 0$ if $p_1 > \beta(p_0)$; $d_2 = 1$ if $p_1 < \beta(p_0)$. At $p_0 = \beta(p_0)$, firm 2 is indifferent between $d_2 = 0$ and $d_2 = 1$.

Proof: Straightforward calculation. ∎

■ $p_0 \in (\hat{p}_0, \bar{p}_0]$

SPNE candidates include $D = 10$ and $D = 11$. The value of p_1 at which firm 2 is indifferent between $d_2 = 0$ and $d_2 = 1$, is given by a function $\alpha(p_0)$. Define

$$\Omega_0^\alpha \equiv [\hat{p}_0, \bar{p}_0], \quad \Omega_1^\alpha \equiv [p_{1z}, \bar{p}_1].$$

$\pi_2^{11}(p_1)$ is strictly decreasing in Ω_1^α, $\pi_2^{10}(p_0)$ is strictly decreasing in Ω_0^α. Moreover, $\pi_2^{11}(p_{1z}) > \pi_2^{10}(\hat{p}_0), \pi_2^{11}(\bar{p}_1) = \pi_2^{10}(\bar{p}_0)$. Therefore, $\alpha(p_0)$ is well-defined:

$$\alpha(p_0) \equiv \pi_2^{11^{-1}} \circ \pi_2^{10} \equiv \frac{a+b-2ac}{2(1-c)} - \frac{2(a+b-2ac-2p_0+2cp_0)\sqrt{2(1-c)}}{2(3-4c)(1-c)}.$$

For any p_0 incurred as its cost in the Cournot game with $D = 10$, $\alpha(p_0)$ is the maximum cost p_1 that firm 2 is willing to bear in order to switch to the Stackelberg game with $D = 11$. Lemma 11 and Figure 3.10 below have this situation illustrated.

Lemma 11 *In region R, $p_1 = \alpha(p_0)$ is a strictly increasing function from Ω_0^α to Ω_1^α. Moreover, from all $p_0 \in \Omega_0^\alpha$,*
i. $\pi_2^{11}(\alpha(p_0)) = \pi_2^{10}(p_0)$;
ii. $\pi_2^{11}(p_1) > \pi_2^{10}(p_0)$ if $p_1 < \alpha(p_0)$ and $\pi_2^{11}(p_1) < \pi_2^{10}(p_0)$ if $p_1 > \alpha(p_0)$;
iii. $d_2 = 0$ if $p_1 > \alpha(p_0)$; $d_2 = 1$ if $p_1 < \alpha(p_0)$. At $p_0 = \alpha(p_0)$, firm 2 is indifferent between $d_2 = 0$ and $d_2 = 1$.

Proof: Straightforward calculation. ∎

Basic observations on $\alpha(p_0), \beta(p_0)$ are collected below.

Observation 5 $\alpha(\hat{p}_0) > \beta(\hat{p}_0) > p_{1z}$; $\frac{d\alpha(p_0)}{dp_0} > \frac{d\beta(p_0)}{dp_0} > 0$; $\alpha(\bar{p}_0) = \bar{p}_1, \beta(p_{0z}) > p_{1z}$.

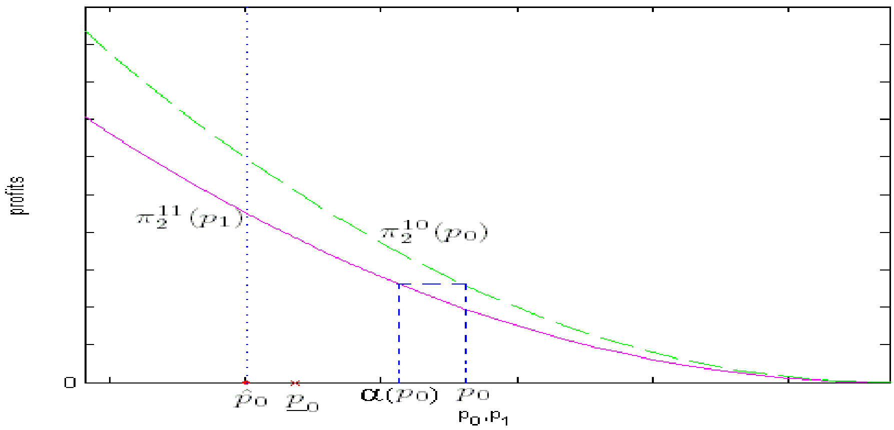

Figure 3.10: $\pi_2^{10}(p_0)$ and $\pi_2^{11}(p_0)$

■ $p_0 = \hat{p}_0$

Given that $d_2 = 0$, since $\pi_1^{10}(\hat{p}_0) = \pi_1^{00}(\hat{p}_0)$, firm 1 is indifferent between $d_1 = 0$ and $d_1 = 1$, and firm 2 knows this.

Lemma 12 *Suppose* $p_0 = \hat{p}_0$*. If* $p_1 \leq \beta(\hat{p}_0)$*,* $d_2 = 1$*; if* $p_1 \geq \alpha(p_0)$*,* $d_2 = 0$*; if* $\beta(p_0) < p_1 < \alpha(p_0)$*, it is either* $d_2 = 0$ *or* $d_2 = 1$*.*

Proof: By Lemma 7, if $d_2 = 1$, then firm 2's ensuing profit is $\pi_2^{00}(\hat{p}_0)$. By Lemma 8, given that $d_2 = 0$, firm 1 is indifferent between $d_1 = 0$ and $d_1 = 1$, thus firm 2's ensuing profit is either $\pi_2^{10}(\hat{p}_0)$ or $\pi_2^{00}(\hat{p}_0)$. Follows from Observation 5, Lemma 10 and Lemma 11, if $p_1 \leq \beta(\hat{p}_0)$, we have $\pi_2^{10}(\hat{p}_0) < \pi_2^{11}(\hat{p}_0), \pi_2^{00}(\hat{p}_0) \leq \pi_2^{11}(\hat{p}_0)$, thus $d_2 = 1$ is a dominant strategy for firm 2; if $p_1 \geq \alpha(\hat{p}_0)$, we have $\pi_2^{10}(\hat{p}_0) \geq \pi_2^{11}(\hat{p}_0), \pi_2^{00}(\hat{p}_0) > \pi_2^{11}(\hat{p}_0)$, thus $d_2 = 0$ is a dominant strategy for firm 2. For $\beta(p_0) < p_1 < \alpha(p_0)$, $\pi_2^{10}(\hat{p}_0) < \pi_2^{11}(\hat{p}_0) < \pi_2^{00}(\hat{p}_0)$, thus in pure strategy, firm 2 will either play $d_2 = 0$ if it believes $d_1 = 0$, or play $d_2 = 1$ if it believes $d_1 = 1$. ■

The Regime Diagram

The SPNE outsourcing pattern candidates for different regimes of (p_0, p_1) are stated by Lemma 8, Lemma 10 and Lemma 11. To tell if they are the outcome of backward induction in stage two and three, one more thing left to check is, given that $d_2 = 1$, wether or not will firm 1 deviate from $d_1 = 1$ to $d_1 = 0$. The reason is, although $D = 01$ is ruled out from the SPNE candidates, it is possible that in some regime of prices, after firm 2 has outsourced to firm 1, firm 1 is optimal setting $d_1 = 0$. That is, with $d_2 = 1$ under a low p_1, if p_0 is not too high, it can be more profitable for firm 1 to outsource to firm 0 in stead of producing inside. Whenever this happens in region R, it will disqualify $D = 11$ as an SPNE candidate and hence question our analysis above. Therefore it is important to find the regime in which firm 1 will choose $d_1 = 0$ after $d_0 = 1$.

When $D = 01$, both firm 1 and firm 2 are producing positive quantities only when $z(p_0) < p_1 < h(p_0)$. Observations on these two functions are collected below.

Observation 6 *Both $h(p_0), z(p_0)$ are strictly increasing in p_0. Moreover, $h(\underline{\underline{p}}_0) > \bar{p}_1$, $p_{1z} < z(p_0^{00}) < \bar{p}_1$, $p_{1z} < z(\hat{p}_0) < \beta(\hat{p}_0)$.*

Connecting with observation 5, both $\beta(p_0)$ and $\alpha(p_0)$ lie between $z(p_0)$ and $h(p_0)$. Define

$$\Omega_0^f \equiv [\underline{\underline{p}}_0, \bar{p}_0], \;\; \Omega_1^f \equiv \{p_1 | z(p_0) \leq p_1 \leq \bar{p}_1, p_0 \in \Omega_0^f.\}$$

Figure 3.11 illustrates profits of firm 1 when $D = 11$ and $D = 01$. For $p_1 \in \Omega_1^f$, $\pi_1^{11}(p_1)$ is strictly increasing and concave in p_1, shown by the solid line; $\pi_1^{01}(p_0, p_1)$ is strictly increasing in p_1, shown by three dashed lines. Furthermore, $\pi_1^{01}(p_0, p_1)$ is strictly decreasing in p_0. Define

$$f(p_1) \equiv \pi_1^{01^{-1}} \circ \pi_1^{11}(p_1) \equiv 7a - 6p_1 - \sqrt{\frac{\Delta}{1-c}},$$

with $\Delta = 24ac^2p_1 - 12a^2c^2 - 12c^2p_1^2 + 60acp_1 - 36a^2c - 12bcp_1 + 12abc - 24cp_1^2 + 36p_1^2 - 84ap_1 + b^2 + 12bp_1 + 49a^2 - 14ab$. Because $\pi_1^{11}(\bar{p}_1) < \pi_1^{01}(\bar{p}_1, \underline{\underline{p}}_0)$, $\pi_1^{11}(z(\bar{p}_0)) > \pi_1^{01}(z(\bar{p}_0), \bar{p}_0)$, $\pi_1^{11}(z(\underline{\underline{p}}_0)) > \pi_1^{01}(z(\underline{\underline{p}}_0), \underline{\underline{p}}_0)$, $f(p_1)$ is well defined.

Lemma 13 *(Figure 3.11) $p_0 = f(p_1)$ is a strictly increasing function from Ω_1^f to Ω_0^f. Moreover, for all $p_1 \in \Omega_1^f$,*
i. $\pi_1^{11}(p_1) = \pi_1^{01}(f(p_1), p_1)$;
ii. $\pi_1^{11}(p_1) > \pi_1^{01}(p_0, p_1)$ if $p_0 > f(p_1)$ and $\pi_1^{11}(p_1) < \pi_1^{01}(p_0, p_1)$ if $p_0 < f(p_1)$;
iii. Suppose $d_2 = 1$. Then $d_1 = 0$ if $p_0 < f(p_1)$ and $d_1 = 1$ if $p_0 > f(p_1)$. When $p_0 = f(p_1)$, firm 1 is indifferent between $d_1 = 0$ and $d_1 = 1$.

Proof: Straight forward calculation. ∎

Observation 7. $\underline{\underline{p}}_0 < f(\bar{p}_1) < \hat{p}_0$, $f(p_{1z}) < \underline{\underline{p}}_0$. Moreover, $f(\beta(\underline{\underline{p}}_0)) < \underline{\underline{p}}_0$.

The last inequality in observation 7 is important. It tells us that both $\beta(p_0)$ and $\alpha(p_0)$ lie to the right of $p_0 = f(p_1)$, so that the SPNE candidates solved above are valid for region R except the northwest corner. As will be shown below, the northwest corner of R defined by $\{(p_0, p_1) | \underline{\underline{p}}_0 \leq p_0 \leq f(p_1), p_{1z} \leq p_1 \leq \bar{p}_1\}$, is irrelevant to our analysis. Therefore, the region of (p_0, p_1) relevant to our SPNE searching is reduced to the one containing $\beta(p_0)$ and $\alpha(p_0)$, defined by

$$R' \equiv \{(p_0, p_1) | p_0 \in [\max\{f(p_1), \underline{\underline{p}}_0\}, \bar{p}_0], p_1 \in [p_{1z}, \bar{p}_1]\}.$$

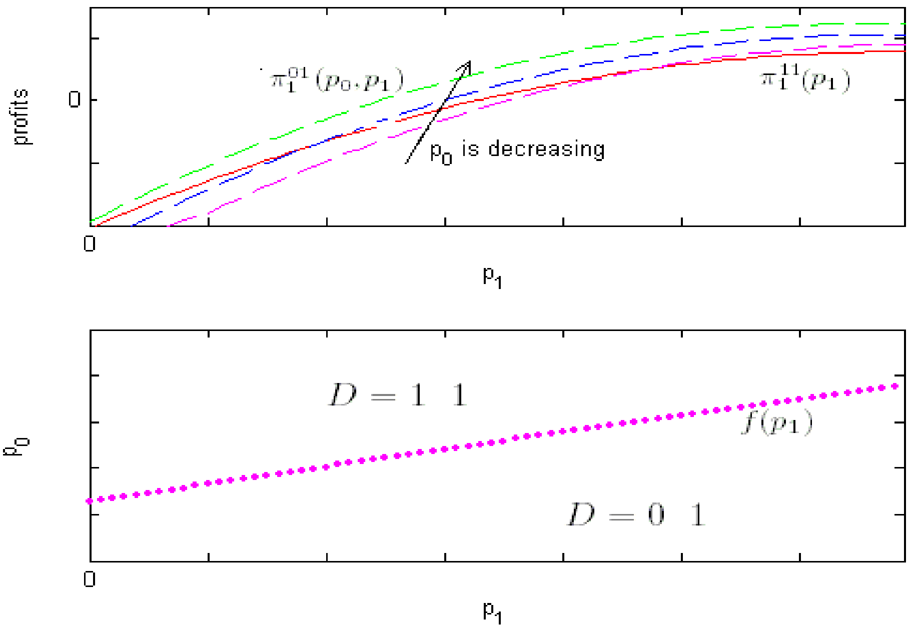

Figure 3.11: $\pi_1^{11}(p_1)$, $\pi_1^{01}(p_0, p_1)$ and $f(p_1)$.

It turns out that in R' whenever $d_2 = 1$, $d_1 = 1$ must be true in any SPNE. Thus for R' the SPNE candidates we found above is the outcome of backward induction for stage two and three. Lemma 14 and Figure 3.12 depict this conclusion.

Lemma 14 *(Figure 3.12)For $(p_0, p_1) \in R'$, the SPNE outsourcing patterns for subgame in stage two and three are:*
i. When $p_0 > \hat{p}_0$, $D = 10$ if $p_1 > \alpha(p_0)$; $D = 11$ if $p_1 < \alpha(p_0)$; if $p_1 = \alpha(p_0)$, $D = 10$ and $D = 11$ are the multiple equilibria outsourcing pattern.
ii. When $p_0 < \hat{p}_0$, $D = 00$ if $p_1 > \beta(p_0)$; $D = 11$ if $p_1 < \beta(p_0)$; if $p_1 = \beta(p_0)$, $D = 00$ and $D = 11$ are the multiple equilibria outsourcing pattern.
iii. When $p_0 = \hat{p}_0$, $D = 11$ if $p_1 < \beta(\hat{p}_0)$; if $\beta(\hat{p}_0) \leq p_1 < \alpha(\hat{p}_0)$, there are two pure strategy equilibria outsourcing pattern, $D = 00$ and $D = 11$; at $p_1 = \alpha(\hat{p}_0)$, there are three pure strategy equilibria outsourcing pattern, $D = 00$, $D = 10$ and $D = 11$; if $p_1 > \alpha(\hat{p}_0)$, there are two pure strategy equilibria outsourcing pattern, $D = 00$ and $D = 10$.

Proof: Comes from Lemma 8, Lemma 10, Lemma 11, Lemma 13, and observation 7. At $p_0 = \hat{p}_0$, when $\beta(\hat{p}_0) < p_1 \leq \alpha(\hat{p}_0)$, firm 2's strategy is either $d_2 = 0$ or $d_2 = 1$, according to its correct anticipation of firm 1's choice between $d_1 = 0$ and $d_1 = 1$; when $p_1 > \alpha(\hat{p}_0)$, $d_2 = 0$, then firm 1 is indifferent between $d_1 = 0$ and $d_1 = 1$. ∎

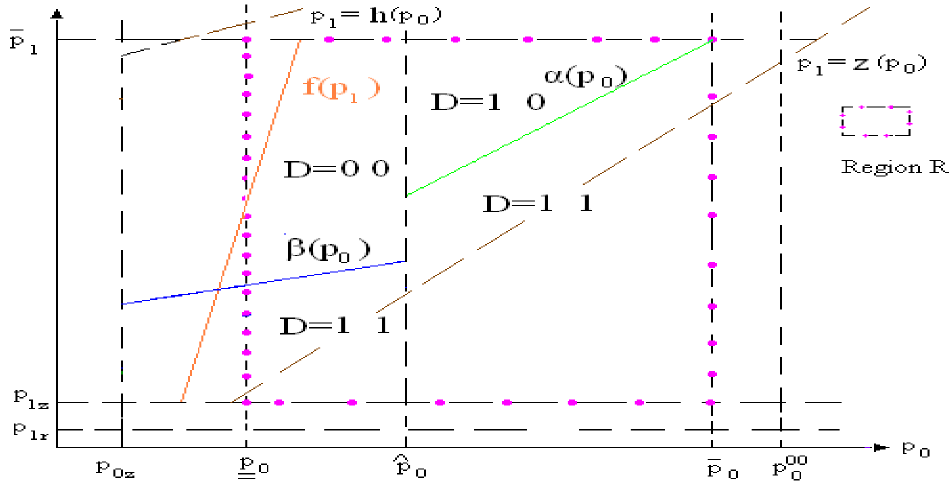

Figure 3.12: The Regime Diagram.

3.4.2 Strategies in Stage One

From firm 1's respect, there are two strategic considerations underlying its determination of p_1. On the one side, under economies to scale firm 1 wants to provide 2, not only to make profits from the intermediate market, but also to decrease its average cost for the final product. On the other side, being a provider incurs firm 1 the follower's disadvantage, which dampens firm 1's incentive to compete firm 0 hard in the intermediate market. Thus firm 1 is willing to supply firm 2 only when the first effect dominates the second, i.e. when p_1 is high enough. However, in this scenario firm 0 will cut p_1 off by charging a low p_0 to attract firm 2, and firm 1, due to the second effect, may not be willing to further decrease p_1 in their price competition. Furthermore, when firm 0's economies of scale are not too small, firm 0 may be willing to charge a p_0 which is also attractive to firm 1, since now providing both firm 1 and firm 2 can be more profitable for firm 0. In this case, the SPNE outcome will be that both firm 1 and firm 2 outsource to firm 0.

Detailed analysis for the equilibrium value of p_1 is done according to either $p_0 > \hat{p}_0$ or $p_0 \leq \hat{p}_0$.

■ When $p_0 > \hat{p}_0$.

Suppose $p_0 > \hat{p}_0$. In this regime, $\alpha(p_0)$ gives the threshold of p_1 between the SPNE outsourcing patterns $D = 10$ and $D = 11$. Firm 1 faces a tradeoff between a higher profit in the intermediate market when $D = 10$, and a higher profit in the final-product market when $D = 11$. For a given p_0, the value of p_1 at which firm 1's gain through providing exactly remedies its loss, is given by $\lambda(p_0)$. In other words, $\lambda(p_0)$ is the lowest p_1 at which firm 1 is willing to supply firm 2. Define $\lambda(p_0) : \Omega_0^\alpha \to \Omega_1^\alpha$ as

$$\lambda(p_0) \equiv \pi_1^{11^{-1}} \circ \pi_1^{10}(p_0)$$
$$\equiv \frac{a+b-2ac}{2(1-c)} - \frac{\sqrt{3}\sqrt{(a+b-2ac-2p_0+2cp_0)(8bc-7b+2p_0-2cp_0+5a-6ac)}}{3(1-c)(3-4c)}.$$

With $\pi_1^{11}(p_1)$ strictly increasing in p_1, $\pi_1^{10}(p_0)$ strictly decreasing in p_0, $\pi_1^{11}(\bar{p}_1) = \pi_1^{10}(\bar{p}_0)$, $\pi_1^{11}(p_{1z}) < \pi_1^{10}(\hat{p}_0)$, $\lambda(p_0)$ is well defined. Note that $\lambda(\bar{p}_0) = \bar{p}_1$, $\lambda(\hat{p}_0) > p_{1z}$.

Lemma 15 *For $p_0 \in \Omega_0^\alpha$, $\lambda(p_0) : \Omega_0^\alpha \to \Omega_1^\alpha$ is strictly increasing in p_0. Moreover, $\pi_1^{11}(\lambda(p_0)) = \pi_1^{10}(p_0)$, $\pi_1^{11}(p_1) > \pi_1^{10}(p_0)$ for $p_1 > \lambda(p_0)$ and $\pi_1^{11}(p_1) < \pi_1^{10}(p_0)$ for $p_1 < \lambda(p_0)$.*

Proof: Straight forward calculation. ∎

Lemma 16 *Suppose $p_0 > \hat{p}_0$. In any SPNE if $D = 11$, then*
i. $p_1 = \alpha(p_0)$; ii. $p_1 \geq \lambda(p_0)$.

Proof: i. By Lemma 11, it must be true that $p_1 \leq \alpha(p_0)$ if in any SPNE $D = 11$. Suppose $p_1 < \alpha(p_0)$. By increasing p_1 to $p_1 + \epsilon < \alpha(p_0)$, the SPNE is still $D = 11$ but firm 1 improves since $\pi_1^{11}(p_1)$ is strictly increasing for $p_1 \leq \bar{p}_1$. A contradiction.

ii. Suppose $p_1 < \lambda(p_0)$. By Lemma 15, $\pi_1^{11}(p_1) < \pi_1^{10}(p_0)$, thus firm 1 is better off not providing firm 2 by deviating to $p_1 \geq \bar{p}_1$. A contradiction. ∎

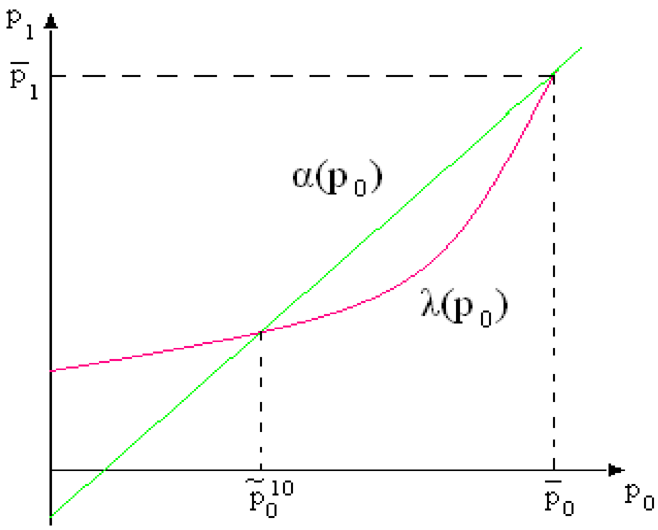

Figure 3.13: $\alpha(p_0)$ and $\lambda(p_0)$.

Figure 3.13 shows the relationship between $\alpha(p_0)$ and $\lambda(p_0)$. At $p_0 = \tilde{p}_0^{10}$, $\alpha(p_0) = \lambda(p_0)$, with $\tilde{p}_0^{10}$ defined by

$$\tilde{p}_0^{10} \equiv \frac{a + 13b - 14bc - 12ac + 12ac^2}{2(1-c)(7-6c)}.$$

Lemma 17 *For $p_0 \in \Omega_0^\alpha$, $\alpha(p_0) < \lambda(p_0)$ for $p_0 < \tilde{p}_0^{10}$, $\alpha(p_0) > \lambda(p_0)$ for $\tilde{p}_0^{10} < p_0 < \bar{p}_0$, and $\alpha(p_0) = \lambda(p_0)$ for $p_0 = \tilde{p}_0^{10}$ or $p_0 = \bar{p}_0$.*

Proof: Follows from definitions of $\alpha(p_0)$ and $\lambda(p_0)$. ∎

Define

$$\hat{c} \equiv \frac{2-\sqrt{2}}{4} \approx 0.15.$$

Observation 8. $\beta(\hat{p}_0) < \tilde{p}_0^{10} < p_0^{10} < \bar{p}_0$; $\tilde{p}_0^{10} > \underline{p}_0$ if and only if $c < \hat{c}$.

Lemma 18 *For $p_0 \in \Omega_0^\alpha$, in any SPNE if $D = 10$, then*
i. $p_1 = \alpha(p_0)$; ii. $p_1 \leq \lambda(p_0)$; iii. $c \leq \hat{c}$.

Proof: i. By Lemma 11, it must be true that $p_1 \geq \alpha(p_0)$ if in any SPNE $D = 10$. Suppose $p_1 > \alpha(p_0)$. By observation 1, firm 0 is better off increasing p_0 to $p_0' = p_0 + \epsilon$ such that $p_1 \geq \alpha(p_0')$, $D = 10$ is still the SPNE but firm 0 has a higher profit. A contradiction.

ii. Suppose $p_1 > \lambda(p_0)$. Let firm 1 decrease p_1 to $p_1' = p_1 - \epsilon$, so that $p_1' > \lambda(p_0)$ and $p_1' < \alpha(p_0)$ are satisfied. By Lemma 14, $D = 11$ is the SPNE outsourcing pattern, and firm 1 is better off by Lemma 15. A contradiction.

iii. From i, ii and Lemma 17, if in any SPNE $D = 10$, it must be true that $p_0 \leq \tilde{p}_0^{10}$. On the other side, for firm 0 to be willing to provide firm 2, $p_0 \geq \underline{p}_0$ must hold. By observation 8, it must be that $c \leq \hat{c}$. ∎

Furthermore, by observation 5, given $p_1 = \alpha(p_0)$, firm 0 by deviating to $p_0 \leq \hat{p}_0$ can get $D = 00$ to be the SPNE outsourcing pattern. If in any SPNE $D = 10$, it must be that firm 0 has no incentive to lower p_0 in order to provide more with both firm 1 and firm 2 as its purchasers. By Observation 2, $\pi_0^{00}(p_0)$ is strictly increasing for $p_0 \leq \hat{p}_0$. Connecting with Lemma 14, if firm 0 deviates, it should set p_0 arbitrarily close to $\hat{p}_0$ to guarantee that $D = 00$. Take $p_0 = \hat{p}_0$ in deviation. Notice that $\pi_0^{10}(p_0)$ is strictly increasing in p_0 for $p_0 \leq \tilde{p}_0^{10}$. Therefore, the sufficient condition for $D = 10$ to be an SPNE outsourcing pattern, is that $\pi_0^{10}(\tilde{p}_0^{10}) \geq \pi_0^{00}(\hat{p}_0)$, otherwise firm 0 for sure deviates to $p_0 = \hat{p}_0$.

Define

$$\underline{c} = 0.07010997262.$$

Observation 9. $\pi_0^{10}(\underline{p}_0) < \pi_0^{00}(\hat{p}_0)$; $\pi_0^{10}(\tilde{p}_0^{10}) \geq \pi_0^{00}(\hat{p}_0)$ if and only if $c \leq \underline{c}$.

The value of $\pi_0^{00}(\hat{p}_0) - \pi_0^{10}(\tilde{p}_0^{10})$ is strictly increasing in c. For $c \leq \underline{c}$, there exists a $p_0 \in (\underline{p}_0, \tilde{p}_0^{10}]$ which solves $\pi_0^{10}(p_0) = \pi_0^{00}(\hat{p}_0)$. Define the solution as $p_0^c : (0, \underline{c}] \to (\underline{p}_0, \tilde{p}_0^{10}]$:

$$p_0^c \equiv \pi_0^{10^{-1}} \circ \pi_0^{00}(\hat{p}_0),$$

which is strictly increasing in c. Note that $p_0^c = \tilde{p}_0^{10}$ at $c = \underline{c}$.

Lemma 19 *For $p_0 \in \Omega_0^\alpha$, if $D = 10$ is an SPNE outsourcing pattern, it must be that $c \leq \underline{c}$, with $p_0 \in [p_0^c, \tilde{p}_0^{10}]$.*

Proof: Suppose in some SPNE $D = 10$ and $c > \underline{c}$. By Lemma 18, $p_1 = \alpha(p_0), p_0 \leq \tilde{p}_0^{10}$ and $c \leq \hat{c}$. Note that $\underline{c} < \hat{c}$. By Observation 5, any p_1 given by $\alpha(p_0), p_0 \in \Omega_0^\alpha$ is bigger than any p_1 given by $\beta(p_0), p_0 \in \Omega_0^\beta$. By observation 9, $\pi_0^{10}(p_0) < \pi_0^{00}(\hat{p}_0)$ for all $p_0 \in [\hat{p}_0, \tilde{p}_0^{10}]$, hence firm 0 will deviate from $p_0 \in [\hat{p}_0, \tilde{p}_0^{10}]$ to $p_0 = \hat{p}_0 - \epsilon$, with ϵ arbitrarily small. By Lemma 14 the ensuing outsourcing pattern after deviation is $D = 00$, and firm 0 is better off. A contradiction. Furthermore, under $c \leq \underline{c}$, $\pi_0^{10}(p_0) \geq \pi_0^{00}(\hat{p}_0)$ only when $p_0 \in [p_0^c, \tilde{p}_0^{10}]$. If $p_0 < p_0^c$ firm 0 will again deviate to $p_0 = \hat{p}_0 - \epsilon$. Thus for $D = 10$ to be an SPNE outsourcing pattern, it must be $c \leq \underline{c}$, $p_0 \in [p_0^c, \tilde{p}_0^{10}]$. ∎

Since $\underline{c} < \hat{c}$, Lemma 19 gives a more restrictive condition on c than Lemma 18, for $D = 10$ to be an SPNE outsourcing pattern.

■ When $p_0 \leq \hat{p}_0$

Firm 1 compares $\pi_1^{00}(p_0)$ and $\pi_1^{11}(p_1)$ to determine p_1. Since $\pi_1^{11}(p_1)$ is strictly increasing in $p_1 \in \Omega_1^\beta$, $\pi_1^{00}(p_0)$ is strictly decreasing in $p_0 \in \Omega_0^\beta$, $\pi_1^{00}(\underline{\underline{p}}_0) < \pi_1^{11}(\bar{p}_1)$, $\pi_1^{00}(\hat{p}_0) > \pi_1^{11}(p_{1z})$, the function $\gamma(p_0) : \Omega_0^\beta \to \Omega_1^\beta$ is well defined:

$$\begin{aligned}\gamma(p_0) &\equiv \pi_1^{11^{-1}} \circ \pi_1^{00}(p_0) \\ &\equiv \frac{9(a + b - 2ac) - 2\sqrt{\Delta'}}{18(1 - c)},\end{aligned}$$

with $\Delta' = 15a^2 - 54ab + 12ca^2 + 27b^2 + 24ap_0 - 24acp_0 + 12cp_0^2 - 12p_0^2$.

Lemma 20 *For $p_0 \in \Omega_0^\beta$, $\gamma(p_0) : \Omega_0^\beta \to \Omega_1^\beta$ is strictly decreasing in p_0. Furthermore, $\pi_1^{11}(\gamma(p_0)) = \pi_1^{00}(p_0)$, $\pi_1^{11}(p_1) > \pi_1^{00}(p_0)$ if $p_1 > \gamma(p_0)$ and $\pi_1^{11}(p_1) < \pi_1^{00}(p_0)$ if $p_1 < \gamma(p_0)$.*

Proof: Straight forward calculation. ∎

We begin from the subcase when $p_0 < \hat{p}_0$.

Lemma 21 *Suppose $p_0 < \hat{p}_0$. In any SPNE if $D = 00$, then*
i. $p_1 = \beta(p_0)$; ii. $p_1 \leq \gamma(p_0)$.

Proof: i. By Lemma 10, it must be true that $p_1 \geq \beta(p_0)$ if in any SPNE $D = 00$. Suppose $p_1 > \beta(p_0)$. Then by observation 2, firm 0 can improve by increasing p_0 to $p_0' = p_0 + \epsilon$, with ϵ small so that $p_1 \geq \beta(p_0')$ to guarantee $D = 00$. A contradiction.

ii. Suppose $p_1 > \gamma(p_0)$. Let firm 1 decrease p_1 to $p_1' = p_1 - \epsilon$, so that $p_1' > \gamma(p_0)$ and $p_1' < \beta(p_0)$ are achieved. By Lemma 10, in SPNE $D = 11$ and by Lemma 20, firm 1 is better off. A contradiction. ∎

Lemma 22 *(Figure 3.14)For $p_0 \in \Omega_0^\beta$, $\beta(p_0) < \gamma(p_0)$.*

Proof: Follows from Lemma 10, Lemma 20, and the observation $\gamma(\hat{p}_0) > \beta(\hat{p}_0)$. ∎

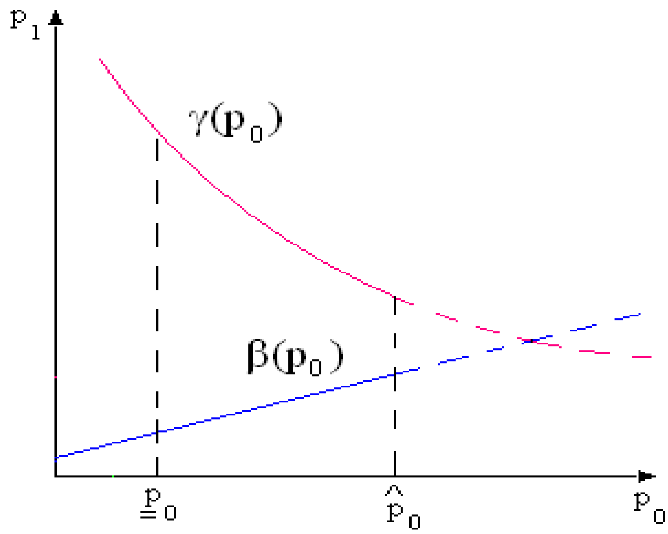

Figure 3.14: $\beta(p_0)$ and $\gamma(p_0)$.

By Lemma 22, as long as $p_1 = \beta(p_0)$, it must be true that $p_1 < \gamma(p_0)$. I.e. in Lemma 21, part ii is implied by part i.

Secondly, suppose $p_0 = \hat{p}_0$. By Lemma 14, pure strategy equilibrium outsourcing pattern exists for $p_1 \leq \beta(\hat{p}_0)$, with the ensuing $D = 11$ ensuing, or $p_1 \geq \alpha(p_0)$, with $D = 00$ or $D = 10$.

Lemma 23 *There does not exist any SPNE in which $p_0 = \hat{p}_0$, then $D = 11$ or $D = 10$.*

Proof: Firstly, suppose $p_0 = \hat{p}_0$ and $D = 11$ is in SPNE, by Lemma 14, it must be $p_1 \leq \alpha(\hat{p}_0)$. If $p_1 \leq \beta(\hat{p}_0)$, by Lemma 15, Lemma 20 and Lemma 22, firm 1 is at least better off deviating to $p_1 = \bar{p}_1$ to achieve either $D = 10$ or $D = 00$, a contradiction. If $p_1 \in (\beta(\hat{p}_0), \alpha(\hat{p}_0)]$, then firm 0 will deviate to p_0 a little bit lower for $D = 00$, a contradiction. Secondly, suppose $p_0 = \hat{p}_0$ and $D = 10$ is in SPNE, then we have $p_1 \geq \alpha(\hat{p}_0)$. Since $\hat{p}_0 < \underline{p}_0$, firm 0 is losing money. It is better off charging p_0 a little bit lower to get $D = 00$ and a positive profit, again a contradiction. ∎

For $D = 00$ to be in SPNE with $p_0 = \hat{p}_0$ and $p_1 \geq \alpha(\hat{p}_0)$, firstly notice that firm 1 has no incentive to deviate either to a lower or a higher p_1. Secondly, it is easy to see that firm 0 has no incentive to decrease p_0, but it may have incentive to increase p_0 if p_1 is big enough, in order to get $D = 10$ with a higher profit. Given some p_1, the highest p_0 it can deviate with $D = 10$ ensuing is $p_0 = \alpha^{-1}(p_1)$. If under such a deviation, $\pi_0^{10}(\alpha^{-1}(p_1)) > \pi_0^{00}(\hat{p}_0)$, firm 0 will deviate. Therefore we expect an upper bound for the value of p_1 for $D = 00$ to be in SPNE at $p_0 = \hat{p}_0$.

Define

$$\tilde{c} \equiv 0.1808334279.$$

Observation 7 *If and only if $c < \tilde{c}$, $\pi_0^{10}(p_0^{10}) > \pi_0^{00}(\hat{p}_0)$ is true.*

Since p_0^{10} is the optimal price for firm 0 when $D = 10$, according to Observation 7, we need to take the trouble looking for that upper bound of p_1 only when $c_1 < \tilde{c}$. For a given p_1, if firm 0 deviates, it will deviate to a p_0 along the curve $\alpha(p_0)$, in order to maximize its profit under $D = 10$. Thus for $c \in (0, \tilde{c})$, there exist a $p_1 \in (\alpha(\hat{p}_0), \alpha(p_0^{10}))$, defined as $p_1^c : (0, \tilde{c}) \to (\alpha(\hat{p}_0), \alpha(p_0^{10}))$,

$$p_1^c \equiv \alpha[(\pi_0^{10})^{-1} \circ \pi_0^{00}(\hat{p}_0)],$$

which gives the highest p_1 at which firm 0 has no incentive to deviate to a higher p_0 to get $D = 10$, hence guarantees $D = 00$ to be in SPNE. Note for $c \leq \underline{c}$, by definitions of p_0^c and p_1^c, we have $p_1^c = \alpha(p_0^c)$.

Lemma 24 *At $p_0 = \hat{p}_0$, in SPNE $D = 00$ if p_1 satisfies*

$$\begin{cases} p_1 \in [\beta(\hat{p}_0), p_1^c] & \textit{if} \quad c < \tilde{c} \\ p_1 \geq \beta(\hat{p}_0) & \textit{o.w.} \end{cases}$$

However, firm 1 with $p_0 = \hat{p}_0$ in fact is indifferent between $d_1 = 0$ and $d_1 = 1$ in the last stage. Thus such a SPNE stated above is not stable in that, if firm 1 by some "error" plays $d_1 = 1$ in the last stage, firm 0 will either get zero profit with $D = 11$, or lose money with $D = 10$, since $\hat{p}_0 < \underline{p}_0$. Nevertheless, firm 0 by lowering p_0 by an arbitrarily small value can guarantee $D = 00$, together with a positive profit. Therefore, although $D = 00$ under $p_0 = \hat{p}_0$ can be in SPNE as stated above, it is not robust to any positive probability for firm 1 to play $d_1 = 1$, which in fact, is also an optimal outsourcing decision for firm 1 in the last stage.

3.4.3 Proof to the Main Results

Theorem 5 *Under assumption A1, there exist SPNE of* $\Gamma^{imp}(a,b,c)$ *indexed by* E^{00} *and* E^{10}.
I. In E^{00}*, provider prices satisfy*

$$\{(p_0, p_1 = \beta(p_0)) : p_0 \in [\underline{\underline{p}}_0, \hat{p}_0)\}$$

or

$$\begin{cases} (\hat{p}_0, p_1 \in [\beta(\hat{p}_0), p_1^c]) & \textit{if} \quad c < \tilde{c} \\ (\hat{p}_0, p_1 \geq \beta(\hat{p}_0)) & \textit{o.w.} \end{cases}$$

and $D = 00$.
II. In E^{10}*, provider prices satisfy*

$$\{(p_0, p_1 = \alpha(p_0)) : p_0 \in [p_0^c, \tilde{p}_0^{10}]\},$$

for $c \leq \underline{c}$*, and* $D = 10$.

Proof: Firstly we show that E^{00} is an SPNE for any value of c. Under (p_0, p_1) given in E^{00}, by Lemma 14, $D = 00$ is the ensuing equilibrium in stage two and three. What left is to show that none will deviate in stage one.

At $p_0 = \hat{p}_0$, the proof is given by Lemma 24.

Suppose $p_0 < \hat{p}_0$. Given $p_1 = \beta(p_0)$, firm 0 is winning a non-negative profit for $p_0 \in [\underline{\underline{p}}_0, \hat{p}_0)$. If firm 0 deviates to a higher p_0, by Lemma 14, $D = 11$ is the following equilibrium. Firm 0 ends up with a zero profit by deviation; if firm 0 deviates to a lower p_0, $D = 00$ is still the ensuing equilibrium. But $\pi_0^{00}(p_0)$ is strictly increasing in $p_0 \in [\underline{\underline{p}}_0, \hat{p}_0)$, thus firm 0 is worse off. Hence firm 0 will not deviate. On the other side, given p_0, if firm 1 deviates to $p_1 > \beta(p_0)$, $D = 00$ will not be changed, neither will 1's profit; if firm 1 deviates to $p_1 < \beta(p_0)$, by Lemma 14, $D = 11$ is ensuing but firm 1 is worse off, followed by Lemma 20 and Lemma 22. firm 1 will not deviate, either.

Secondly, we show that E^{10} is an SPNE only if $c \leq \underline{c}$. The only part followed by Lemma 19. By Lemma 14, $D = 10$ is the ensuing equilibrium for the combination of (p_0, p_1) in E^{10}, thus we only need to prove that in stage one none will deviate. The proof is similar as for E^{00}. ∎

Corrolary 3 *Under assumption A1, there does not exist any SPNE in which firm 1 and firm 2 both outsource to firm 1.*

Proof: Suppose in some SPNE $D = 11$. When $p_0 < \hat{p}_0$, by Lemma 14, it must be true that $p_1 \leq \beta(p_0)$. However, by Lemma 20 and Lemma 22, firm 1 is better off not providing, a contradiction. At $p_0 = \hat{p}_0$, it must be true that $p_1 \leq \alpha(p_0)$. The same reason as above rules out the case that $p_1 \leq \beta(\hat{p}_0)$. If $p_1 \in (\beta(\hat{p}_0), \alpha(\hat{p}_0)]$, firm 0 by deviating to $p_0 = \hat{p}_0 - \epsilon$ with ϵ small, by Lemma 14, can get a positive profit with $D = 00$ ensuing, again a contradiction. When $p_0 > \hat{p}_0$, by Lemma 16 and Lemma 17, it must be true that $p_0 \geq \tilde{p}_0^{10}$. Furthermore, $p_1 = \alpha(p_0)$. Thus firm 0 will deviate to $p_0 = \hat{p}_0 - \epsilon$ to get $D = 00$. ∎

Theorem 6 *Under assumption A1, there does not exist other SPNE for* $\Gamma^{imp}(a, b, c)$ *than* E^{00} *and* E^{10}.

Proof: Firstly, by Lemma 7 and Corollary 3, $D = 01$ and $D = 11$ are off equilibrium; Secondly, for $D = 00$ and $D = 10$ to be in any SPNE, prices (p_0, p_1) must be indexed by E^{10} and E^{11}, followed by Lemma 18, Lemma 19, and Lemma 24. ∎

3.4.4 When Firm 0 Has Cost Disadvantage

Our major conclusion in former sections can be summarized: in any SPNE for $\Gamma^{imp}(a, b, c)$, firm 2 is outsourcing to firm 0. Although it is derived from the assumption that firm 0 and firm 1 have the same cost for the intermediate product, it is still true even when firm 0 has moderate cost disadvantage compared to firm 1. Assume that 0's cost function is

$$C_0(q) = \lambda C_1(q), \quad \text{with } \lambda \in [1, \frac{a}{b}). \tag{3.3}$$

By assuming $\lambda < \frac{a}{b}$, A1 is true with firm 0's new cost function. Denote this new game in which firm 0 has cost disadvantage as $\Gamma^{imp}(a, b, c, \lambda)$.

When firm 0 has a higher cost, the lower bound of p_0 for firm 0 to be willing to provide will increase. Now the lowest p_0 for firm 0 to provide in any SPNE depends on the value of λ, denoted by $\underline{p}_0(\lambda)$ for E^{10} and $\underline{\underline{p}}_0(\lambda)$ for E^{00}, both are increasing in λ. On the other side, $p_0^c(\lambda)$ is decreasing in λ, and $\hat{p}_0, \tilde{p}_0^{10}$ are fixed by firm 1's strategy. Define

$$\lambda^{00} \equiv \frac{3a\sqrt{1-c} - 6b\sqrt{(1-c)} - 3a + 4ac}{4ac\sqrt{1-c} - 3b\sqrt{1-c} - 3b + 4bc - 4bc\sqrt{1-c}},$$

$$\lambda^{10} \equiv \frac{12ac^2 - 12ac - 14bc + a + 13b}{2(1-c)(7b - 4bc - 2ac)}.$$

We have

$$\hat{p}_0 \geq \underline{\underline{p}}_0(\lambda) \Leftrightarrow \lambda \leq \lambda^{00}, \tilde{p}_0^{10} \geq \underline{p}_0(\lambda) \Leftrightarrow \lambda \leq \lambda^{10}.$$

λ^{00} is increasing in c whereas λ^{10} is decreasing in c. Furthermore, we have

$$\lim_{c \to 0} \lambda^{00} = 1, \ \lim_{c \to 0} \lambda^{10} = \frac{a + 13b}{14b},$$

$$\lambda^{10}|_{c=\hat{c}} = 1, \;\; \lambda^{00}|_{c=\frac{b}{2a}} < \frac{a}{b}.$$

At a given $\lambda > 1$, by comparing $\pi_0^{10}(\tilde{p}_0^{10}, \lambda)$ and $\pi_0^{00}(\hat{p}_0, \lambda)$, we can find the upper bound of c as $\underline{c}(\lambda)$, at which these two are equal, as the necessary condition for $D = 10$ to be in SPNE. Define $\tilde{\lambda} \equiv \underline{c}^{-1}(\lambda)$. At a given value of c, $\tilde{\lambda}$ solves

$$\pi_0^{10}(\tilde{p}_0^{10}) = \pi_0^{00}(\hat{p}_0).$$

Since λ^{00} solves

$$\pi_0^{10}(\underline{\underline{p}}_0) = \pi_0^{00}(\hat{p}_0),$$

λ^{10} solves

$$\pi_0^{10}(\tilde{p}_0^{10}) = \pi_0^{00}(\underline{p}_0),$$

these three cures, λ^{00}, λ^{10}, $\tilde{\lambda}$, intersect at the same value of c.

Theorem 7 *(Figure 3.6)Under A1 and (3.3), as long as $\lambda < \max\{\lambda^{10}, \lambda^{00}\}$, firm 2 outsources to firm 0 in any SPNE for $\Gamma^{imp}(a, b, c, \lambda)$.*

Proof: Firstly, the change in firm 0's cost has no effect on firm 1 and firm 2's choice for any given $\{p_0, p_1\}$, therefore our findings from backward induction for stage two and three are not affected. On the other side, firm 0's choice on p_0 depends on λ, and $\underline{p}_0(\lambda)$, $\underline{\underline{p}}_0(\lambda)$ are strictly increasing in λ. However, whenever $\lambda \leq \lambda^{00}$, $\underline{\underline{p}}_0(\lambda) < \hat{p}_0^{10}$ is true and $D = 00$ is still in SPNE. For $D = 10$ to be in SPNE with $\lambda > 1$, two conditions need to be satisfied: $\tilde{p}_0^{10} \geq p_0^c(\lambda)$ and $\tilde{p}_0^{10} \geq \underline{p}_0(\lambda)$. Since $p_0^c(\lambda)$ is decreasing in λ, the first condition in not binding. Moreover, by solving $\tilde{p}_0^{10} = p_0^c(\lambda)$ for $\tilde{\lambda}$, the first condition is true for $\lambda \geq \tilde{\lambda}$. And the second condition is true when $\lambda \leq \lambda^{10}$. From Figure 3.6, these three curves always intersect at the same point. For $d_2 = 0$ in any SPNE, strict inequality is needed. Therefore, when $\lambda < \max\{\lambda^{10}, \lambda^{00}\}$, firm 2 outsources to firm 0 in any SPNE. ∎

3.5 Robustness of the Main Results

3.5.1 Sequential Price Announcements in Stage One

Let $\Gamma_{0,1}^{imp}(a, b, c)$ as a new game obtained from $\Gamma^{imp}(a, b, c)$ with one modification: In stage one firm 0 announces p_0 at first, then firm 1 announces p_1 after it observes the value of p_0.

Recall that when $c > (\leq)\underline{c}$, $\pi_0^{00}(\hat{p}_0) > (\leq)\pi_0^{10}(\tilde{p}_0^{10})$. We have a theorem below.

Theorem 8 *There exists SPNE for $\Omega_{0,1}^{imp}$.*
I. When $c > \underline{c}$, any SPNE is indexed by $(p_0 = \hat{p}_0, p_1 \geq \beta(p_0))$, then $D = 00$;

II. When $c < \underline{c}$, there exists a unique SPNE indexed by $(p_0 = \tilde{p}_0^{10}, p_1 = \alpha(\tilde{p}_0^{10}))$, and $D = 10$;
III. When $c = \underline{c}$, both $D = 00$ or $D = 10$ can be in SPNE, with corresponding prices described by I and II.

Proof: We prove I by two parts.

i. The strategies stated above are SPNE. Firstly, given prices $(p_0 = \hat{p}_0, p_1 \geq \beta(\hat{p}_0))$, by Lemma 14, $D = 00$ is an ensuing equilibrium. Secondly, given $p_0 = \hat{p}_0$, if firm 1 deviates to $p_1 < \beta(\hat{p}_0)$, $D = 11$ is ensuing. By Lemma 20 and Lemma 22, firm 1 is worse off. Thus firm 1 will not deviate. Lastly, firm 0 will not deviate to any other p_0. If firm 0 deviates to a lower p_0, by observation 2, $\pi_0^{00}(p_0)$'s profit is strictly increasing for $p_0 \leq \hat{p}_0$, hence it is worse off; on the other side, if firm 0 deviates to $p_0 > \hat{p}_0$, then either $D = 10$ or $D = 11$ is ensuing and firm 0 is always worse off.

ii. There does not exist any other SPNE. Firstly, with (p_0, p_1) given, we want to show that $D = 00$ is the unique ensuing equilibrium. By Lemma 14, we only need to rule out the possibility that $D = 11$. Suppose $D = 11$ is the ensuing equilibrium, which implies that firm 0 achieves a zero profit. Let firm 0 deviate to $p_0 = p_0' \in (\underline{p}_0, \hat{p}_0)$, then firm 1 will react with $p_1 \geq \beta(p_0')$. The reason is, by Lemma 14, if $p_1 < \beta(p_0')$, $D = 11$ is the outcome in the following stages. However, by Lemma 20 and Lemma 22, firm 1 is worse off providing firm 2 than outsourcing to firm 0 together with 2. Thus firm 1 will set $p_1 \geq \beta(p_0')$, which implies that $D = 00$ will ensue and firm 0 achieves a positive profit with p_0'. A contradiction to that $D = 11$ is in SPNE. Secondly, given $p_0 = \hat{p}_0$, in SPNE firm 1 will set $p_1 \geq \beta(\hat{p}_0)$ to ward off firm 2. Lastly, in any SPNE firm 0 will choose $p_0 = \hat{p}_0$, followed by the monotonicity of $\pi_0^{00}(p_0)$ in $p_0 \leq \hat{p}_0$.
Prove to the rest part is similar to part I hence is omitted here. ∎

Consider the second case: Suppose now firm 1 is the one who announces its price for the intermediate good before firm 0's announcement. let $\Gamma_{1,0}^{imp}(a, b, c)$ denote the modified game. Now another strategic consideration arises for firm 1: by announcing a high p_1, it gives firm 0 spaces to also announce a high p_0, which on the one side guarantees that firm 2 outsources to firm 0, and on the other side yields both firms 0 and 1 higher profits. For firm 0, its profit improves from $p_0 = \tilde{p}_0^{10}$ to $p_0 = p_0^{10}$ with $D = 10$ ensuing; for firm 1, it is better off in the final product market through strategically increasing firm 2's cost.

It is easy to check that $\pi_1^{00}(\underline{p}_0) < \pi_1^{10}(p_0^{10})$ is always true under assumption A1. Since $\pi_0^{00}(\hat{p}_0) \leq \pi_0^{10}(p_0^{10})$ for $c \leq \tilde{c}$, we have a theorem below.

Theorem 9 *There exists SPNE of $\Omega_{1,0}^{imp}$:*
I. When $c > \tilde{c}$, the SPNE is indexed by $(p_0 = \underline{p}_0, p_1 = \beta(\underline{p}_0))$ and $D = 00$;
II. When $c \leq \tilde{c}$, the SPNE is indexed by $(p_0 = p_0^{10}, p_1 \geq \alpha(p_0^{10}))$ and $D = 10$.

Proof: Firm 1 is more profitable letting firm 0 to provide with a p_0 advantageous to firm 1, than providing firm 2 by itself. Because $\pi_1^{00}(p_0)$ is strictly decreasing in $p_0 \in [\underline{p}_0, a]$, and $\pi_1^{10}(p_0)$ is strictly increasing in $p_0 \in [\underline{p}_0, p_0^{10}]$, firm 1 prefers $p_0 = \underline{p}_0$ when $D = 00$ and p_0^{10} when $D = 10$.

It is obvious that given the prices in I and II, the following outsourcing pattern is in SPNE. When $c > \tilde{c}$, given $p_1 = \beta(\underline{p}_0)$, any deviation of firm 0 ends up with either negative or zero profit, hence firm 0 has no incentive to deviate. Moreover, firm 1 will not deviate. Suppose it deviates to $p_1 < \beta(\underline{p}_0)$. Because firm 0 never sets $p_0 < \underline{p}_0$, $D = 11$ will be the outcome and firm 1 is worse off by Lemma 20 and Lemma 22. On the other side, suppose firm 1 deviates to p_1 slightly bigger than $\beta(\underline{p}_0)$, denote it as p_1', so that by Lemma 10 there exists p_0' suchgiven by $\beta(p_0') = p_1'$, which satisfies $p_0' > \underline{p}_0$. Firm 0 will set $p_0 = p_0' > \underline{p}_0$, such that $D = 00$ is the following equilibrium and F_1 is worse off. If firm 1 deviates to a price even higher, since $\pi_0^{00}(\hat{p}_0) \leq \pi_0^{10}(p_0^{10})$ for $c \leq \tilde{c}$, firm 0 will set $p_0 = \hat{p}_0$ then $D = 00$ follows, and firm 1 is again worse off.

For $c \leq \tilde{c}$, it is easy to see that firm 0 will set $p_0 = p_0^{10}$ to maximize its profit under $D = 10$, given that firm 1 has set $p_1 \geq \alpha(p_0^{10}))$. For firm 1, since $\pi_1^{00}(\underline{p}_0) < \pi_1^{10}(p_0^{10})$ is always true, it is always worse off by deviating from $p_1 \geq \alpha(p_0^{10}))$. ∎

3.5.2 When There Are Several Outside Providers

In this modification, assume that there are more than one firm 0 who are providing good I and are out of the market of good F. Denote them as $0, 0', 0''$.... Assume they are symmetric. All other constructions for the benchmark game are kept the same. Denote this modified game as $\Gamma^{nimp}(a, b, c)$.

Lemma 25 *When there are several outside providers, E^{10} is no longer an SPNE for any c.*

Proof: Suppose $c \leq \underline{c}$. Suppose E^{10} is an SPNE. From Theorem 5, $p_0 \in [p_0^c, \tilde{p}_0^{10}]$. Competition between the providers will drive p_0 down to p_0^c. Suppose it is an SPNE that firm 0 is charging $p_0 = p_0^c$, then firm 2 outsources to firm 0 with firm 1 producing inside. All other outside providers end up with zero profit. Let one of them, say $0'$, deviate to $p_0 = \underline{p}_0 + \epsilon$. Since $\pi_2^{00}(\underline{p}_0) > \pi_2^{10}(p_0^c)$, with ϵ small enough, in stage two firm 2 will outsource to $0'$, and by Lemma 14, $D = 00$ is achieved. Firm $0'$ achieves a positive profit by cutting firm 0 off, a contradiction. ∎

Theorem 10 *Under A1, there exists SPNE for $\Gamma^{nimp}(a, b, c)$. In any SPNE, at least one outside provider charges $p_0 = \underline{p}_0$, with $p_1 \geq \beta(\underline{p}_0)$, then firm 1 and firm 2 both outsource to the same outside provider under $p_0 = \underline{p}_0$.*

Proof: By Lemma 25, only E^{00} can be SPNE, in which $p_0 \in [\underline{p}_0, \hat{p}_0]$. Competition among the outside providers will drive the price down such that at least one of them set $p_0 = \underline{p}_0$. On the other side, if more than one of them set such a price, then they are not losing money only when both firm 1 and firm 2 outsource to the same outside provider. ∎

3.5.3 When There Are Trembling Hands in Prices

Modify the benchmark model in that, assume there is trembling hand in $\{p_0, p_1\}$, so that with some possibility the real value of prices, realized after the outsourcing decisions of firm 1 and firm 2 have been made, is larger than the announced prices. Even so, our conclusion that firm 2 will always outsource to firm 0 in any SPNE stands true. Firstly, for E^{00}, with $p_0 \in [\underline{p}_0, \hat{p}_0)$ and $p_1 = \beta(p_0)$, it is true that $|\frac{d\pi_2^{00}(p_0)}{dp_0}| < |\frac{d\pi_2^{11}(p_1)}{dp_1}|$, the change in firm 2's profit when mistake happens in p_0 is smaller, compared to when $D = 11$ and the same mistake happens in p_1. Thus firm 2 will stick to $d_2 = 0$; secondly, for E^{10}, with $p_0 \in [p_0^c, \tilde{p}_0^{10}]$ and $p_1 = \alpha(p_0)$, $|\frac{d\pi_2^{10}(p_0)}{p_0}| < |\frac{d\pi_2^{11}(p_1)}{p_1}|$ is true for almost all $c \leq \underline{c}$, except those values of c very close to $\underline{c}$. Again firm 2 will stick to $d_2 = 0$.

3.6 A Game with Perfect Information

Every setting for this new game is the same as for $\Gamma^{imp}(a, b, c)$, except that in the last stage, firm 1 observes firm 2's strategy whether firm 2 is outsourcing to firm 1 or to firm 0. Thus firm 1 has perfect information when making its decision. Denote this game as $\Gamma^p(a, b, c)$, in which firm 1 is always willy-nilly accommodating firm 2's production by acting as a follower, and its unwillingness to provide firm 1 due to the follower's disadvantage in the benchmark model has been eliminated. In this modified game, firm 1 has stronger incentive to compete firm 0 for providing firm 2.

The game is solved by backward induction for sub-game perfect Nash equilibrium (SPNE) in pure strategy, with the proviso that none plays weakly dominated strategy.

3.6.1 Major Conclusions

Define

$$p_0^* \equiv a - \frac{a - b}{\sqrt{1 - c}}$$

.

Theorem 11 *(see Figure 3.15)Under assumption A1, there exist SPNE of* $\Gamma^p(a,b,c)$ *indexed by* E^{00}*: provider prices satisfies* $\{(p_0, \beta(p_0)) : p_0 \in [\underline{\underline{p}}_0, p_0^*]\}$*, and then* $D = 00$.

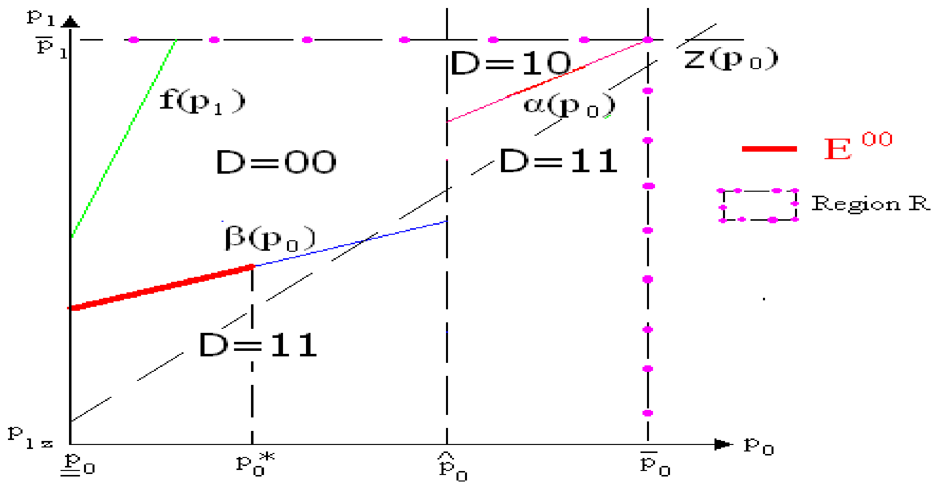

Figure 3.15: E^{00}

Theorem 12 *Under A1, there does not exist any SPNE other than* E^{00}.

With firm 1's cost function for good I as $C_1(q)$, the same as in the benchmark model, assume that firm 0's cost function is

$$C_0(q) = \lambda C_1(q), \quad \lambda \in [1, \frac{a}{b}). \tag{3.4}$$

Now $\underline{\underline{p}}_0(\lambda)$ is strictly increasing in λ.

Define

$$\bar{\lambda} \equiv \frac{4(a - b - a\sqrt{1-c})}{3c(a-b) - 4b\sqrt{1-c}}.$$

Observation 8 $\bar{\lambda}$ *is strictly increasing in* c*. Moreover,* $\lim_{c\to 0} \bar{\lambda} = 1, \lim_{c \to \frac{b}{2a}} \bar{\lambda} = \frac{4aX}{bY} < \frac{a}{b}$.

Here X, Y are defined by $X \equiv \sqrt{2}a\sqrt{\frac{2a-b}{a}} - 2(a-b), Y \equiv 4\sqrt{2}a\sqrt{\frac{2a-b}{a}} - 3(a-b)$.

Theorem 13 *Under assumption A1 and firm 0's cost function given by (3.4), as long as* $\lambda < \bar{\lambda}$*, SPNE exists and in any SPNE* $D = 00$*, with prices indexed by* $\{(p_0, \beta(p_0)) : p_0 \in [\underline{\underline{p}}_0(\lambda), p_0^*]\}$.

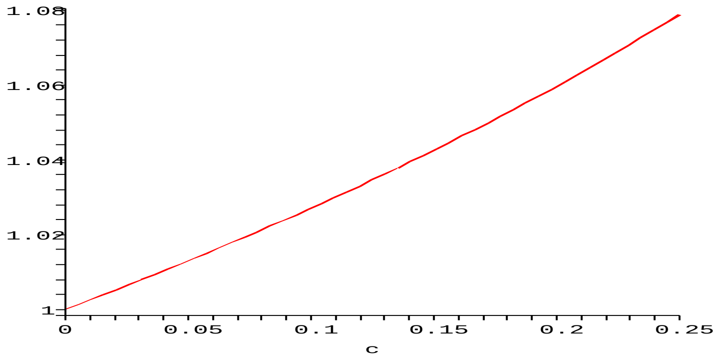

Figure 3.16: $\bar{\lambda}$ is increasing in c.

3.6.2 When There Are $n > 2$ Firms in the Final-product Market

Assume that in stead of duopoly, there are $n > 2$ firms competing in good F, denoted as firm 1, firm 2,..., firm n. Among them only firm 1 can produce the intermediate product good I inside. All firm 2 to firm n are symmetric and neither of them can produce good I, thus will have to either outsource to firm 0, or to firm 1.

The game is different with $\Gamma^p(a, b, c)$ only in the second stage, in which firm 2 to firm n after observing prices announced in stage one, simultaneously decide to which one, firm 0 or firm 1, to outsource, together with their quantities to outsource. Denote the modified game as $\Gamma^p(a, b, c, n)$. Demand and cost functions are the same as before, with assumption $A1$ satisfied.

We focus on symmetric strategy SPNE for this game, with symmetric strategy in the sense that firms 2,...,n have uniform outsourcing decisions. The proviso that no player is using a weakly dominated strategy in any SPNE is also employed.

Let d_n denote firm 2,...,n's outsourcing decisions.

$$d_n = \begin{cases} 0, & \text{if firm } 2,\ldots,n \text{ outsources to firm } 0 \\ 1, & \text{if firm } 2,\ldots,n \text{ outsource to firm } 1 \end{cases}$$

Connecting with firm 1's outsourcing decision in the last stage, represented by d_1, outsourcing pattern of this game is again

$$D = d_1 d_n \in \{11, 10, 00, 01\}.$$

Major Conclusion

The analysis here is similar as for the original game, hence details are omitted. Our major conclusion relies on several important functions and observations. The first one is about the threshold $\hat{p}_0(n)$, for firm 1 to be indifferent between $d_1 = 1$ or

$d_1 = 0$ given that $d_n = 0$; the second is on two functions, $\alpha(p_0)$ and $\beta(p_0)$, which are the same as in the benchmark game; the third one is on the function $f(p_1, n)$, which is the threshold for firm 1 to be indifferent between $d_1 = 0$ or $d_1 = 1$, given that $d_n = 1$. The fourth one is about the threshold of p_1 for firm 1 to be indifferent between $D = 10$ and $D = 00$, which depends on the value of n, and is given as $p_1 = \lambda(p_0, n)$; aslo the threshold of p_1 for firm 1 to be indifferent between $D = 00$ and $D = 11$ is $p_1 = \gamma(p_0)$. We also define the lowest p_0 for firm 0 to provide in $D = 00$ as $\underline{\underline{p}}_0(n)$, which solves $\pi_0^{00}(p_0) = 0$. Basic observations on these values and functions for $n > 2$ are listed below.

Define

$$\underline{\underline{p}}_0(n) \equiv \frac{2n(b-ac)+ac}{2n(1-c)+c},$$

$$\hat{p}_0(n) \equiv \frac{(2\sqrt{1-c}bn - \sqrt{1-c}a + 2\sqrt{1-c}ac - \sqrt{1-c}b - 2\sqrt{1-c}bcn + a - 3ac + 2ac^2)}{(-1+c)(2c - 2\sqrt{1-c}n + 2\sqrt{1-c} - 1)}.$$

Observation 9 $\frac{d\hat{p}_0(n)}{dn} > 0$, $\frac{d\underline{\underline{p}}_0(n)}{dn} < 0$;
$\lim_{n\to\infty} \hat{p}_0(n) = b$, $\lim_{n\to\infty} \underline{\underline{p}}_0(n) = \frac{b-ac}{1-c} > 0$.

Observation 10 $\alpha(\hat{p}_0(n)) > \beta(\hat{p}_0(n))$, $f(\beta(\underline{\underline{p}}_0(n))) < \underline{\underline{p}}_0(n)$.

Observation 11 $\alpha(p_0) > \lambda(p_0, n)$ *for* $p_0 > \hat{p}_0(n)$.

Observation 12 $\beta(p_0)$ *intersects* $\gamma(p_0, n)$ *at* $p_0 = p_0^*$. $\beta(p_0) > \gamma(p_0, n)$ *if and only if* $p_0 > p_0^*$.

Recall that $\hat{p}_0(n) > \underline{\underline{p}}_0(n)$ at $n = 2$ in the benchmark model. By Observation 9, $\hat{p}_0(n) > \underline{\underline{p}}_0(n)$ is true for all $n \geq 2$. All the important properties for the regime diagram is kept with $n > 2$.

Theorem 14 *Under assumption* $A1$*, there exists SPNE of* $\Gamma^p(a, b, c, n)$ *indexed by* $E^{00}(n)$*:* $D = 00$*, with provider prices* $\{(p_0, \beta(p_0)) : p_0 \in [\underline{\underline{p}}_0(n), p_0^*]\}$*, where* $\underline{\underline{p}}_0(n)$ *is decreasing in* n*. And there does not exist any other SPNE.*

Proof: Proof is similar as for Theorem 11 and Theorem 12. ∎

3.6.3 When Firm 0 Has Cost Disadvantage

With firm 1's cost function for good F kept the same, assume now that firm 0's cost is given by (3.4) as

$$C_0(q) = \lambda C_1(q), \quad \lambda \in [1, \frac{a}{b}).$$

Our argument for the benchmark model stands here in that, as long as λ is less than some threshold, Theorem 14 is still true, with this threshold given by solving

$$p_0^* \geq \underset{=0}{p}(n, \lambda) \Rightarrow \lambda \leq \bar{\lambda}(n),$$

with

$$\bar{\lambda}(n) = \frac{2n(a\sqrt{1-c} - a + b)}{2nb\sqrt{1-c} - (a-b)c(2n-1)}.$$

We have some observations on $\bar{\lambda}(n)$:

Observation 13 $\bar{\lambda}(n) > 1, \frac{d\bar{\lambda}(n)}{dn} > 0.$

Observation 14 $\lim_{c \to 0} \bar{\lambda}(n) = 1, \lim_{n \to \infty} \bar{\lambda}(n) = \frac{a\sqrt{1-c}-a+b}{b\sqrt{1-c}-(a-b)c}.$

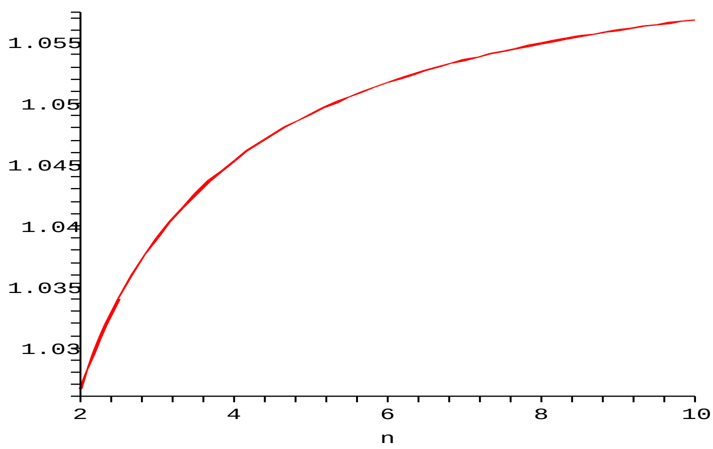

Figure 3.17: $\bar{\lambda}(n)$

Theorem 15 *Under A1 and (3.4), as long as* $\lambda < \bar{\lambda}(n)$*, SPNE for* $\Gamma^p(a, b, c, n)$ *exists. In any SPNE, prices are indexed by* $\{(p_0, \beta(p_0)) : p_0 \in [\underset{=0}{p}(\lambda(n)), p_0^*]\}$ *and* $D = 00$*. Moreover,* $\bar{\lambda}(n)$ *is increasing in* n*.*

Proof: Similar as for Theorem 13. ∎

3.7 Conclusion

If firm 1 can not observe firm 2's quantity when firm 2 outsources to firm 0, two reasons drive firm 2 to outsource to firm 0. Firstly, if firm 2 outsources to firm 1, since firm 1's average cost is decreasing in the quantity it produces, firm 2 is making its rival stronger by helping it to decrease its average cost. Firm 2 is unwilling to outsource to firm 1; Secondly, firm 1 incurs the disadvantage as being a Stackelberg follower when providing firm 2, which can be avoided by either producing only for itself or outsourcing to firm too. Firm 1 is not eager to provide firm 2. In all the SPNE firm 2 is outsourcing to firm and for big range of parameters, in the SPNE firm 1 is outsourcing to firm too.

If firm 1 can observe firm 2's quantity whenever firm 2 outsources to firm 0 or to firm 1, the second strategic reason above is no longer true. However, only economies of scale works to lead to the phenomenon that in any SPNE both firm 1 and firm 2 outsource to firm 0, but now with a smaller range of p_0.

In both of these two cases, our conclusions are true even when firm 0 has some cost disadvantage compared to firm 1.

3.8 Appendix of Chapter 3

1. The optimal $q_i^D(p_0,p_1), \pi_i^D(p_0,p_1), i=1,2$ for each outsourcing pattern.

D=11. Firm 2 outsources to 1, then firm 1 produces inside.

In stage three, firm 1 is maximizing

$$\pi_1(q_1,q_2) = (a-q_1-q_2)q_1 + p_1q_2 - b(q_1+q_2) + c(q_1+q_2)^2$$
$$\text{s.t.} \quad q_1 \geq 0.$$

Since $\frac{d^2\pi_1(q_1,q_2)}{dq_1^2} = -2(1-c) < 0$, the best reaction of firm 1 is $q_1^{11}(q_2)$:

$$q_1^{11}(q_2) = \begin{cases} \frac{a-b-q_2+2cq_2}{2(1-c)} & \text{if} \quad q_2 < \frac{a-b}{1-2c} \\ 0 & \text{o.w.} \end{cases}$$

Note that when $q_1^{11}(q_2) > 0$, $-1 < \frac{dq_1^{11}(q_2)}{dq_2} = -\frac{1-2c}{2(1-c)} < 0$. Firm 2's profit is

$$\pi_2(q_2) = (a - q_1^{11}(q_2) - q_2)q_2 - p_1q_2 \text{ with } q_2 \geq 0$$
$$= \begin{cases} \frac{(a+b-2ac-q_2-2p_1+2cp_1)q_2}{2(1-c)} & \text{if} \quad 0 \leq q_2 < \frac{a-b}{1-2c} \\ (a-q_2-p_1)q_2 & \text{o.w.} \end{cases}$$

Note $\frac{d^2\pi_2(q_2)}{dq_2^2} = -\frac{1}{1-c} < 0$ when $0 \leq q_2 < \frac{a-b}{1-2c}$. By first order condition, the optimal q_2 is solved as a function of p_1:

$$q_2^{11}(p_1) = \begin{cases} 0 & \text{if} \quad p_1 \geq \bar{p}_1 \\ \frac{a+b-2ac-2(1-c)p_1}{2} & \text{if} \quad p_{1z} < p_1 < \bar{p}_1 \\ \frac{a-b}{1-2c} & \text{if} \quad p_{1r} \leq p_1 \leq p_{1z} \\ \frac{a-p_1}{2} & \text{o.w.} \end{cases}$$

Here $\bar{p}_1 = \frac{a+b-2ac}{2(1-c)}$, $p_{1z} = \frac{4ac^2+3b-2bc-a-4ac}{2(1-2c)(1-c)}$, $p_{1r} = \frac{2b-a-2ac}{1-2c}$. When $p_1 < p_{1r}$, firm 1's production is blocked by firm 2 through its monopoly quantity; when $p_1 > p_{1z}$, firm 2 is accommodating 1's production and producing the leader's quantity; when p_1 lies in the middle range, firm 2 is deterring firm 1 from production by producing more than its monopoly quantity. Substituting $q_2^{11}(p_1)$ into $q_1^{11}(q_2)$, we solve the optimal q_1 as a function of p_1:

$$q_1^{11}(p_1) = \begin{cases} \frac{a-b}{2(1-c)} & \text{if} \quad p_1 \geq \bar{p}_1 \\ \frac{a-3b+4ac(1-c)+2bc}{4(1-c)} + \frac{p_1}{2}(1-2c) & \text{if} \quad p_{1z} < p_1 < \bar{p}_1 \\ 0 & \text{o.w.} \end{cases}$$

The maximized profits for 1, firm 2 are $\pi_1^{11}(p_1)$ and $\pi_2^{11}(p_1)$, respectively:

$$\pi_1^{11}(p_1) = \begin{cases} \pi_{1M}^{11} = \frac{(a-b)^2}{4(1-c)} & \text{if} \quad p_1 \geq \bar{p}_1 \\ \pi_{1f}^{11}(p_1) = \frac{(12ca^2-12c^2a^2+12abc+b^2+a^2-14ab)}{16(1-c)} & \\ +\frac{3}{4}(a+b-2ac)p_1 - \frac{3}{4}(1-c)p_1^2 & \text{if} \quad p_{1z} < p_1 < \bar{p}_1 \\ \frac{(a-b)(ac+bc-b+p_1-2cp_1)}{(1-2c)^2} & \text{if} \quad p_{1r} \leq p_1 \leq p_{1z} \\ \frac{1}{4}(a-p_1)(ac-2b+2p_1-cp_1) & \text{o.w.} \end{cases}$$

$$\pi_2^{11}(p_1) = \begin{cases} 0 & \text{if} \quad p_1 \geq \bar{p}_1 \\ \pi_{2l}^{11}(p_1) = \frac{(a+b-2ac-2p_1+2cp_1)^2}{8(1-c)} & \text{if} \quad p_{1z} < p_1 < \bar{p}_1 \\ \frac{(a-b)(b+2cp_1-2ac-p_1)}{(1-2c)^2} & \text{if} \quad p_{1r} \leq p_1 \leq p_{1z} \\ \pi_{2M}^{11} = \frac{(a-p_1)^2}{4} & \text{if} \quad p_1 < p_{1r} \end{cases}$$

Here π_{1M}^{11} and π_{2M}^{11} are firm 1 and 2's monopoly profit in pattern 11, respectively; $\pi_{1f}^{11}(p_1), \pi_{2l}^{11}(p_1)$ are firm 1 and 2's profits when they are the follower and the leader in pattern 11, respectively. We have $\frac{d^2\pi_{1f}^{11}(p_1)}{dp_1^2} = \frac{3}{2}(c-1)$, $\frac{d^2\pi_{2l}^{11}(p_1)}{dp_1^2} = 1-c$. $\pi_{1f}^{11}(p_1)$ is increasing and strictly concave in p_1; $\pi_{2l}^{11}(p_1)$ is decreasing and strictly convex in p_1. Furthermore, when $p_1 \leq p_{1z}$, $\pi_1^{11}(p_1)$ is negative and is strictly increasing in p_1.

D=10. Firm 2 outsources to 0, then firm 1 produces inside.

Firms firm 1 and firm 2 are setting quantities simultaneously. Firm 1 is maximizing

$$\pi_1(q_1, q_2) = (a - q_1 - q_2)q_1 - bq_1 + cq_1^2$$
$$\text{s.t.} \quad q_1 \geq 0$$

and firm 2 is maximizing

$$\begin{aligned} &\pi_2(q_1, q_2) = (a - q_1 - q_2)q_2 - p_0 q_2 \\ &\text{s.t.} \quad q_2 \geq 0 \end{aligned}$$

Denote $\bar{p}_0 = \frac{a+b-2ac}{2(1-c)}$, $p_{0z} = 2b - a$, their best quantities and profits are:

$$q_1^{10}(p_0) = \begin{cases} \frac{a-b}{2(1-c)} & \text{if} \quad p_0 \geq \bar{p}_0 \\ \frac{a+p_0-2b}{3-4c} & \text{if} \quad p_{0z} < p_0 < \bar{p}_0 \\ 0 & \text{o.w.} \end{cases}$$

$$q_2^{10}(p_0) = \begin{cases} 0 & \text{if} \quad p_0 \geq \bar{p}_0 \\ \frac{a+b-2ac-2p_0+2cp_0}{3-4c} & \text{if} \quad p_{0z} < p_0 < \bar{p}_0 \\ \frac{a-p_0}{2} & \text{o.w.} \end{cases}$$

$$\pi_1^{10}(p_0) = \begin{cases} \pi_{1M}^{10} = \frac{(a-b)^2}{4(1-c)} & \text{if} \quad p_0 \geq \bar{p}_0 \\ \pi_{1c}^{10}(p_0) = \frac{(a+p_0-2b)^2(1-c)}{(3-4c)^2} & \text{if} \quad p_{0z} < p_0 < \bar{p}_0 \\ 0 & \text{o.w.} \end{cases}$$

$$\pi_2^{10}(p_0) = \begin{cases} 0 & \text{if} \quad p_0 \geq \bar{p}_0 \\ \pi_{2c}^{10}(p_0) = \frac{(a+b-2ac-2p_0+2cp_0)^2}{(3-4c)^2} & \text{if} \quad p_{0z} < p_0 < \bar{p}_0 \\ \pi_{2M}^{10} = \frac{(a-p_0)^2}{4} & \text{o.w.} \end{cases}$$

Here $\pi_{1c}^{10}(p_0)$ is 1's Cournot profit when $D = 10$, which is convex and increasing in p_0; $\pi_{2c}^{10}(p_0)$ is 2's Cournot profit when $D = 10$, which is concave and decreasing in p_0.

D=00. Firm 2 outsources to 0, then firm 1 outsources to firm 0 too.

In this pattern firm 1 and firm 2 are symmetric. Firm 1 is maximizing

$$\begin{aligned} &\pi_1 = (a - q_1 - q_2)q_1 - p_0 q_1 \\ &\text{s.t.} \quad q_1 \geq 0, \end{aligned}$$

and firm 2 is maximizing

$$\begin{aligned} &\pi_2 = (a - q_1 - q_2)q_2 - p_0 q_2 \\ &\text{s.t.} \quad q_2 \geq 0. \end{aligned}$$

By first order condition, the optimal reactions and profits are

$$q_1^{00}(p_0) = q_2^{00}(p_0) = \begin{cases} \frac{a-p_0}{3} & \text{if} \quad p_0 < a \\ 0 & \text{o.w.} \end{cases}$$

$$\pi_1^{00}(p_0) = \pi_2^{00}(p_0) = \begin{cases} \pi_c^{00}(p_0) = \frac{(a-p_0)^2}{9} & \text{if} \quad p_0 < a \\ 0 & \text{o.w.} \end{cases}$$

$\pi_c^{00}(p_0)$ is strictly concave and decreasing in p_0.

D=01. Firm 2 outsources to 1, then firm 1 outsources to 0.
Firm 1's profit is

$$\begin{aligned} &\pi_1(q_1, q_2) = (a - q_1 - q_2)q_1 + p_1 q_2 - p_0(q_1 + q_2) \\ &\text{s.t.} \quad q_1 \geq 0. \end{aligned}$$

Its optimal reaction is

$$q_1^{01}(q_2) = \begin{cases} \frac{a-p_0-q_2}{2} & \text{if} \quad p_0 < a, q_2 < a - p_0 \\ 0 & \text{o.w.} \end{cases}$$

Firm 2's profit is

$$\begin{aligned} \pi_2(q_2) &= (a - q_1^{01}(q_2) - q_2)q_2 - p_1 q_2 \ \text{ s.t. } \ q_2 \geq 0 \\ &= \begin{cases} (a - \frac{a-q_2-p_0}{2} - q_2 - p_1)q_2 & \text{if} \quad p_0 < a, 0 < q_2 < a - p_0 \\ (a - q_2 - p_1)q_2 & \text{o.w.} \end{cases} \end{aligned}$$

Assume $p_0 < a$. Denote $h(p_0) = \frac{p_0+a}{2}, z(p_0) = \frac{3p_0-a}{2}, r(p_0) = 2p_0 - a$. firm 1 and 2's optimal quantities and their corresponding profits are solved as functions of $\{p_0, p_1\}$:

$$q_2^{01}(p_0, p_1) = \begin{cases} 0 & \text{if} \quad p_1 \geq h(p_0) \\ \frac{a+p_0-2p_1}{2} & \text{if} \quad z(p_0) \leq p_1 < h(p_0) \\ a - p_0 & \text{if} \quad r(p_0) \leq p_1 < z(p_0) \\ \frac{a-p_1}{2} & \text{o.w.} \end{cases}$$

$$q_1^{01}(p_0, p_1) = \begin{cases} \frac{a-p_0}{2} & \text{if} \quad p_1 \geq h(p_0) \\ \frac{a+2p_1-3p_0}{4} & \text{if} \quad z(p_0) \leq p_1 < h(p_0) \\ 0 & \text{o.w.} \end{cases}$$

$$\pi_2^{01}(p_0, p_1) = \begin{cases} 0 & \text{if} \quad p_1 \geq p_{1h}(p_0) \\ \pi_{2l}^{01}(p_0, p_1) = \frac{(a+p_0-2p_1)^2}{8} & \text{if} \quad p_{1z}(p_0) \leq p_1 < p_{1h}(p_0) \\ (a - p_0)(p_0 - p_1) & \text{if} \quad p_{1r}(p_0) \leq p_1 < p_{1z}(p_0) \\ \pi_{2M}^{01} = \frac{(a-p_1)^2}{4} & \text{o.w.} \end{cases}$$

$$\pi_1^{01}(p_0, p_1) = \begin{cases} \pi_{1M}^{01} = \frac{(a-p_0)^2}{4} & \text{if} \quad p_1 \geq p_{1h}(p_0) \\ \pi_{1f}^{01}(p_0, p_1) = \frac{a^2+12p_1(a+p_0-p_1)+p_0(p_0-14a)}{16} & \text{if} \quad p_{1z}(p_0) \leq p_1 < p_{1h}(p_0) \\ (a - p_0)(p_1 - p_0) & \text{if} \quad p_{1r}(p_0) \leq p_1 < p_{1z}(p_0) \\ (p_1 - p_0)\frac{a-p_1}{2} & \text{o.w.} \end{cases}$$

As long as $p_0 < a$, it is true that $\pi_{1f}^{01}(p_0, p_1)$ is concave and increasing in p_1, convex and decreasing in p_0; $\pi_{2l}^{01}(p_0, p_1)$ is convex and increasing in p_0, convex and decreasing in p_1.

2. Firm 0's profits in different outsourcing patterns.

If $D = 10$, when $p_0 < \bar{p}_0$, firm 2 is outsourcing positive quantities to 0.

$$\begin{aligned}\pi_0^{10}(p_0) =&(p_0 - b)q_2^{10}(p_0) + c[q_2^{10}(p_0)]^2\\ =& \frac{(a + b - 2ac - 2p_0 + 2cp_0)[2c^2p_0 + (3p_0 + ac)(1 - 2c) - 3b + 5bc]}{(3 - 4c)^2}.\end{aligned}$$

$\pi_0^{10}(p_0)$ is strictly concave in p_0, and is maximized at $p_0 = p_0^{10}$, solved by first order condition.

$$p_0^{10} = \frac{20ac^2 - 14ac + 9b - 22bc - 8ac^3 + 12bc^2 + 3a}{4(1 - c)(2c^2 - 6c + 3)}$$

The lowest p_0 which F_0 is willing to charge is solved from $\pi_0^{10}(p_0) = 0$ as $\underline{p}_0$:

$$\underline{p}_0 = \frac{2ac^2 + 3b - 5bc - ac}{(2c^2 - 6c + 3)}.$$

If $D = 00$, when $p_0 < a$ both firm 1 and firm 2 are outsourcing positive quantities, otherwise neither of them outsources.

$$\begin{aligned}\pi_0^{00}(p_0) &= (p_0 - b)(q_1^{00}(p_0) + q_2^{00}(p_0)) + c(q_1^{00}(p_0) + q_2^{00}(p_0))^2\\ &= \frac{2}{9}(a - p_0)(2ac + 3p_0 - 2cp_0 - 3b),\end{aligned}$$

which is strictly concave in p_0. The optimal p_0 is solved by the first order condition as p_0^{00},

$$p_0^{00} = \frac{3(a + b) - 4ac}{2(3 - 2c)}.$$

The minimum of p_0 which F_0 is willing to charge is solved by $\pi_0^{00}(p_0) = 0$ as $\underline{\underline{p}}_0$:

$$\underline{\underline{p}}_0 = \frac{3b - 2ac}{3 - 2c}.$$

3. Proof for Lemma 7. (Under $A1$, $D = 01$ is not in any SPNE for $\Gamma^{imp}(a, b, c)$).

Proof: Prove by contradiction. Suppose in some SPNE $D = 01$, then the total quantity produced for good F is $q_1^{01}(p_0, p_1) + q_2^{01}(p_0, p_1)$, simplified as $q_1^{01} + q_2^{01}$.

Firstly, for firm 0 to be willing to provide $q_1^{01} + q_2^{01}$, its profit must be non-negative, i.e.

$$\pi_0^{01}(p_0, p_1) = (p_0 - b)(q_1^{01} + q_2^{01}) + c(q_1^{01} + q_2^{01})^2 \geq 0 \Rightarrow p_0 \geq b - c(q_1^{01} + q_2^{01}). \quad (3.5)$$

Secondly, $p_0 < a$ must hold for either firm 1 or firm 2 to outsource positive quantities. When $p_1 > p_{1z}(p_0)$, it is true that $q_1^{01}(p_0, p_1) > 0, q_2^{01}(p_0, p_1) \geq 0$. When strict inequality in condition (3.5) holds, firm 1's profit is

$$\begin{aligned}\pi_1^{01} &= (a - q_1^{01} - q_2^{01})q_1^{01} + p_1 q_2^{01} - p_0(q_1^{01} + q_2^{01}) \\ &< (a - q_1^{01} - q_2^{01})q_1^{01} + p_1 q_2^{01} - b(q_1^{01} + q_2^{01}) + c(q_1^{01} + q_2^{01})^2,\end{aligned}$$

which means that firm 1 is strictly better off to produce inside $q_1^{01} + q_2^{01}$. A contradiction. When equality holds in condition (3.5),

$$\pi_1^{01} = (a - q_1^{01} - q_2^{01})q_1^{01} + p_1 q_2^{01} - b(q_1^{01} + q_2^{01}) + c(q_1^{01} + q_2^{01})^2.$$

Firm 1 is indifferent between outsourcing or producing $(q_1^{01} + q_2^{01})$. Here q_1^{01} is the optimal quantity solved by first order condition. By condition (3.5),

$$q_1^{01} = \frac{a - p_0 - q_2^{01}}{2} \Rightarrow q_1^{01} = \frac{a - b - q_2^{01} + c q_2^{01}}{2 - c}.$$

However, given $q_2 = q_2^{01}(p_0, p_1)$, by deviating to producing inside, firm 1 is maximizing

$$\pi_1^d = (a - q_1^d - q_2^{01})q_1^d + p_1 q_2^{01} - b(q_1^d + q_2^{01}) + c(q_1^d + q_2^{01})^2.$$

The unique optimal quantity is $q_1^d(q_2^{01}) = \frac{a-b-q_2^{01}+2cq_2^{01}}{2(1-c)} > q_1^{01}$. By deviation, firm 1 is producing more and achieving a higher profit. A contradiction.

Lastly, when $p_1 \leq p_{1z}(p_0)$, we have $q_1^{01}(p_0, p_1) = 0$ and $q_2^{01}(p_0, p_1) \geq 0$. Under $p_0 < a$, firm 1's profit is negative. It is at least better off deviating to $p_1 > a$ for a positive profit. Again a contradiction. Lemma 7 is proved. ∎

4. Proof for Lemma 9. (The following statements are true for any SPNE:
i. $p_0 < \underline{p}_0$ is weakly dominated for firm 0;
ii. $p_0 > \bar{p}_0$ is weakly dominated for firm 0;
iii. $p_1 < p_{1z}$ is weakly dominated for firm 1;
iv. Restricting p_1 to $p_1 \leq \bar{p}_1$ does not affect the SPNE outsourcing pattern.)

Proof: i. $p_0 < \underline{p}_0$ is weakly dominated by $p_0 = \underline{p}_0$. By Lemma 7, 8, and Observation 4, when $p_0 \leq \underline{p}_0$, in any SPNE either $D = 00$ or $D = 11$. Suppose $p_0 < \underline{p}_0$. If $D = 11$, by increasing p_0 to $\underline{p}_0$, the outsourcing pattern will not change and firm 0 is indifferent. If $D = 00$, firm 0 gets negative profit with $p_0 < \underline{p}_0$. However, at $p_0 = \underline{p}_0$, whatever the outsourcing pattern is, either $D = 11$ or $D = 00$, firm 0 gets a zero profit. Thus $p_0 = \underline{p}_0$ weakly dominates $p_0 < \underline{p}_0$.
ii. $p_0 > \bar{p}_0$ is weakly dominated by $p_0 = \bar{p}_0 - \epsilon$, with ϵ a small positive value. Suppose $p_0 > \bar{p}_0$. Since $\bar{p}_0 > \hat{p}_0$, in any SPNE either $D = 10$ or $D = 11$. If $D = 10$, by observation 1, $\pi_0^{10}(p_0) = 0$. At $p_0 = \bar{p}_0 - \epsilon$ with ϵ small, the outsourcing pattern will

not change, but $\pi_0^{10}(p_0) > 0$ and firm 0 is better off; if $D = 11$, 0's profit is zero. If it is still $D = 11$ at $p_0 = \bar{p}_0 - \epsilon$, firm 0 is indifferent; if it happens that $D = 10$, firm 0 gets positive profit and is better off. Thus $p_0 = \bar{p}_0 - \epsilon$ weakly dominates $p_0 > \bar{p}_0$.
iii. $p_1 < p_{1z}$ is weakly dominated by $p_1 = p_{1z}$. Suppose $p_1 < p_{1z}$. Firstly, if $D = 00$ or $D = 10$, the outsourcing pattern will not change when $p_1 = p_{1z}$, thus firm 1 is indifferent; secondly, if $D = 11$, it is true that $\pi_1^{11}(p_{1z}) = \frac{(2c^2-2c-1)(a-b)^2}{2(1-c)(1-2c)^2} < 0$. Furthermore, $\pi_1^{11}(p_1)$ is strictly increasing in p_1 for $p_1 \leq p_{1z}$. When $p_1 = p_{1z}$, if $D = 11$ is still the SPNE outsourcing pattern, firm 1 is better off; if the outsourcing pattern changes to $D = 10$ or $D = 00$, firm 1 can at least improve to a zero profit by letting $q_1 = 0$. Therefore, $p_1 = p_{1z}$ weakly dominates $p_1 < p_{1z}$.
iv. Suppose in some SPNE $p_1 > \bar{p}_1$. Firstly, if $D = 00$ or $D = 10$ is in some SPNE , it will still be in true when $p_1 = \bar{p}_1$. The reason is, under $p_1 \geq \bar{p}_1$, 2's profit is always zero if $d_2 = 1$. Therefore when p_1 decreases from $p_1 > \bar{p}_1$ to $p_1 = \bar{p}_1$, firm 2 has no incentive to switch to $d_2 = 1$. Then in the last stage, firm 1 will not change its outsourcing decision with $d_2 = 0$ and p_0 fixed. Secondly, if $D = 11$ is in some SPNE, then with a lower p_1, it is still in SPNE. In fact, for $p_1 \geq \bar{p}_1$, $\pi_2^{11}(p_1) = \pi_0^{11} = 0, \pi_1^{11}(p_1) = \pi_{1M}^{11}$. $p_1 > \bar{p}_1$ yields the same profit for each player as $p_1 = \bar{p}_1$. Therefore, if any outsourcing pattern appears to be in some SPNE, it will still in SPNE when p_1 is restricted to $p_1 \leq \bar{p}_1$. ∎

Chapter 4

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